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AND

ASSURANCE MAGAZINE.

"I hold every man a debtor to his profession, from the which as men of course do seek to receive countenance and profit, so ought they of duty to endeavour themselves by way of amends to be a help and ornament thereunto."—BACON.

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CONTENTS OF VOL. XXIII.

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On the Mortality observed amongst the various Classes of Bonus Policies in the British Empire Mutual Assurance Company. By George F. Hardy, F.I.A.

[Read before the Institute, 28 March 1881.]

THE object of the present paper is simply to bring to the notice of the members of the Institute the results of an investigation into the mortality prevailing amongst the different classes of bonus policyholders in the above-named Company, and to point out what appears to be the practical bearing of these results. There are also one or two points in connection with the collecting of the experience which may be worth mentioning.

In the company in question (in common, I believe, with many societies) an option is given to the policyholders, on the occasion of the first division of profits in which they are to participate, of receiving the value of their bonus in one of three ways:—

I. Cash.
II. Reversionary Additions.
III. Reduction of future Premiums (either for life or for three years).

The selection has usually been understood to be made once for all, but, as a matter of fact, the option has been occasionally allowed at subsequent distributions.

There appeared to be some reason for anticipating a difference in the relative mortality of these three classes of policyholders in consequence of the exercise of this option. This seemed at least a point worth testing by an appeal to actual experience; and it was mainly with the view of instituting such a test that the investigation referred to was commenced.

It should be stated that the reductions of premium were mostly for three years only, those for life being but few; that the bonus divisions were triennial; that the experience in every instance commenced from the date of the first option exercised by the policyholder; and that this occurred, on the average, about three years from entry. Surcharged or invalid lives (which were not numerous) were included at their "rated-up" ages; and, finally, a policy was always considered to remain in the class originally selected, subsequent transfers to another class (as, for instance, from Reversionary to Cash) being treated as new risks, and a separate account kept of all such transfers. By this means an exact measure was obtained of the value of the lives at the date of the first option in each class, while it was also possible, by eliminating the transferred policies (*i.e.*, by treating the transfers from one class to another as "discontinuances" in the former class), to obtain the mortality at each age among all the policies *for the time being* in either class; as also to discover whether the value of the option differed at subsequent periods in the history of the policy, and to trace the effect of its exercise, in the continued transfers from one class to another, in the mortality of each.

The experience was taken in respect of policies, and not lives; and as this mode of proceeding has occasionally been objected to, though I am not aware that its actual effect has been investigated, I may perhaps be pardoned a slight digression in justification of its adoption in an enquiry such as the present, where only the general effect of the mortality upon the annuity-values is in question, and where the rate of mortality at individual ages is not wanted. Two alternate suppositions are possible—1st. That the policyholders assured under more than one policy will be subject to the same mortality as the remaining policyholders. 2nd. That they form a special class, having a different mortality from the average. Taking the latter alternative first, it appears to me to be a reason for retaining, and not eliminating, second and subsequent assurances, when investigating the experience of a society; for an insurance office, or any similar institution, is concerned simply with financial results. Its experience actually works out, in practice, with policies, and not with lives; and where a policyholder is assured

under two policies, there will be two distinct risks and two claims to pay when he dies. On the first supposition, that no essential difference need be anticipated between the mortality amongst the duplicate policies and the remaining experience, the only effect their retention can have will be somewhat to increase the irregularity of our final tables, by magnifying the accidental deviations from the general average. This in itself is certainly not desirable: it will, however, occur to so slight an extent as to be scarcely worth mentioning. The effect in any given case is easily estimated. Thus, to take an extreme case,—Suppose 10,000 policy-holders of the same class, as respects their average mortality, of whom, however, 5,000 hold each a second policy. The effect of including these 5,000 policies in an investigation will be to leave the broad results of the experience unaltered, but to increase the magnitude of the accidental irregularities in the ratio of (about) 19 : 18, or rather over 5 per cent. Again, if 1,000 out of the 10,000 mentioned above held six policies each, the "roughness" of the results would be increased in the ratio of about 7 : 5. In neither, therefore, of these extreme cases would the effect of including the duplicate assurances be at all serious; though, of course, at particular ages at the extreme end of the table it would produce more decided irregularities.

By the adoption of a simple plan in estimating the ages, all fractions in the ages and numbers at risk were rendered unnecessary, the cards giving the nearest ages at entry, discontinuance and for the existing, and the age last birthday for the dead.

To resume: it will be as well to deal first with the general experience, taking all three classes together. The main results are set out in the following table, comparing the actual and "expected" deaths (H^M mortality) at various groups of ages.

TABLE I.—*British Empire Company, all Classes of Bonus Policies.—Mortality Experience to Sept.* 1879.

Ages.	At Risk.	"Expected" Deaths, H^M.	Actual Deaths.	"Expected" =100, Actual=
Under 30	6,202	43	30	79
30–39	43,755	306	248	81
40–49	53,829	658	573	87
50–59	42,462	858	786	92
60–69	17,799	714	640	90
70–79	3,843	318	276	87
80 and over	354	61	65	107
All Ages .	168,244	2,958	2,618	88

TABLE II.

Actual and Expected Deaths (by B.E.M. Experience) among the Three Classes of Bonus Policies.

Ages.	CASH BONUS POLICIES.				REVERSIONARY BONUS POLICIES.				REDUCTION OF PREMIUM POLICIES.				Ages.
	Original.		Transferred.		Original.		Transferred.		Original.		Transferred.		
	Actual.	Expected (B.E.M.).	Actual.	Expected (B.E.M.).	Actual.	Expected (B.E.M.).	Actual.	Expected (B.E.M.).	Actual.	Expected (B.E.M.).	Actual.	Expected (B.E.M.).	
Under 20	...	...	...	...	...	...	...	...	...	...	...	...	Under 20
20	...	...	...	...	...	...	...	...	...	...	...	...	20
21	1	·5	...	·1	1	·9	...	...	...	·4	...	...	21
22	...	·5	...	·1	2	·9	...	...	...	·4	...	...	22
23	...	·8	...	·1	2	1·6	...	...	...	·6	...	...	23
24	2	·9	...	·1	1	1·5	...	...	...	·6	...	...	24
25	1	1·7	1	·2	4	2·7	...	·1	...	·9	...	·1	25
26	...	·8	...	·1	1	1·3	...	...	...	·4	...	·1	26
27	...	·7	...	·1	2	·9	...	...	2	·4	...	...	27
28	...	·7	...	·1	2	·9	...	...	...	·4	...	·1	28
29	3	3·4	...	·4	5	4·0	...	·2	1	1·6	...	·3	29
30	1	2·6	...	·4	6	3·1	...	·2	...	1·2	1	·2	30
31	6	7·7	1	1·2	11	8·7	...	·5	3	3·6	1	·6	31
32	3	4·8	2	·8	5	5·2	2	·4	4	2·2	...	·4	32
33	5	9·0	2	1·6	10	9·2	...	·7	7	3·8	...	·7	33
34	9	9·0	1	1·6	12	9·0	...	·8	1	3·8	...	·7	34
35	8	9·7	2	1·7	8	9·4	1	·9	7	4·0	...	·8	35
36	12	11·9	3	2·0	12	11·2	2	1·2	4	5·0	1	1·0	36
37	17	17·5	4	3·0	19	16·2	3	1·9	5	7·2	2	1·5	37
38	17	17·1	3	3·1	18	13·7	5	1·9	3	7·2	1	1·4	38
39	14	15·0	2	2·8	16	13·5	1	1·7	5	6·4	2	1·3	39
40	20	17·4	1	3·4	13	15·6	3	2·1	7	7·5	2	1·6	40
41	13	17·3	3	3·4	16	15·2	1	2·1	11	7·4	1	1·7	41
42	17	14·4	4	2·9	12	12·5	3	2·0	4	6·1	3	1·4	42
43	31	32·2	6	6·6	28	27·2	1	4·6	14	13·5	4	3·1	43
44	27	29·6	5	6·2	29	24·7	6	4·4	11	12·6	...	2·8	44
45	23	25·8	2	5·6	23	21·0	2	3·9	12	10·9	...	2·5	45
46	25	28·0	4	6·2	24	22·8	5	4·5	14	12·2	1	2·9	46
47	24	30·5	7	6·9	25	24·4	5	5·1	19	13·0	4	3·3	47
48	28	29·8	5	6·9	24	23·7	5	5·1	14	12·6	4	3·3	48
49	17	29·6	4	7·0	32	23·2	2	5·1	16	12·3	6	3·3	49

50	37	37·8	9	9·2	32	29·6	6	6·8	14	15·6	3	4·3	50
51	36	33·8	6	8·1	24	26·3	7	6·1	12	14·0	2	4·0	51
52	40	35·1	3	8·3	23	27·3	10	6·4	16	14·6	2	4·3	52
53	28	29·8	9	7·4	26	23·1	2	5·3	12	12·5	1	3·7	53
54	35	36·1	5	9·2	21	28·1	5	6·5	15	15·0	1	4·6	54
55	35	38·2	10	10·2	41	29·7	10	7·0	15	15·8	6	4·9	55
56	40	38·5	7	10·4	31	30·0	12	7·1	13	15·6	5	4·9	56
57	37	39·8	14	11·3	37	31·0	10	7·4	12	16·4	4	5·2	57
58	35	39·1	4	11·1	32	29·5	4	7·4	8	16·3	3	5·2	58
59	35	32·0	13	9·1	26	24·0	10	6·0	8	13·2	7	4·4	59
60	31	33·7	15	9·6	25	24·8	7	6·3	16	13·7	2	4·6	60
61	38	32·7	10	9·4	24	24·0	10	6·0	8	13·4	1	4·7	61
62	25	31·3	16	9·0	28	22·7	10	5·9	14	12·9	4	4·6	62
63	39	34·0	10	9·6	23	24·1	8	6·5	10	13·7	4	4·6	63
64	23	29·7	11	8·2	24	21·2	8	5·6	15	11·4	6	4·2	64
65	34	33·6	10	9·3	20	23·8	5	6·4	16	12·4	9	4·6	65
66	20	20·0	8	5·6	16	14·0	5	3·7	5	7·2	2	2·7	66
67	31	32·8	9	9·0	21	23·0	6	6·1	15	11·1	5	4·8	67
68	33	32·7	13	8·9	23	22·8	7	6·4	10	10·4	6	4·8	68
69	30	25·9	7	7·0	20	18·7	5	4·9	3	8·4	2	3·7	69
70	20	25·9	8	7·2	24	18·2	7	5·0	9	8·2	2	3·8	70
71	23	23·0	6	6·0	18	15·3	7	4·6	3	6·1	4	3·6	71
72	16	14·8	3	4·0	9	9·1	5	2·7	3	4·0	1	2·2	72
73	25	19·9	3	4·0	6	9·2	3	2·6	7	4·3	3	2·1	73
74	11	12·2	7	3·9	11	8·3	3	2·3	2	3·3	2	1·7	74
75	16	12·2	1	3·4	6	7·7	3	2·2	1	3·1	3	1·6	75
76	8	9·3	2	2·9	6	5·9	2	2·1	4	2·8	...	1·2	76
77	7	10·2	5	3·2	10	6·2	5	2·3	2	2·6	2	1·2	77
78	8	9·3	1	2·5	6	6·4	3	1·9	3	2·2	...	1·0	78
79	8	7·3	3	2·3	4	4·3	...	1·4	1	1·4	...	·9	79
80	9	7·6	3	2·6	4	5·5	2	2·0	2	1·9	2	·9	80
81	7	8·3	1	2·7	3	5·7	1	1·9	6	2·0	...	1·0	81
82	1	2·7	1	·8	3	2·0	...	·6	1	·3	1	·2	82
83	14	10·1	2	3·0	4	6·7	3	2·2	...	1·2	1	·4	83
84	3	2·6	...	1·0	1	2·6	1	·5	2	·8	...	...	84
85	1	·4	...	·4	...	·5	...	·1	...	·1	...	...	85
86	...	...	...	...	...	...	...	...	...	...	...	...	86
87	...	...	...	...	...	...	...	...	...	...	...	...	87
88	...	·2	1	·8	1	·8	...	...	...	...	...	...	88
89	...	·2	1	·8	1	·8	...	...	...	...	...	...	89
90 & upwards	...	·3	2	1·7	2	1·7	...	...	...	...	...	...	90 & upwards
All Ages	1163	1212·0	301	307·7	998	938·8	239	207·6	457	466·1	129	145·7	All Ages

The general mortality, especially if it is remembered that the first three years of assurance are excluded, will be seen to be extremely light, and would appear to show that a greater variety is to be looked for in this respect in the experience of different companies than has sometimes been thought possible. It also shows the difficulty of setting up any arbitrary standard of solvency with respect to life offices, regardless of the special experience of the society under examination. In illustration of this point, I have tested the difference between the reserves required by a valuation at 3 per-cent by the H^M Table, and a valuation by the society's own experience as summarized above—taking the date of the last investigation, and including only the participating policies. The premiums valued are the same in both cases (as, of course, they would be in any valuation made as a test of solvency), namely, the office premiums less 20 per-cent.

The first three years of the society's experience being excluded, its use in a valuation would correspond more nearly to the severer test of an $H^{M(5)}$ valuation; yet the reserve brought out at 3 per-cent is more than 10 per-cent less than by the H^M Table. This result seems to me important, as if we admit that in some societies the reserves actually required to meet their liabilities may be as much as 10 per-cent below the sum, as estimated by an average table, we must also admit that in other societies the true reserve may again be just as much in excess, and we thus have a possible range of 20 per-cent between two societies to all outward appearances similar.

With respect to the reasons for the extremely light mortality observed, I am unable to offer any suggestions. The policies are for small average amounts, about £300, and the members principally tradesmen. The medical examinations I believe to have been very strictly conducted throughout the history of the company.

The most interesting features of the experience are brought out, however, when the various classes of bonus policies are separated. The general results of this analysis are given in Table II (pp. 4 and 5), where the heading "Original" refers to policies in which the selection of the particular class of bonus was made at the first option afforded the policyholder; the "Transferred" being those transferred at some subsequent date from one of the other classes. The column headed "B.E.M." gives the expected deaths according to the average mortality of the whole of the policies taken together.

Speaking generally, the conclusion to be drawn from the above figures is that the option given to the assured to select the manner in which his bonuses shall be received *does* exercise an influence upon the future rates of mortality prevailing in the various classes of bonus policies which are thus created; and that the value of this option, as measured by its effect upon the mortality, is greater the later it is made. Thus, taking the "Original" policies, both the Cash and Reduction of Premium classes are much better lives than the Reversionary, while the difference in the mortality of the various classes is much more marked in the "Transferred" than in the "Original" policies. It is further worth noting that, as regards the latter, they would appear to differ but little at the older ages, as the following comparison will show:—

TABLE III.

Deaths after 60 *Years of Age.*

Class.	Actual.	Expected (B.E.M.).
Cash . . .	481	481
Reversionary . .	342	340
Reduction . .	158	160

I am quite unable to account for the Original Reduction of Premium policies having a higher rate of mortality than the "Cash", the "Transferred" being by far the best of any of the six classes in Table II. The figures in Table II do not, of course, enable us to form any exact conclusion as to their effect upon the annuity values at the various ages; and as this is the most important feature of the enquiry, I have calculated the annuity values for each age at entry (*i.e.*, at the date the selection is made,) for each of the six classes given in Table II. This has been done with quite sufficient accuracy for the purpose by 3 fig. logs throughout. The mortality for the first five years was taken from that portion of the experience only, and that after five years from entry, from the remainder of the experience; thus any effect of selection was duly brought out, and was found to be considerable in the Cash and Reduction of Premium policies. As only the general results would have any interest, the following table will contain all that it is necessary to give. No attempt was made to graduate the original facts: as the results, however, would be very irregular at the isolated ages, I have thought it best to give the average

of the annuity-values at different groups of ages. The annuities headed "B.E.M," refer to the whole of the experience—the first three years having been included.

TABLE IV.

Ages.	Mean Annuity-Values.							
	B.E.M.		Cash Bonus.		Reversionary Bonus.		Reduction of Premium.	
	"Select" (at Date of Entry).	At Date of 1st Bonus.	Original.	Transferred.	Original.	Transferred.	Original.	Transferred.
20–29	21·44	21·27	21·89	...	20·58	...	22·00	...
30–39	19·44	19·29	19·53	19·30	18·80	18·48	19·55	20·04
40–49	16·32	16·15	16·32	16·43	15·77	16·00	16·27	17·00
50–59	13·10	12·86	12·88	13·07	12·61	11·61	13·37	13·67
60–69	9·96	9·66	9·67	9·84	8·02	8·36	10·41	10·40

These differences in the annuity-values may be represented with quite sufficient nearness in practice, either by assuming a constant difference at all ages, or making a constant addition to or deduction from the age. I do not know that any special advantage attaches to either plan in preference to the other. If we adopt the latter plan, we may take as our guide the following table:—

TABLE V.

Ages.	Addition to Age represented by the Mean Annuity-Values, as compared with B.E.M. "Select" Annuities.					
	Cash Bonus.		Reversionary Bonus.		Reduction of Premium.	
	Original.	Transferred.	Original.	Transferred.	Original.	Transferred.
20–29	−2	...	+4½	...	−3	...
30–39	− ½	+½	+2	+3	− ½	−2
40–49	...	−½	+2	+1	...	−2
50–59	+ ½	...	+1½	+4½	−1	−2
60–69	+1	+½	+6	+5	−1½	−1½

It will be seen from this table that, except at the young ages, the Cash Bonus lives are of about the same quality as those recently selected; but there would appear to be a tendency to deteriorate. Generally speaking, the results are too rough to be relied on at any particular age, and I do not consider much

importance can be attached to the group of ages 60–69. We may, however, deduce from the tables the following rough rules for ordinary use (the meaning of the abbreviations will be obvious) :—

1. In turning R.B. or Red$^{n.}$ P. into C.B., deduct one year from the age.
2. In turning R.B. or C.B. into Red$^{n.}$ P., deduct two years from the age.
3. In converting C.B. or Red$^{n.}$ P. into R.B., add three years to the age.

It is clear that the following options (Cash to R.B., and Red$^{n.}$ P. to C.B. or R.B.) can only be allowed at any time subsequent to the first distribution, when it is known the life is in fair average health.

It follows, from the foregoing facts, that an additional reserve should be made for reversionary bonus additions in companies where a portion of the bonus is paid in other ways; and further, that a similar addition should be made to the reserve for reductions of future premiums. An examination of the annuity-values in these classes after five years, shows that the case may be met by rules 2 and 3.

The subject of this paper evidently has some bearing on the question of surrender values, as the conversion of a Reversionary Bonus into Cash is the nearest approach we can have to the surrender of a policy. If we make the supposition that the value of the selection is the same in both instances, we may apply the above rules to the calculation of surrender values. If we deducted one year from the age on account of the selection, and a year from the duration to meet "initial" expenses, the effect would be practically the same as taking the duration of the policy as two years less than the true time. This would give a very convenient practical formula for surrender values, which would have the advantage of giving the small values known to be necessary for policies but recently effected, while the values for policies of long duration would be much fairer to the assured than those estimated by taking a fixed percentage of the tabular value at all ages.

It may be objected, that the facts presented in this paper, being drawn from the experience of a single company, cannot be generally applied. It will be seen, however, that I have not assumed that the mortality amongst Reversionary Bonus policies in every company will be the same as that observed among the policies dealt with in the present experience; but merely that in

any company giving similar options as to Bonus to its assured, the *relative mortality* of the different classes of bonus-policies will be similar to that brought out by the above investigation; this, I think, is not an improbable assumption. It may be useful just to summarize the main results arrived at. They are—

1st. The option exercised by a policyholder as to the manner in which his Bonus shall be received, exercises an influence over the future mortality experienced among each class of Bonuses.

2nd. That it is necessary to take this fact into account in calculating the amounts of such bonuses in the first instance, and in the valuation of the Reversionary Bonuses and Reductions of Premium subsequently.

In conclusion, I wish to express my obligation to the officials of the Company, with which I am no longer connected, for their courteous permission to extract the necessary materials from the books of the Company, and to publish the foregoing facts.

Discussion.

The President (Mr. A. H. Bailey)—Most of you know the amount of labour required to get out the results which are shown here, and I am sure you will all appreciate Mr. Hardy's industry. I have always thought that the experience of an individual society, especially where it relates to anything like a well-defined class, is of considerable interest. I am doubtful whether the combined information derived from various companies obtaining their business from different sources, and whose connections are among different classes of society, is altogether suitable for the determination of the actual effect of selection in an individual society. Mr. Hardy says "the experience was taken in respect of policies and not lives," and remarks upon the objections that have been taken to that course. I am not quite sure that I understand him, but if I read his paper rightly, I think he has missed the force of the objection. I do not know that it ever was supposed that lives assured by more than one policy are subject to a different mortality. The real objection was that the ratios are disturbed. As, for instance, take such an age as forty, where the rate of mortality is about 1 per-cent, if you have 500 lives passing through the age of forty to forty-one, the deaths among those 500 lives will be 5 or thereabouts. If you take a possible, but an extreme case, one of those deaths may occur on a life insured under four policies; and if that were the only disturbing cause, the effect would be that there would be 8 claims for 503 policies, and that the ratio

of 5 to 500, or 1 per-cent, would be altered to 8 to 503, or something more than $1\frac{1}{2}$ per-cent. I know that there has been a remark made, and a just one, that what assurance companies are concerned with is the ratio of claims to policies, and that, therefore, for their purposes it might be that experience derived from policies was more valuable. But as regards the purely scientific aspect of a table of mortality, that would not hold. I think, therefore, that it is very satisfactory to know, as we now know, that the results of what are called the Seventeen Offices' Experience, and the results of the experience collected by the Institute, are for all practical purposes identical, although one is the experience of policies and the other the experience of lives. As to the difficulty of setting up any arbitrary standard of solvency, I do not think that anyone has proposed to set up such a standard. That notion, perhaps, may have arisen from the rules laid down in one of the Life Assurance Companies Acts as to how, in case of liquidation, policies are to be valued; but that is a different thing entirely from a test of solvency. I know that in one or two cases an attempt has been made to apply that rule in a Court of Chancery proceedings, but I do not think that it will be attempted again. The light mortality in the particular society under investigation is, undoubtedly, interesting. It is stated that "The policies are for small average amounts—about £300—and the members principally tradesmen." It is surprising that they should show such a light mortality, and the cause is worth further enquiry. I think that Mr. Hardy's Table No. 5 requires some qualifications. The variations at the younger ages are certainly startling; but then the number of deaths are very few, so that I think not much dependence is to be placed upon the results. Never having made such elaborate investigations as Mr. Hardy, but merely noting matters in my daily experience, my conviction is that these options of cash bonus and reversionary bonus depend to a very great extent on the pecuniary circumstances of the claimant. A man asks for a cash bonus because he wants money, not because he is in good health. Amongst the many anomalies which competition has drawn us into, this question of cash bonus is one. What an absurdity it is for a man paying a premium to provide a sum at his death, periodically—once in three, or five, or seven years—to take back some of his money, which has, in the majority of cases, passed through the hands of certain agents who have been paid something more or less thereupon. Why does not he pay less in the first instance? Cash bonus and life assurance appear to me to be a contradiction in terms.

Mr. G. Humphreys—With regard to the words "at risk" in Mr. Hardy's Table 1. That means the years of life, does it not?

Mr. G. F. Hardy—Yes.

Mr. Humphreys—But you have not given the number of lives out of which the deaths occur. I think it may be an advantage to do that.

Mr. Justican—I should like to ask Mr. Hardy one question as to the principle which he has adopted in obtaining his ratios in the case of the duplicate policies. I presume he has taken the ratio of

the square roots of the numbers at risk, to the total numbers; but I have not been able to bring out exactly the figures given.

Mr. R. P. HARDY—If a gentleman comes forward and voluntarily undertakes the mass of work which Mr. Hardy must evidently have done, this Institute is bound to return its best thanks, whatever the actual merits of the paper; but I think we can say something more than that, because, if Mr. Hardy has not actually answered some of these questions which we are putting to ourselves every day, he has supplied us with some figures for their more formal consideration. The conclusions which, I take it, he wishes us to accept are that we should, according to the old-fashioned principle, rate up the ages when distributing a bonus, or deal especially with the ages, having regard to the way in which the bonus had been originally selected. It has sometimes seemed to me that those principles, if admitted, would require us not only to apportion the individual profit, having regard to a special rate of mortality made upon that particular class, but also that we should in the valuation make a reserve, determined in like way. If that were to be the case, it seems to me that it would present interminable practical difficulties. Every policy would require its own mortality table. I think that there would be a great deal of official trouble in that, and I should gladly find some practicable rule, such as lowering the rate of interest or taking a larger portion off the loading. I am by no means clear that there is much of real option in selecting the bonus alternative. I know one society where I am quite sure at least one-half of the policyholders are prevented from exercising any voice in the matter. The bonus there does and must go by reversion. I am therefore by no means sure that there is what Mr. Hardy calls an option in the matter, neither can I see that there should be any difference in the mortality where cash or reduction of premium had been chosen. A portion of these results may be effected according to the practice of the office, for instance, by allowing the agent when reduction of premium is taken a commission on the full premiums. Unless that is done, he invariably encourages the taking of cash or reversion. I am somewhat surprised at the all-round rate of mortality that Mr. Hardy shows, and I should like to ask him whether these expected deaths have been taken according to the true method, or whether he has simply multiplied the actual exposures by the rate of mortality? [Mr. G. F. HARDY—Yes.] Well, I would say with all submission I believe that is an incorrect method. The true way of getting out the expected deaths is to bring forward such exposures as would survive according to the table by which you are testing. Unless you do that, the results show a different mortality to the real estimate, and they would be consequently defective. The true process is very laborious and troublesome. I can find no short method myself, but I think that if Mr. Hardy has the time to give to it, he will find that his expected deaths, if estimated truly, will come a little more closely to his actual. With regard to the vexed question as to the standard of reserve in the case and other special circumstances, it has always seemed to me important that effect should be given to what are probably the actual facts. The cases where it is necessary to take a modified view are, generally speaking, those of offices in such a

condition that no average table will apply. It generally happens that when they wish to fall back upon such an argument their position is something worse than the $H^{M(5)}$. I therefore think that this range of 10 per-cent which Mr. Hardy indicates could not actually apply to the cases where it would be necessary to try it.

Mr. G. King—Mr. Hardy has investigated the mortality amongst the different classes of policyholders of a particular company, and brought out some results which are certainly remarkable; but I think such an investigation can be carried a little further. In this company which Mr. Hardy has investigated, the option is only once exercised with regard to the bonus. A very interesting investigation could take place where the option is allowed at every allotment. [The President—The common way, I think.] Yes. I think that if the mortality experience of the bonuses alone were taken out and compared with the mortality experience of the policies alone, it would be worth doing. We must not go to such an investigation with a foregone conclusion; and even if negative results were arrived at, those negative results would be of some value. There is one curious point which I should like to see brought out, and that is whether there is any difference between the mortality of bonuses, if I may use the expression, and the mortality of policies? It may seem rather startling to make that suggestion, because the bonuses are on the same lives as policies, and one might say that they ought to follow the same mortality, but it does not necessarily follow that they do. If the unhealthy lives select a reversionary bonus, and the healthy lives take a cash bonus, then the rate of mortality among the reversionary bonuses will be higher than among the policies, because there will not be the same number of good lives among the bonuses to keep down the rate of mortality. In the case of the British Empire, that investigation cannot take place because the bonus and the policy are practically the same thing. The practical suggestions of Mr. Hardy are of some interest. He says that we must take account of the facts which he brings out in calculating the amount of such bonuses in the first instance, and in the valuation subsequently of the reversionary bonuses and reductions of premiums. I think it is a common, if not a universal practice, to take account of the facts in calculating the bonus. It is usual to assume that there is some option exercised, and to rate up the lives somewhat, either by taking a higher age or a lower rate of interest in allotting the bonus. But so far as my investigations of the blue-books show, it is not usual to take any account of the same facts in a valuation, and it seems to me that that is a mistake. If we rate up the lives in allotting the bonus, I do not see any reason why we should not rate up the lives when we value the bonus. In fact, when we value the bonus by the ordinary reversion, we throw into the general fund all the extra reserve which we made in allotting the bonus. I do not think that is right. Still more, I think that it is objectionable to do as some do—to value the bonuses at a higher rate of interest than the policies.

Mr. R. P. Hardy—I would ask Mr. King whether, in the case of a policy carrying a reversionary bonus, he would value the premium

as well as the bonus, as if the life were subject to a higher mortality than the tabular rate?

Mr. King—No.

Mr. R. P. Hardy—Why would not the argument apply to the value of premiums as well as the bonus? If one ceases the other is payable.

Mr. King—I explain it in this way—that selection has taken place as to the bonuses, and they depend on unhealthy lives, and consequently the average rate of mortality among the bonuses would be different from the average rate of mortality among the policies, among which there are healthy lives to keep down the rate. Although, in individual cases, the falling in of the bonus and of the policy would depend upon the same event, yet the denominator of the rate of mortality would be different.

Mr. R. P. Hardy—As a good deal seems to turn upon this question practically, I should like Mr. King to explain himself further. I think the same circumstance that causes the bonus to be payable makes the premium stop. If you are going to assume that certain conditions measure the value of the bonus, why should not the same circumstances be applied in measuring the premium? Why will you use two annuities, one for one side of the question and another for the other?

Mr. King—I think I may take an example to explain that point. Suppose we have an office whose policies follow the rate of mortality of the $H^{M(5)}$ Table—that table which Mr. Sprague, in his late paper, divided into two portions, the lives still select and the lives which have become deteriorated—the whole average of the policies is represented by the $H^{M(5)}$ Table, and therefore the sums assured and the premiums under all the policies must be valued by that table. But in the bonuses, let us suppose that an option has been exercised, and that they are all on deteriorated lives, none being on the lives still select. Therefore the bonus will follow an entirely different law from the policies, and we must value them by Mr. Sprague's unhealthy table. In the one case the average mortality is represented by the $H^{M(5)}$, in the other case by the unhealthy table.

Mr. R. P. Hardy—You mean to say, then, that the bonus which is payable with the sum assured falls in at a quicker rate than the sum assured?

Mr. King—Yes; it may do so.

Mr. C. D. Higham—One or two gentlemen have congratulated Mr. Hardy on his paper, but I think we may also congratulate the office on the skill with which its medical examination is conducted, and the success with which the officials and agents keep the business together; for there must be some strong force at work when the rate of mortality is so small in a company insuring sums of £300 on the lives of small tradesmen. If I may make a suggestion to Mr. Hardy, I think his Table No. 2 would be greatly improved if he were to insert the percentages which the actual bear to the expected deaths. Whether he take the H^M or the British Empire mortality, I think matters little; but if there were a percentage of some sort, it would be so much more easy for the eye to read the table. The only point to which I am inclined to take exception,

is the clause near the end of the paper, in which he says that though the figures are possibly wrong, through the smallness of the numbers made use of, yet as he is dealing with relations rather than with results, it is not of much importance. It seems to me, sir, that if the figures are wrong, it is as likely as not that the error will be on the one side as on the other; and if you have an extreme variation on different sides, you will get a double error which may be much more serious. One ought to think very carefully before dealing with figures founded on such a limited experience as this seems to be.

Mr. T. G. Ackland—The results which the author has brought out are certainly remarkable; and although we should be disposed to imagine that there was no option of any consequence with reference to these bonuses, yet when we find that the cases of "transferred" reversionary bonuses are 116 per-cent of the general mortality of the office, while the "transferred" reduction of premium cases are 87 per-cent of the average mortality, it would certainly appear as if there was a distinct selection against the office in the one case, and in favour of the office in the other. I notice that in the paper before us the word "selection" is used in three distinctly different senses—sometimes referring to the choice of a bonus option, sometimes to the medical selection of the life by the office, and sometimes to the selection which the assured makes against the office when he exercises the option. Perhaps by the choice of a different word the matter might be made a little clearer. As it stands, it is the cause of some confusion to one's mind. The matter referred to towards the close of the paper—that of surrender values—bears a special interest, I think, with regard to the reduced or paid-up policies which are issued by many offices. The assured who are offered a reduced or paid-up policy at death, as a matter of choice, compared with a cash surrender value, will probably exercise a very considerable option against the company, somewhat similar to the option exercised in this paper by the reversionary bonus cases. In some cases it is the practice to give a very large paid-up policy, amounting to the whole of the premiums paid, and in that case, the assured will probably have an even greater inducement to select the reduced policy, especially if in bad health, and it would be very interesting to get out the mortality of that class if it were possible.

Mr. Searle—We, of course, all agree with the full explanation given by yourself of the error which is caused by taking into account the number of policies instead of the number of lives, but after that I was somewhat surprised to hear Mr. King, as I think, fall into that identical error—not exactly on the face of it, but it is the same error. Taking the policies instead of the lives is precisely the same thing as is done when we take the mortality by the sums assured. We usually take our expected claims as well as our expected deaths, and the taking of the expected claims is precisely the same thing in principle as taking the mortality by policies; that is to say, you have different amounts of risk for a single life—a larger amount on some and a less amount on others. Where I think Mr. King fell into that same mistake, was in recommending that the return should be taken out of expected bonus claims. That would be taking out a larger risk on one life than it would upon another; and in order to take it to any

practical purpose, you would have to reduce the amount of bonus on each life to the same amount, and that would come precisely to the same thing that Mr. Hardy has done in taking out the mortality of the lives on which the bonuses ran. The next observation that occurs to me, sir, is in reference to the Tables 3 and 5. As you have pointed out, the $+4\frac{1}{2}$ in the first line of Table 5 is not much to be depended on owing to the scarcity of the number of lives. The other large figures happen to occur in the bottom line—the $+6$ and the $+5$; and I have been unable to reconcile them with the figures which occur in Table 3, where the actual deaths after sixty years of age are shown to agree exactly almost with the expected. If they agree exactly as in Table 3, why should there be that difference in the value of the annuity, when you come to the last line of Table 5? Perhaps Mr. Hardy will give an explanation of it. There is a third point on which I think I can throw a little light—as to the small mortality among these small tradesmen who insure for £300 a-piece. The fact is that there is a large number of teetotalers among them. Mr. Ralph Hardy, I think, has shown, elsewhere, that the mortality of teetotalers is less than the mortality of the general population, and there is no doubt that the lives in this particular office (I say it from some experience of them—not on behalf of the office, but outside) are selected from those whose habits of life are exceedingly good, and I believe that a large portion of this light mortality really turns on the excellence of the habits of life of the persons assured in this company. [The President—Do I understand you to say that small tradesmen are teetotalers?] These are not small tradesmen selected at random, but small tradesmen selected through certain particular agencies which lead the company to insure large numbers of that class rather than others.

Mr. Capern, representing the British Empire, wished to say that he did not think the number of teetotalers appreciably affected the returns of mortality. The company was undoubtedly very particular on the question of temperance. In fact, the occurrence of the word "teetotaler" in a proposal caused an amount of suspicion that was not complimentary to the teetotalers. It was the object of the company in such cases, as far as possible, to find out how long the proposer had been a teetotaler, and what were his habits previously to his becoming such, and it not unfrequently turned out that he had been in the habit of indulging a little too often prior to the period of total abstinence.

Mr. Whittall—In Table 4 Mr. Hardy names his annuity values—those which represent the experience of the whole of the society—"selection (at date of entry)", and I think in speaking of the table verbally he also spoke of these annuities as being select annuities. Now if this annuity value 9·96, which is opposite the ages 60–69, represents the value of annuities on lives which enter at those ages, will it not account for the very large difference which appears in Table 5 opposite to those ages under the head of reversionary bonus? If the annuities on these reversionary-bonus lives are compared with the annuity values of those who have entered only at the later ages, it is very probable that they would show a very much worse mortality.

Mr. G. F. Hardy, in replying, said that his facts had not been selected to suit any particular theory, and that no attempt even had been made to graduate the original results. He continued—I think that, possibly, in the latter respect the results have been somewhat spoiled by the resulting irregularities, and the President has noticed that in the early ages in Table 5 the figures are rather large. If, however, the rules are referred to that I deduce from those figures—Rules Nos. 1, 2, and 3—it will be seen that I have practically ignored the first and last groups of ages, simply from the fact that the number of risks is extremely small, and that no great importance can be attached to the results at those ages. With respect to the question of duplicate policies, I agree that the results may be thrown out very considerably at individual ages; and I do not think that for a mortality table, like the Institute Table, where we wish for an accurate estimate of the mortality at each particular age, such a mode of procedure would be justifiable; but I simply wish to justify it in the case of an investigation like the present, where the object is not to obtain the mortality at any particular age, but simply annuity values in which, of course, any slight irregularities at one age or another might be supposed to fairly balance each other on the final result. As to the way in which the effect of including the duplicate cases was calculated, Mr. Justican was quite right in supposing that they were done on the principle that the probable accidental error in the number of deaths would be roughly proportionate to the square root of the number of deaths. But I think that the difference between his result and mine may arise from the fact that he has taken the probable deviation or error in sums of two sets of problems as the sum of the separate deviations, instead of taking the square roots of the sums of the respective squares. If this is done the result comes out as in the paper. Mr. Bailey considers that the options are dependent especially upon the pecuniary circumstances of the assurer. I think there is no doubt that this must be so in a great majority of cases. I noticed, in fact, in getting out the present experience, that by far the larger number of reversionary bonuses were on young lives, and that, apparently, they had been attracted by the great difference between the stated amounts of cash bonus and reversionary, to accept the latter. But it is not so much that an option is exercised in every case, as that in certain special cases the option is exercised, and it is these cases that really throw out the mortality. With respect to the valuation of reversionary-bonus policies, the way I look at it is somewhat in this light. There is no doubt that, if we wished to value the reversionary-bonus policies alone, we ought to adopt the same standard of mortality as for reversionary bonuses. But it is assumed in the valuation of a company that, though these policies may be worse than the average, yet, as a result of this fact, there must be other policies better than the average which makes up for them, these being the cash-bonus and the reduction-of-premium policies; and we may therefore assume an average rate of mortality for the whole sums assured; though if we are dealing with only a single class, as is the case in valuing reversionary bonuses, I think we should take into account the special mortality which we might

suppose to prevail amongst that class. An interesting point was raised by Mr. Searle; the deaths after sixty being nearly equal in all the classes of bonus policies, it is a somewhat extraordinary thing that the annuities differ so greatly. I may say that the annuities which are termed "selection" annuities should be termed "select" annuities. I borrowed that term from Mr. Sprague, because they were calculated in precisely the same way in which he calculated his annuities, with the exception that the rates of mortality were not separated for the first five years of assurance, but the mean mortality was taken over the whole five years for each age, while after the five years that portion of the experience corresponding to the $H^{M(5)}$ Table in the Institute Experience was used to continue the annuities to the end of life.

The Position of the Insurance Press in relation to Insurance Offices and Insurance Interests. By CORNELIUS WALFORD, F.I.A., F.S.S., F.R.Hist. Soc., *Barrister-at-Law.*

[Read before the Institute of Actuaries, 28 March 1881.]

I THINK the time has arrived when the subject indicated in the title of this paper may be fairly and fully considered. It is certainly one which must frequently have presented itself to the managerial mind; and there can be no reason why this question should not be discussed with as much philosophic calmness as any of the many theoretical problems, or points in practice, which continually present themselves for reflection, and perchance for decision.

The point may indeed arise—whether I am the proper person to introduce the topic. I take the individual responsibility of deciding in the affirmative. I have, on the one hand, been as frequently assailed by the insurance press, as any one, and, on the other, received as much kindness and friendly recognition as any man can desire, and more than I claim to deserve. It may be that in either case the extreme has been reached, or passed. I have the advantage of having been a writer upon the press, insurance and general, from the days of my youth, and I say at once that my sympathies are largely on that side. But I think that the familiarity which draws me to the side of its virtues, also renders me, at least in some degree, cognizant of its short-comings. I have the further advantage of having been on various occasions consulted by managers on the one hand, and by editors on the other, upon the points which I now proceed to discuss.

It is a recognized fact, that no great interest long exists in this

country (or, indeed, in any other) without involving either the aid or the hostility of the press. The process is usually this: the general press either openly attacks, or unskilfully criticizes, a particular interest; some one who knows more of its aims and objects than ordinary writers, comes forward as exponent or champion; he finds his efforts appreciated, and shapes himself into the required line of thought. In this manner he becomes familiar with men and things in such connection, and so comes to the front on special occasions, almost as a matter of course. Then, as ramifications of interests arise, there come to be opportunities for varying views. Others join in on such occasions; their indications of partizanship become encouraged by the varying factions; and hence every shade of interest, in process of time, has its champion, ready to do battle for the cause it has espoused.

The field of special journalism thus legitimately created, is very apt to become a little redundant. In periods of strife, all may find something to say: all may appear to have a path of duty; but when quiet times recur, the champions may be more numerous than the requirements of the cause. Then ensues the struggle for individual existence, and in that struggle is sometimes evoked experiences and expedients which do not redound to the common good. In the abstract, all have a right to live; but if the life is to be long, the means by which it is to be sustained should be entirely legitimate. In our ordinary daily life, we meet with two great classes of persons—the one composed of those who live by the force of their industry and ability, who mark out a course of independent usefulness, and become prosperous citizens; the other class is composed of those who live, as it were, by sufferance; who have no self-sustaining force; who never become citizens of the world in the true sense; they are always helpless and dependent, and not unfrequently complain of how little the world has done for them!

The same is the case with the press, whether class or general. There are journals which strike out on a course of obvious utility from the day of their first issue,—which fall into a line of acknowledged usefulness from the beginning,—and run a prosperous and independent career. There are those also that are almost from the beginning mere helpless shams; they originate nothing, they uphold nothing; they have no animation, no enterprize, and become mere parasites upon the interests to which they are professedly attached. I know of no more lamentable, more pitiable position, to drift into.

Now, in the management of monetary institutions, those placed in control are usually, and should always be, so placed because they are believed to be competent to guide and protect the interests confided to them. They are supposed to take a broad business view of all that pertains to their position. They must not only give a financial, but a moral account of their stewardship.

And to such a man as we have last supposed, the conviction is certain to arise—"In all that I do, in all that I sanction, I must see and see only the interests of those that have placed me in this position of trust." And who can or ought to complain that it should be so? It is the cultivation of this wholesome precept that has made us a great mercantile people.

On the other hand, it may be asked by mental interrogation, who expects anything otherwise? We all have to do the best in the position we are placed; we all have to respect the trusts reposed in us. We all have to show cause, commercially or morally, for every act of our daily business lives. Transactions conducted on these principles are alike honourable and satisfactory to all concerned; operations conducted in subornation of such principles are alike unsatisfactory, not to say morally degrading, to all concerned.

Now, we may safely apply these self-evident maxims to the dealings of managers of insurance offices, with the proprietors or managers of the insurance press. The latter, having created a legitimate influence by independence, uprightness, and ability, take their stand upon the ground of public utility. They claim to render essential service to the cause they have espoused. Their journals are the organs of communication with the outer world. The announcements contained in their advertizing columns reach the eyes of those whose minds have been trained by the well-designed articles in the other columns, to understand their significance. A word of editorial comment will be understood to be sincere, whether in praise or censure. The editor will have shown his readers that he is worthy of their confidence, and the circulation of his journal will represent a real radius of power.

An editor who knows that he stands in this rightful position, will feel under no submissive obligation to the insurance office that gives him its business patronage. He will feel that he is giving value for money; he will not, therefore, sacrifice his independence; for by so doing he would most assuredly lose his influence. I know of few more proud positions than that of an editor who, by the force of his own high teaching, has achieved such a degree of

independence. He becomes the centre of communication with an ever-extending circle. The knowledge his position enables him to acquire he imparts to his readers, accompanied by words of friendly caution: by suggestions and advice which they know well how to profit by. The power of such a man is immense, and deservedly so; the trust is correspondingly great. And that trust he will know must never, under any circumstances, be abused; for once forfeited, it can never be regained.

So far, our course runs very smooth. I have kept upon the lines of theory and analogy. I have admitted that every interest finds itself espoused by a section of the press, which adapts itself to the requirements of that interest. I have admitted that this special advocacy may be made beneficial to the cause in view; and that so long as it is beneficial there is a mutuality of interest which leads to and upholds a position of entire independence. For emoluments rendered on the one side, there is absolute requital in the way of service on the other.

When we descend from theory to practice, matters are apt to assume a somewhat modified form. It may be asked, is not this assumed protection of class interests apt to be a little overdone? May there not be a little too much championship? And, in that case, does not the independence become a little subverted? It must be admitted that such fears are not altogether groundless—that there is such a thing as overstraining the very best designs. And it has to be admitted that, upon such points, it is the privilege, as indeed it is the duty, of every manager to form his own judgment, and act upon it! On the other hand, the judgment of the manager may be captious; he may be anticipating more than can be ordinarily accomplished. He may be expecting direct manifestations of benefits which, in their very nature, can only operate in a somewhat indirect manner.

In this position of things, it may be well at once to try and arrive at a common understanding as to the services which the insurance press may render to the insurance offices—anything that it may accomplish beyond being so much to the good. I will enumerate a few of the more prominent heads.

1. The press may dwell unceasingly upon the fact that, in all times past, from the earliest dawn of social life, the principle of association in the bearing of each others' burdens has been in operation; and this is the essential principle upon which Insurance, in all its multiform branches, proceeds.

2. It may go further, and contend that in all periods of advancing civilization, as evinced by the development of commerce, Insurance, in some form or other, has gone hand in hand with such development, giving to it that one element of protection without which business ventures become mere gambling transactions, and ruin may await the most seemingly prosperous. And it may enforce these views in a hundred pleasant examples drawn from the history of the great nations of the past.

3. It may contend that, to-day, Insurance is as equal to the protection of our trade and commerce, in all its varied ramifications, and also to meet the contingencies of our daily life, as it ever was. That it is in this respect as much relied upon as it has been at any time in the world's history. And that it affords the increasing advantage of providing for the perpetuation of our social enjoyments in all their pleasant surroundings. Life insurance, during the many centuries of its known existence, was never more active for good than it is to-day—never more capable of upholding the coveted fabric of social independence.

4. It may teach legislators the lesson, that association for the common good is a principle inherent to mankind—that it existed anterior to legislation itself; and that, on the whole, it is exceedingly doubtful whether legislation—by the fiscal burdens it has imposed, and by the unmeaning restrictions it has too often set up—has not been more productive of harm than good. Further, it may safely assert, that whenever legislators attempt to take Insurance under their direct patronage, they do this to its great detriment. State insurance schemes may be broadly asserted to be failures. Insurance thrives best on free soil.

5. It may point out that, side by side with the development of Insurance, have grown up fraudulent devices, aiming not so much at its subversion as to the turning its good uses to the pecuniary advantage of wrong-doers. That this same principle of fraud is as rampant now as it ever was; but it is only those who are practically concerned in the business, and in the various branches of the business, who fathom its machinations to the full extent.

6. In this connection it may ask legislators to lend a helping hand in the framing of needful laws of repression. It may admonish judges against petulance, and other acts of bad taste, when insurance associations deem themselves compelled to invoke the aid of the law, or when they are under the necessity of defending themselves against unjust demands. It may be said broadly that no insurance association was ever mad enough to attempt to resist

a claim without seeing very good reasons for doing so; and the directors and managers have better means of judging than mere outsiders can ever have.

7. In this same connection it may advocate the enactment of laws giving the most ample powers for investigating claims deemed to be fraudulent—such, for instance, as reforming the abuses of the present coroners' courts; and giving ample powers for investigating the causes of deaths, as also the causes of fires, before effective tribunals.

8. In the matter of the protection of our cities and towns against fire, it may make it clear that this is as much a municipal duty as attempting to keep away the plague, or the providing a proper supply of water. Where the fire offices are called upon to support brigades, it is throwing upon the thrifty, *who insure*, the burden of protecting the unthrifty, who do not.

9. It may render familiar, by frequent elucidation, the laws of health, as applied to communities; and the principles of longevity, as applied to individuals.

10. And in furtherance of these good offices, it may enforce the necessity of provident habits amongst a people who would be permanently in the van of civilization and progress.

11. It may be specially useful in the case of attacks from without. The public press may be said to be always at war with class interests. This constitutes a portion of its public duty. It is for the journals which assume to protect class interests to answer such attacks, and to do so with judgment and skill.

12. It should gather up with diligence all the statistics bearing upon the branches of the business of which it assumes to be the champion. The maxim that "knowledge is power" applies with marked effect to the underwriter; with him, to be fore-warned is to be fore-armed. All that is doing in other countries in the same connection becomes an essential branch of knowledge, and should become known to him with promptness. The press *should* be the medium of communication.

13. It should be prompt in the announcement of all new works on insurance and cognate subjects, wherever issued, and should point out their aim and purpose. A detailed table of contents is frequently the best mode of accomplishing this. All pamphlets bearing upon insurance, controversial and otherwise, should be noticed, either critically or generally—the latter preferred. I would even say that fresh issues of the prospectuses of insurance associations should be referred to, when new features are intro-

duced. The offices would readily supply this latter information, I assume.

A journal conducted on such principles, and distinguished by such teachings, will have a wonderful effect in improving the moral tone of its readers. Agents so instructed will soon become of a very different stamp from the ordinary men of that class. Such a journal, too, will find outside readers. It is one of the objections frequently urged against class journals, and particularly against insurance journals, that they do not operate outside their class. I do not know that this is a very effective argument, provided it can be shown that they do good service amongst those for whose benefit they are primarily designed. But they must either protect the cause from within, or advance it from without, if they are to lay claim to be of real utility. Having laid down my views of what an insurance journal should be and do, let me with equal frankness state what I think it should not be or do.

(*a*). It should never attempt to set class against class, *i.e.*, office against office—for even agents do not always discriminate, or even remember accurately, which is the alleged offender. And as to the outside public, evil things said of one office are apt to be applied to them all, and thus to engender a spirit of distrust.

(*b*). It should never be violently partizan in its advocacy. In most well-regulated minds this of itself suggests feelings of distrust. No man is always right, or always wrong; and in the same way, no company is infallible in its management. There are, indeed, occasions when its voice should be loud and emphatic; but these occasions are few and far between, and mostly arise in the case of attacks from without.

(*c*). It should never proceed to form final conclusions upon *ex parte* statements. The press can always afford to wait until both sides have expressed themselves, and then the truth will usually be within its grasp. In matters affecting the interests of insurance offices—and these are the interests the insurance press primarily professes to protect—the office side of the question may always be very readily learned.

(*d*). It should especially show fair play to young offices. The task is difficult enough, even with fair play, to place a young company upon a sound business footing. Perhaps there is, at the present time, no human enterprize surrounded with greater difficulties. Many of the largest insurance offices of to-day have

passed through periods when a very small amount of adverse criticism would have killed them. It is not the province of the press to kill; and curative processes are always gentle. It is cowardice amounting to infamy, when the assumed protectors of insurance interests lead the attacks upon the weak and the struggling.

(*e*). Again, when companies are attacked by enemies from without—known by the expressive name of "wreckers",—the insurance journals should not be the first to play into the hands of the enemy by exclaiming, in effect, "we always thought so." If it had always thought so, it ought to have said so—supporting the assertion by reasonable evidence, which is always available in such cases at very small cost. A journalist has no special means of judging of the solvency or otherwise of an insurance office. This can only be effectively determined by those who have had considerable actuarial training and experience. To be small is not necessarily to be insolvent. It is as unpardonable as it is unjustifiable to make continued insinuations, having no foundation in fact. Associations that have been so treated can hardly feel very sympathetic with the press—their traducers! Again, it is not the young offices which have brought disgrace upon the business of insurance.

Of course, I do not here attempt to make my arguments exhaustive; the points advanced must be regarded as suggestive only. I do not want to set up too high a standard of ability in the management of an insurance journal. It is not the most able that are the most successful, or the most honourable; and those that claim the greater superiority are not by any means, as a rule, those most entitled to it. There is a conscientious principle involved in true journalism which always manages to make itself felt, and this apart from the mere pretensions of the paper and its editorial utterances. In truth, the editor that is usually correct in his views does not need to proclaim the fact to his readers. *They know it already;* while, if it be not the fact, the assumption of it is only to make matters ridiculous! The one cardinal principle in the conduct of a journal, insurance or otherwise, is fairness—the doing the just thing; in other words, *absolute impartiality.* Without this pervading spirit, the press, instead of being a blessing, becomes a curse. The ability in such a case only magnifies the capacity for evil. In a fairly conducted journal, if an injustice has been done, the editor himself will be the first to make

reparation. In this manner, confidence is established all round. A man unfairly represented, by direct or indirect means, knows that it is but for the moment; the next issue will correct the evil, and put things in the proper light. With all our really great journals, the principle of fairness has been made a cardinal principle. It says something for human nature that it should be so. And when this mode of conduct is departed from, happily the journal soon ceases to be great—that is, to be influential.

Out of the honesty of conduct I have last spoken of, comes *independence*. An honest man is always independent in spirit. So it is with an honest journal. Aiming only at what is right, the independence comes as a matter of course.

The mention of independence in journalism opens up another consideration; this is usually—but not necessarily—associated with freedom of speech. Where there is freedom of speech, there is always the greater necessity for being certain as to the facts. And there is another danger—and that is the *Law of Libel*. Under the plea of "protecting the liberty of the subject", laws against freedom of speech, more severe than those to be found in almost any other country, have been enacted here. The law of libel is essentially the rascal's paradise. An honest and honourable man will, and can, bear with a good deal of abuse before he seeks to put such cruel machinery into force; but a man of ill-repute, whose conscience renders every harsh word as of magnified import, knows of no remedy but the almost savage vindictiveness of a criminal prosecution under the law of libel. The insurance press, much less frequently than the general press, falls within the meshes of this law. This, I am bound to say, is not always because the exemption is deserved; it is rather because those attacked feel that the actual facts of their career are known to their professional brethren, and mere misrepresentation can do no permanent injury. On the other hand, the press itself, where it has failed to speak out when plain duty required it to do so, seeks shelter behind the possible terrors of the law.

There is still another view concerning the insurance press, and this is, that having espoused the cause of insurance interests in general, these are to be regarded as even of wider significance than the mere interests of individual offices. The interests of insurance may happily be advanced without regard to particular offices, who carry on the business merely as a matter of profit. It would be well, perhaps, if the press could take this view. But this implies that the insurance press should then look to the general public for support, rather than to the insurance associations.

I confess to thinking that the highest position the press could take would be to address the general public, and advocate insurance because it is for the common good. There would then be no appeals to offices for patronage. The offices would seek the journals, and advertize in them, merely because they deemed it their interest to do so. The journals might remain insurance journals in the true sense of expounding all that relates to insurance; but they would no longer assume the *rôle* of being the organs of the insurance offices. Perhaps the relations between the offices and the press will never be entirely satisfactory until this stage be reached!

I will, for the moment, glance at the possible mode of conducting an insurance journal under such conditions. It would have to be made much more of a newspaper—news would be its main feature; news of all that related to the interests of insurance. It must chronicle all legislative intervention, and keep true record of its effects; it must warn the public of the legislative failures in the past, giving time, place, and circumstance. This would not be at all difficult. It must report all law cases upon insurance contracts before the courts of the world, and especially criticize and condemn all miscarriages of justice. I have seen a judge on the bench wince under the well-directed lash of a sagacious journalist, who saw the real bearing of the case, while all but its technical points had escaped judicial cognizance. It must report all cases of coroners' inquiries, where insurance interests are at stake, and where foul play is suspected. Where the inquiry has been but a sham and a delusion, as is too often the case, it must say so, in order that other modes of inquiry may be adopted. It must deal in the same way with mysterious fires; with fraudulent claims upon accident, plate-glass, and other companies—too often the victims of much injustice at judicial and other hands. It must have the true "Argus eye" for seeing all that has a bearing upon insurance interests. Its reports must be furnished by those who have some knowledge of the matters under consideration. Its "leaders" must be short and to the point, indicating the parts of the report which contain the gist of the subject. Leaders on merely technical points can usually be dispensed with—the scientific journals deal with these. Points of practical management may be best discussed by opening the columns freely to those who understand them—I mean the managers of the offices. The correspondence in such a journal becomes at once one of the most interesting and instructive features; points in practice always con-

tinue to crop up. To-day it is surrender values; next week the proper rate for a pianoforte manufactory; and the week after, the extra premium required for residence in China. The skilful editor will keep points useful and ripe for discussion constantly in view.

The ever-extending application of insurance must be duly chronicled; its capacity to meet emergencies which can in no other manner be so well provided against must be continually reiterated and illustrated by example. The views, indeed, must be cosmopolitan; the aim, to furnish the reader with information which will be of present as also of permanent interest to him.

Insurance journalism on these broad-gauge principles has been attempted on the other side of the Atlantic, and I believe I may say with reasonable success. A large and extending body of readers has been obtained, chiefly amongst insurance agents—who are a much more independent body there than here—but also extending to outside circles, as merchants and others. It is no part of my purpose now to point to particular journals; but it is always instructive to know what has happened and is happening elsewhere. In this connection I may note an experiment which has been tried in the United States, and which certainly embodies the idea of commercial fairness. The insurance companies (that is, some of them) set apart each year a definite sum—constituting, I assume, a percentage upon the new premium income of the preceding year—and that sum, and no more, they expend with the insurance and other journals in the way of securing publicity. This plan recognizes the fact that, by means of like publicity, such results were at least in part attained. By this arrangement, too, the journals, insurance and others, live upon results; and the new aspirants can hardly be looked upon as implying any additional tax upon the office.

I know that much more remains to be said; but hope that points of value may be elicited in the discussion. To be of value, the discussion must be free, *i.e.*, unreserved. I know of no reason why it should not be so. Latent dissent is much worse than candidly expressed dissatisfaction. The latter presents opportunities for explanation and amendment. The press will have its own means and opportunities of expressing its views. I trust an amicable understanding may be arrived at. The simple fact of my volunteering this paper may fairly be construed into an expression of my individual belief that the relation of the insurance press to the insurance offices and to insurance interests is not entirely satisfactory.

Notes on the Mortality of the Danish Clergy, from 1650 *to* 1878.
By HARALD WESTERGAARD, *of Copenhagen.**

[Read before the Institute, 25 April 1881.]

A CONSIDERABLE number of investigations on mortality in times past has already been published, and without scarcely any exception they all go to show the same thing, that the mortality now-a-days is much smaller than it used to be. Some of these results are, however, only found by a sort of induction, in considering the conditions of life and the state of medical science in past centuries, and most of the statistical facts do not go farther back than to 1750; for more remote periods, the statistical information which has been gathered is scanty, untrustworthy, and dispersed.

I therefore undertook the task of investigating a distinct class of society during as long a period as possible, hoping thus to be able to confirm or correct our notions on the variations of mortality. The materials were taken from an elaborate biographical work by the Rev. S. V. Wiberg, published 1870–71, with a supplement from 1879.

I confess that the general impression of the results was that of a disappointment. In fact, the materials appeared so incorrect, that distinct numerical conclusions were not admissible. Still, I have thought it of interest to publish the chief results, hoping that, by-and-by, more correct materials may be at hand, and that the readers will consider the results here laid down as merely preliminary.

One curious fact assists us greatly in investigating this particular class of society; and if not for that, we should probably never have been able to go so far back as we have done here. A few decenniums after the abolition of the Catholic Church and the celibacy of the priests, in 1536, it was—*de facto,* if not *de jure*—almost a necessary condition of appointment to marry. Very frequently, the predecessor's widow, or at least a relation of his, was the obligatory bride; or the rectorship was inherited from father to son. At all events, a great many clergy-families arose in this way, whose pedigrees were rather easy to trace.

It is hardly necessary to dwell upon the conditions of life in these remote periods. Every country in Europe has, to a certain

* For fuller information, we may refer to a greater work by the same author, now under the press, entitled *Die Lehre von der Mortalität und morbilität.* (Gustav Fischer in Jena.)

degree, the same social history, only the dates may be somewhat different; and the clergymen shared the lot of society in general. In the 17th century the Danish clergy, on the whole, were poor, uneducated, rude, and intemperate, like most of their contemporaries, though we must make allowance for a number of highly educated and serious men. In the next century, the conditions of life improved, and it will be seen that the effect on the mortality was striking. In the present century, finally, Denmark has entered a new stage, in almost every respect opposite to that of the past centuries, and the conditions of life are altogether different from what they used to be.

Before proceeding to state the results of our investigation, we may consider the methods and the accuracy of the materials. Births, deaths, and appointments were assumed to be distributed uniformly over the year; or, which amounts to nearly the same thing, to take place in the middle of the year. The persons, therefore, who were born, for instance, in 1760, were, at the beginning of the year 1790, on an average $29\frac{1}{2}$ years old, and the rates of mortality will therefore hold good for the ages $29\frac{1}{2}$ to $30\frac{1}{2}$, &c., instead of from 30 to 31, &c. It is necessary to state this, in order to avoid mistakes, for statists frequently err in this respect.

Such a laborious work as Wiberg's cannot be altogether correct. A considerable number of errors must always be caused by reading and writing incorrectly; and even when we deal most scrupulously with the materials in computing the number of deaths and of years of life, we cannot avoid errors.

But there is another cause which has a much wider bearing. A great number of particulars are not to be had. Thus, where the church registers are burnt, it may be difficult to find out the years of birth, and in many cases the years of death or appointment are unknown. Not even in this century are the materials quite pure. I doubt whether any investigation is possible without dealing, to a greater or smaller extent, with such unsafe experiences. Generally, statistical investigators pass silently by these difficulties, without stating how they have been overcome. The best plan is, evidently, to divide the materials into safe and unsafe experiences, so that everybody may judge as to the reliability of the results.

It frequently happened that the year of birth was wanted, whereas we knew when the person entered university. It was found that, with great approximation, 20 years was the average age for the students at the beginning of the first term. Nine out of ten

unsafe experiences belonged to this class, with unknown birth year, and in one-fourth part of these cases we knew the year of entering university. As to the remaining cases, a few persons were noticed as dead at an uncertain age, whereas the year of dismissal was known; and it was found that, on an average, six years lapsed between dismissal and death. 110 persons, besides, had no certain death-year, and the remaining persons were chiefly those who had no fixed date of appointment, wherefore an average age was used deduced from the following and preceding years.

I was greatly at a loss with regard to the 110 persons whose death-years were unknown. Where there are only a few unsafe experiences of this description, it is, of course, indifferent what assumptions we make use of. But this is not the case here. We may, indeed, assume that the mortality among these persons has been nearly the same as among the others, and thus compute the ages at death according to this hypothesis. But we must then bear in remembrance that it is totally indifferent whether we include the numbers which we compute in our calculations, or leave them out of consideration; the rates of mortality must be exactly the same, at least if we do not make use of the rough expedient to assume that all persons die at the same age, but distribute the ages according to what we learn from the remaining materials. In the original work, I have chosen to give a complete list of all these 110 cases, so that future investigators may deal with these materials as they like.

The following are the chief results :—

1840–1878.—*Safe Experience (only* 9 *Unsafe Cases).*

Age.	Years of Life.	Deaths.	Force of Mortality.	Mortality of Male Population in Denmark, 1865-74.		English Clergy,† 1860-61 and 1871.	English Male Population,† 1860-61 and 1871.
				Cities and Towns.	Rural Districts.		
25*	1333	7	0·53	0·89	0·63	0·46	0·98
35	7110·5	55	0·77	1·43	0·83	0·63	1·30
45	10645·5	133	1·25	2·49	1·36	1·32	1·85
55	9735·5	230	2·36	4·45	2·80	2·27	3·22
65	5770	334	5·79	7·45	5·90	5·21	6·68
75	2020·5	218	10·8	14·38	12·89	15·02	16·58
85	314·5	69	21·9	27·42	28·65		
Total	36929·5	1046	...	...	...	...	...

* This means, for Danish clergy, 24½–34½. † Dr. Farr's Tables.

In computing these rates of mortality, we must remember two difficulties. Firstly, the numbers are not great enough to give any degree of accuracy to so many decimal points. According to the calculus of probabilities, the true rate at the age 55–65 may as well be, for instance, 2·21 or 2·51 as 2·36. But we only require a general idea of the main features of mortality among the Danish clergy, and this we may fairly say that we have attained.

Again, in comparing these rates with the rates for the general population, or the English clergy, we must interpolate them $\frac{1}{20}$ forward, in order to get corresponding classes of age, *i.e.*, in order to proceed from $24\frac{1}{2}$–$34\frac{1}{2}$, &c., to 25–35, &c. After doing this, we find that the mortality among the clergy is much more favourable than in the urban districts, and, on the whole, smaller than in the rural districts. The mortality is, on the whole, a little higher than among the English clergy, which is in accordance with what we find, if we make use of Hodgson's experience instead of Dr. Farr's.

It will probably be more interesting to compare these mortality rates with those prevailing in former periods. The following table shows the results :—

[*See next page.*]

The unsafe experiences, 1840–78, give 3·1 expected, and 9 actual deaths. It would alter the results very little if we had combined the safe and unsafe experiences for this period.

It is curious that the unsafe experiences show a higher rate of mortality than the other ones. This may easily be understood; for the persons who died young are more likely to die without having made a name than those who die old, not to speak of the fact that if a man dies young he belongs to an earlier period than if he reaches the old age; and we see, on inspection of the tables, that there is a great progress in the accuracy of the materials.

There is the most striking improvement in health from one period to another, whether we consider the safe or unsafe experiences. In the past centuries the mortality was more than double that of the last period (1840–78).

There is reason to believe that the mortality among the 110 persons with unknown death-year is not less than in the safe experiences; and we are therefore fully entitled to the conclusion, that mortality has been greatly decreasing under the influence of the causes which have transformed the society during the present century.

Age.	Safe Experiences.			Unsafe Experiences.			Safe and Unsafe Experiences combined.	
	Years of Life.	Deaths.	Expected Deaths.	Years of Life.	Deaths.	Expected Deaths.	Deaths.	Expected Deaths.
1650–1699.								
25	4057·	41	21·5	4142·5	68	22·0	109	43·5
35	5910·5	117	45·5	9439	317	72·7	434	118·2
45	4030	146	50·4	5122	272	64·0	418	114·4
55	1773·5	110	41·9	1967·5	145	46·4	255	88·3
65	469·5	35	27·2	441	48	25·5	83	52·7
75	67	8	7·2	34·5	9	3·7	17	10·9
85	3	2	9·7	...	...	...	2	0·7
Total	16310·5	459	194·4	21146·5	859	234·3	1318	428·7
1700–1749.								
25	7836·5	90	41·5	942	15	5·0	105	46·5
35	13401	286	103·2	3541	125	27·3	411	130·5
45	10353	348	129·4	3493·5	156	43·7	504	173·1
55	6075	309	143·8	2739·5	195	64·7	504	208·5
65	2661·5	220	154·1	1373·0	169	79·5	389	233·6
75	701·5	113	75·8	302·5	51	32·7	164	108·5
85	42	14	9·2	33·5	9	7·3	23	16·5
Total	41090·5	1380	657·0	12425·	720	260·2	2100	917·2
1750–1799.								
25	6211·5	51	32·9	135	1	0·7	52	33·6
35	14554·5	230	112·1	631·5	14	4·9	244	117·0
45	14624·5	378	182·8	497·5	19	6·2	397	189·0
55	10572·5	530	249·5	433·5	24	10·2	554	259·7
65	5350·5	480	309·8	232·5	23	13·5	503	323·3
75	1554·5	230	167·9	71	12	7·7	242	175·6
85	196·5	55	43·0	13	4	2·8	59	45·8
Total	53064·5	1954	1098·0	2014	97	46·0	2051	1144·0
1800–1839.								
25	4701·5	26	24·9	15·5	...	0·1	26	25·0
35	9887	132	76·1	133	3	1·0	135	77·1
45	10384	214	129·8	210	9	2·6	223	132·4
55	8712·5	349	205·6	197·5	7	4·7	356	210·3
65	4712	361	272·8	166	9	9·6	370	282·4
75	1462	220	157·9	67	14	7·2	234	165·1
85	172	38	37·7	5	2	1·1	40	38·8
Total	40031	1340	904·8	794	44	26·3	1384	931·1

Discussion.

The paper was read by Mr. Sutton, but before doing so, he said—Before I read this paper, perhaps it would be well that I should say a few words with regard to the manner in which it came into my hands. The writer became known to me when he was in England, some two or three years ago, on the subject of the working of the present friendly society system of this country, having been commissioned to come over to England to make some enquiries on the subject; and since then I have had some written communications with him. He is comparatively young, I believe; and from what I have seen, he certainly appears to have shown great industry in making certain actuarial investigations; and the notes which I am going to read to-night I have purposely had printed exactly as he sent them over to me, without any editing whatever. Of course, there are certain imperfections in the language, but I considered it better that his work should in the first instance appear, on the whole, exactly as written; and I think, from this point of view, it will be considered as entitled to great credit. The notes themselves are really a translation of a portion of a large book that he has been writing, of which he gives the title in a note. It is quite clear, from extracts that he has sent me from time to time, that it is in its way a very creditable production. It will no doubt interest the members of the Institute to know that I had a letter from him only last week, to say that this book had been thought so highly of that the Danish University has awarded him a gold medal.

The President, in inviting a discussion on the subject, said—Gentlemen, we are always glad to welcome communications from our foreign friends; and I am sure that, short as this paper is, it is a welcome contribution to a subject in which we must all take an interest in this Institute—the mortality of particular classes. The clergy are a very important class of the community in all countries, and any information which throws light upon the mortality of that particular class is of interest. I am struck with the opening sentence of this paper of Mr. Westergaard's, where he says that, beyond all question, mortality nowadays, to use his own phrase, is much smaller than it used to be. I confess to be somewhat a sceptic on this subject myself, and I think that the information relating to the seventeenth or even the eighteenth century is not altogether to be depended upon. (Hear, hear.) I confess I should like some more information about the sources of the materials from which the facts of the seventeenth century are derived. It is quite incredible to me that while 459 deaths occurred in the seventeenth century, the computed number, by a corresponding table of mortality, in the nineteenth century, should be only 194. I do not believe such a thing for one moment. Again, in the eighteenth century, the expected deaths are about one-half of the actual number. The actual deaths are 1,380, and the expected deaths 657. If that be true, anything like a law of mortality is at an end. As to the clergy, there are two or three noticeable circumstances. There is undoubted evidence that the rate of mortality amongst the clergy is better than in the community generally. It would be interesting if we could get some information about the mortality of

priests in the Church of Rome, where celibacy is obligatory, and about the mortality of priests of the Eastern Church, where marriage is compulsory, and compare them with communities like the clergymen of the Church of England, or other churches, where celibacy is neither compulsory nor otherwise. As to the accuracy of these materials, it is said, "It frequently happens that the year of birth was wanted, whereas we know when the person entered the university." In England, many men enter holy orders late in life. Therefore I think that the date when they enter the university would not be a fair guide as to the date of birth amongst the clergy of the Church of England. It may be in Denmark, but not in England. I shall be very glad to hear any observations which any gentlemen have to make on the subject of this paper or on the subject of mortality generally.

Mr. D. A. Bumsted—I take some interest in this matter, having translated a paper by Herr Stüssi upon the mortality of the clergy, which appeared in the *Journal* a few years ago, and gives a great deal of information about them. I quite agree with you that it is very difficult to ascertain the mortality of the population generally for the seventeenth century. The information in this paper gives very little upon which we can go. There is no doubt that the Danish clergy are in a better condition as regards mortality now than they were in the seventeenth century, because at that time they appeared, from Mr. Westergaard's paper, to have been in a condition within a measurable distance of barbarism. The great majority of them were intemperate, poor, uneducated, and rude. Of course, we can hardly expect such a good rate of mortality among that class of people as amongst those existing at the present day. But I do not think we can draw any conclusion respecting the mortality of the seventeenth century. In the paper that Herr Stüssi wrote, there is a comparison of the mortality of the Greek clergy with that of the Prussian clergy, by Casper's table. The duration of Casper's observations was from 1760 to 1835; and Stüssi's observations of the Greek clergy were from 1820 to 1871; so that there we have two consecutive periods, the first of 75 years, from 1760 to 1835, and the second of 50 years, from 1820 to 1871. From this table, it appears that there is very little difference between the mortality of the clergy in the first period and that in the second period.

The President—The Prussian clergy and the Greek clergy?

Mr. Bumsted—Yes.

The President—They are two different classes of men.

Mr. Bumsted—Yes. But he mentions some important points in which the Greek clergy resemble the Protestants, and gives his opinion—with which I am disposed to agree—that at the present day there is a much greater strain than formerly upon the powers of the clergy (as indeed there is upon all classes), which, so far as the rate of mortality is concerned, more than compensates for any improvement in their condition. Here is a diagram, showing that the mortality of the Greek Catholic clergy, as a rule, runs above Casper's table and below Brune's Prussian table. Of course, the statistics are very small, but as far as they go they seem to be tolerably clear. I am sure we are much obliged to Mr. Sutton for bringing this subject

before us; and I must congratulate the author upon the correctness of his English.

Mr. Walford—Mr. President, I respond to your call, but have been waiting in order that others who know more of this subject than I do should have an opportunity of saying what they know. I dare say we have all, as a matter of interest and professional necessity, paid attention to the mortality of the clergy. It is a very interesting subject, and there is a problem about it which to my thinking has never been entirely cleared, and which, perhaps, hardly admits of being cleared up. If any class of persons in the community ought to live a long life, the clergy ought to do so. If having time for mental contemplation, and being surrounded with everything which makes one's life pleasant, should have any influence on longevity, the clergy ought to get the benefit of that influence, and to all appearance they do seem to have had that advantage. But there are some aspects of the case which seem to point in another direction. There is a society carrying on business in this country which has granted annuities to clergymen out of health—health annuities; and I believe the experience of that society has been so disastrous that they would be very glad indeed to discontinue the business, but they cannot discontinue it because the annuitants will not die or go off the funds, but their health has been so bad as to indicate in some degree an aspect different from their proverbial longevity. I cannot quite account for that. Another fact which presents itself to one's mind may, in some respects, account for the state of health, but hardly for the increased longevity against the failing health. It is this: in families, it frequently happens that the least robust of the male members will be brought up to the church. If there is any selection at all, it is natural, I think, that the least strong and robust should be selected for so easy a profession as the church. I have noticed it in my own family, and in some others; they have been selected for their occupation because their constitutions are rather frail, and an easy life seemed to be desirable. If that selection has existed to any great extent, it is the more remarkable that the clergy should present so good a record as they usually do in the matter of longevity. As to the Danish clergy, there are in this paper several points which strike me as tending to invalidate the broad results stated. This may only have arisen from the obscurity of the language. Perhaps Mr. Sutton, when in correspondence with the author of the paper, will be good enough to take up any points arising in this discussion, and ask for explanation about them. With regard to the assumption in the paper that the general mortality of Europe has improved during the last two or three centuries, I share with you considerable doubt upon that point. I believe that the first person who assumed he had made that fact clear was the late Mr. Griffith Davies, who became very positive on the subject; but it has occurred to me that the data which he has presented to his readers in confirmation was by no means conclusive. Dr. Farr, in some of his writings, has, I think, taken a similar view, and held that the duration of life has increased with each century. I have some doubts about it. It is possible that the absolute duration of life—that is, the number of years lived in the communities—has increased. Medical science has done something

—and a good deal, perhaps,—in saving the lives of sickly children, who used to die young. It may have kept them alive, and brought them up into middle age; but sickly children, I apprehend, never make strong lives, and in the end do not add very much to the longevity of a nation. It may be so, or it may not. That is one of the points that strike me. I have been reading lately a work bearing upon this subject of increased longevity—a somewhat startling book. Dr. Parkin has published part ii. of his work on Epidemiology, and therein he broadly asserts that all the health appliances and other things which have been introduced for the improvement of the sanitary condition of towns have done nothing, and will do nothing, to prolong human life. He says that the laws of life are governed by circumstances altogether beyond the control of human invention. His theory is that mephitic vapours, arising from volcanic action in the earth, produce pestilential epochs at certain recurring periods, and destroy mankind as they always did, and that will be the result up to the end of the chapter. This is a very remarkable and startling theory to be put forth in the present day, when we have been thinking that we have done so much. I have not made myself familiar with all the facts. He gives a great many confirmatory statistics; and I must say that, *prima facie*, he makes out a pretty good case. I have not been able to analyze his theory very carefully at present. Although I have put the subject of plagues and pestilences down for two consecutive years, for a paper to come before the Statistical Society, wherein this point will be specially considered, I have not yet had time to complete it, and, in fact, I have not seen my way to bring the whole subject within the space of an ordinary evening's discussion. If it should be as he says, then I think that we may take it for granted that the probability of the duration of life has not really increased, and that it never will increase. I do not adopt the theory. I report it as I find it. It is one of the interesting points in connection with human life which actuaries are called upon to consider, for it has a considerable bearing upon the problems which, in this room, we are concerned in. I am always glad to welcome an addition to the collected facts, and especially from countries whose social life we are not familiar with. They are valuable in many ways; and the fact that a man like the gentleman who has sent the paper to-night, living in another country, under altered circumstances, will investigate the subject from his point of view, must be productive of good. Every year seems to bring some addition to the facts already existing, and, on the whole, in the end I suppose that we shall have a mass of matured mortality observations such as never existed before; and which ought, if they have any value at all, to render the science of life contingencies a more fixed science than ever before. On the other hand, if Dr. Parkin's view is to be considered as correct, then all the improvements which are effected, whatever beneficial results they seem to promise, will end in delusion. I still think it cannot be so, but there is, at all events, a scientific man of repute who boldly asserts that that is the case. I might go a step further, and point out how civilisation appears to bring its own penalties in the way of intensifying some of the causes of death, such as those resulting from brain diseases, heart disease, and other causes relating to the nervous system, as also from violent

deaths, but I will defer this consideration until another fitting occasion. (Hear, hear.)

Mr. A. Day—I think, Mr. President, that we are rather in a difficulty in discussing a paper about the Danish clergy without anyone here to explain to us the character of that clergy and their form of religion, for we know comparatively little about Denmark at all. He says that in the seventeenth century the Danish clergy were uneducated, rude, and intemperate—like all the rest of the nation, according to this account. But, unhappily, we here do not know much about what the habits of the Danes were—whether they were an especially intemperate race. It is noteworthy that they sent Hamlet to England when he became mad.

Mr. Walford—There was something rotten in the state of Denmark.

Mr. Day—There was something rotten in the state of Denmark, as Mr. Walford says. What struck me as rather astonishing was this—that the author has got a record of such a considerable number of deaths of the clergy in this period. We find that the actual deaths according to the safe experiences between 1650 and 1699 were 459, and that the unsafe were 859. That is about 1,300. There were about 2,000 in the next fifty years, and then there is another quantity of 2,000, and another of 1,800. That seems to be an enormous number of facts to have got together in one limited class, when we know that the population of Denmark is something under two millions. At least, I believe it to be so.

Mr. Sutton—These are the deaths over fifty years of time.

Mr. Walford—All the small villages of Denmark have their clergymen.

Mr. Day—It seems to be a large number of facts to be able to get from such a small kingdom as that. But then, from what the author calls his safe experience, from 1840 to 1878, he brings out a rate of mortality which is almost more favourable than that of the English clergy in the last column but one. Well, that astonishes me. Does not it you?

The President—No.

Mr. Day—I do not think I have any further remark to make. We shall be very glad to see the larger work which Mr. Westergaard proposes to publish.

Mr. Sutton, in replying on behalf of the writer of the paper, said—In communicating, sir, with Mr. Westergaard, I will take special care to mention what you have pointed out. I think that he will probably be able, in some way or other, to give us a short account of the nature of the materials from which he has derived his facts. I have no doubt that at the same time I can get him—I will not say to write a little Danish history—but to explain to us what is the precise nature of the duties and social position, &c., of the Danish clergy, so that we may be able to judge how far they resemble those of our own clergy. With regard, sir, to the general question raised in this paper—the marked improvement in the mortality from century to century, I am bound to say, looking at these figures, that I am rather inclined to be of your opinion—that somewhere or other (where, I do not know) there is something a little wrong.

It has always occurred to me in many similar inquiries, whether this form of stating the comparison is the best It is, perhaps, a complicated subject to go into, and one of some difficulty; but I am not at all certain that this method of taking years of life, putting opposite the actual deaths, and then multiplying those years of life into another rate of mortality to give what are called the expected deaths, and carrying the process right through the table, is the best form of showing the comparison between different periods. I have thought it over myself a good deal, and I am quite sure there is a good deal to be said on the subject. If any member of this Institute were to thoroughly investigate this point, and bring his results before the Institute in the form of a paper on the subject, I am sure that it would very well repay the trouble he might take. Then, sir, going one step more into detail with regard to comparing the mortality of a given occupation in various countries, you can never quite get hold of the exact effect of the occupation upon the rate of mortality, because you cannot bring the Danish clergy over here and make them do their duty in this country under the conditions of climate, salary, and certain other things; and a difficulty always, I think, exists with regard to these comparisons. I have had this brought to my mind to-night through what Mr. Bumsted mentioned with regard to the comparison of the mortality of the Prussian clergy with that of the Greek clergy. There the same difficulty exists. Apart from other things, there is a marked difference between the Prussian climate and the Greek climate, which must have some influence. Mr. Walford mentioned a society whose experience showed a considerable amount of ill-health, and at the same time a remarkable vitality on the part of clergymen. With regard to that, perhaps Mr. Walford can tell us in what way the ill-health was ascertained?

Mr. Walford—From communications published, they will show precisely what I have said. They are annuitants.

Mr. Sutton—Then there is a remark of Mr. Walford's with regard to increased longevity, which brings me back to what I just took occasion to mention as to the proper way of comparing one mortality experience with another. Increased longevity is to me a rather vague phrase. If you take ten thousand men of the same given age, they have all got to die, no matter what the experience is; and the question is, would Mr. Walford, or anyone else, define increased longevity as holding, if a very large proportion attain old age?

The President—If the average duration is greater in the Carlisle Tables than in the Northampton Tables, that would be increased longevity.

Mr. Walford—That is what I meant.

Mr. Sutton—I do not wish, sir, to discuss the matter now, beyond stating that I think it will be proved on close examination not to be such a simple matter as may at first sight appear. I am sure that Mr. Westergaard will be very pleased to hear that his little contribution has been so well received.

Mr. Sutton has since received a communication from Mr. Westergaard in reference to the matters referred to in the above discussion.

Mr. Westergaard writes as follows:—

There appears to have been some doubt among the members of the Institute who took part in the discussion as to the social conditions and history of Denmark. If I were to describe the nation in a few words, I would say that, to obtain a correct idea of it, you should add the English and German nations together, and take an average. In fact, Denmark is now quite a modern country; nor has it ever been much behind other European nations. When saying that the population in the seventeenth century was intemperate, I believe that this was a description applicable to the whole European population, with but few exceptions.

As to the facts, I can only add this, that there are perhaps 1,000 clergymen now in Denmark, and this may very well give 2,000 deaths during a long period (1,046 deaths in 39 years). The duties of a clergyman are not very different from those of your own Protestant ministers, or of the English clergy; and their position in society is much the same, or perhaps better, the Danish clergy being proportionately well paid.

With regard to the so-called Unsafe Experience, I may say that the average age of students on entry was computed from the safe materials, not from other records; and this seems to me to corroborate my conclusions on this point.

As regards the main question, I confess that I am of opinion that mortality is continually undergoing very great changes. Messrs. Bailey and Day have found that not even one-tenth of the children of the Peerage Families die in the first year of life. In Bavaria, on the other hand, it is found that 30 per-cent die, and in some districts 40 or 50 per-cent! Dr. Farr has shown that the rates of mortality among innkeepers, cabmen, potters, &c., are double those of clergymen and lawyers. My opinion is, that whenever there are great social causes acting, mortality will undergo changes from one place, or profession, or period, to another.

The Influence of Marriage on the Death-rate: being Observations on Dr. Stark's Conclusions, by HERBERT SPENCER, Esq., R. A. PROCTOR, Esq., *and* Dr. W. ROBERTSON.

IN our number for July 1880, we reprinted extracts from Dr. Stark's writings on this subject. We did not at the time express any opinion of our own as to how far Dr. Stark's figures, as to married men and bachelors, justify the conclusions he draws from them; but we left our readers to form their own judgment on the subject. We now submit to them some criticisms on Dr. Stark's

conclusions by Mr. Herbert Spencer and Mr. R. A. Proctor; and in doing so we think it well to state that, while Dr. Stark's figures unmistakably show the higher mortality which prevails among the unmarried, it seems to us that there is some want of clearness and consistency in his explanations as to the probable cause of the difference. He very truly points out that it is mostly the robust and temperate who marry, and that the weak, the diseased, and the licentious, do not marry; and that these facts furnish the natural explanation of the difference which exists in the death-rates of the married and unmarried. This view appears to us unquestionable; but it seems inconsistent with the statements that Dr. Stark has made elsewhere, "that the prolongation of life which attends that [the married] state, is a special provision of nature to protect the father of a family, in order that he may provide for his offspring, and superintend their rearing." It seems also quite at variance with his remark that "bachelorhood is more destructive to life than the most unwholesome trades, or than residence in an unhealthy house or district, where there has never been the most distant attempt at sanitary improvements of any kind."

We entirely concur in the views expressed by Mr. Spencer and Mr. Proctor, and the only remark we think it necessary to make, is that, whereas both of those writers speak of the effect of marriage on "longevity", Dr. Stark's statistics say nothing whatever about *longevity,* but only about the *rate of mortality* among married men and bachelors. At first sight it may appear that this is a matter of no consequence,—that, if the rate of mortality is less, the longevity must be greater, and conversely. This would be quite true as regards two permanently distinct bodies of persons; for instance, if a body of women were permanently subject to a less mortality than a body of men, it is clear that their longevity would be greater; but the bachelors of whom Dr. Stark gives statistics are not in this sense a distinct body of men from the married, for there is a constant transfer taking place from the ranks of the bachelors to those of the married. It is, therefore, not correct to speak of the longevity of the bachelors and the married at all. This, however, does not in any way affect the validity of the arguments of Mr. Spencer and Mr. Proctor; for it will be found that if, instead of *longevity,* we substitute *rate of mortality,* their arguments will be unaffected.

We need only add that, if the widowers had been included with the unmarried instead of with the husbands, the differences

between the rates of mortality would have been still greater. It clearly appeared from Mr. Sprague's investigations into the rates of marriage and mortality among the Peerage widowers, that the rate of mortality among those widowers who did not remarry, was extremely heavy,—so much so at the younger ages, that it would scarcely be an exaggeration to say that all the young widowers in good health, remarried within a few years; and that those who did not remarry, died early.—Ed. *J.I.A.*

Extract from "The Study of Sociology", by Herbert Spencer.

A few years ago, Dr. Stark published the results of comparisons he had made between the rates of mortality among the married and among the celibate: showing, as it seemed, the greater healthfulness of married life. Some criticisms made on his argument did not seriously shake it; and he has been since referred to as having conclusively proved the alleged relation. More recently I have seen quoted, from the *Medical Press and Circular*, the following summary of results supposed to tell the same tale:—

"M. Bertillon has made a communication on this subject ('The Influence of Marriage') to the Brussels Academy of Medicine, which has been published in the *Révue Scientifique.* From 25 to 30 years of age, the mortality per 1000 in France amounts to 6·2 in married men, 10·2 in bachelors, and 21·8 in widows. In Brussels the mortality of married women is 9 per 1000, girls the same, and widows as high as 16·9. In Belgium from 7 per 1000 among married men, the number rises to 8·5 in bachelors, and 24·6 in widows. The proportion is the same in Holland. From 8·2 in married men, it rises to 11·7 in bachelors, and 16·9 in widowers, or 12·8 among married women, 8·5 in spinsters, and 13·8 in widows. The result of all the calculations is that from 25 to 30 years of age the mortality per 1000 is 4 in married men, 10·4 in bachelors, and 22 in widows. This beneficial influence of marriage is manifested at all ages, being always more strongly marked in men than in women."

I will not dwell on the fallacy of the above conclusions as referring to the relative mortality of widows—a fallacy sufficiently obvious to any one who thinks awhile.* I will confine myself to the less

* Mr. Spencer has favored us with the following observations on this point:—"The class of widows, taking it as a whole, includes those who having been previously, with their children, supported by a husband earning wages, are left after his death to support themselves and their children; and clearly, the stress brought upon them by this extra exertion, anxiety, and physical privation, is very much greater than that which falls upon any other class. Consequently, they are conditioned in a way which inevitably entails an increase of mortality. Further, it is to be observed that this excess of mortality in the class of widows, consequent upon the condition in which they are left, is increased by the several causes which take away from among them those who are most likely to be long livers. In the first place, those are most likely to be remarried who have best preserved their physical vigour and concomitant attractiveness of appearance; and this implies that, had they remained widows, they would have been among those who lived longest. In the second place, those are most likely to be

conspicuous fallacy in the comparison between the mortalities of married and celibate, fallen into by M. Bertillon as well as by Dr. Stark. Clearly as their figures seem to furnish proof of some direct causal relation between marriage and longevity, they really furnish no proof whatever. There may be such a relation; but the evidence assigned forms no warrant for inferring it.

We have but to consider the circumstances which in many cases determine marriage, and those which in other cases prevent marriage, to see that the connection which the figures apparently imply is not the real connection. Where attachments exist, what most frequently decides the question for or against marriage? The possession of adequate means. Though some improvidently marry without means, yet it is undeniable that in many instances marriage is delayed by the man, or forbidden by the parents, or not assented to by the woman, until there is reasonable evidence of ability to meet the responsibilities. Now of men whose marriages depend on getting the needful incomes, which are the most likely to get the needful incomes? The best, physically and mentally—the strong, the intellectually capable, the morally well-balanced. Often bodily vigour achieves a success, and therefore a revenue, which bodily weakness, unable to bear the stress of competition, cannot achieve. Often superior intelligence brings promotion and increase of salary, while stupidity lags behind in ill-paid posts. Often caution, self-control, and a far-seeing sacrifice of present to future, secure remunerative offices that are never given to the impulsive or the reckless. But what are the effects of bodily vigour, of intelligence, of prudence, on longevity, when compared with the effects of feebleness, of stupidity, of deficient self-control? Obviously, the first further the maintenance of life, and the second tend towards premature death. That is, the qualities which, on the average of cases, give a man an advantage in gaining the means of marrying, are the qualities which make him likely to be a long-liver and conversely.

There is even a more direct relation of the same general nature. In all creatures of high type, it is only when individual growth and development are nearly complete, that the production of new individuals becomes possible; and the power of producing and bringing up new individuals, is measured by the amount of vital power in excess of that needful for self-maintenance. The reproductive instincts, and all their accompanying emotions, become dominant when the

remarried who either have no children, or who are least burdened with children; which again implies that the remarried ones are those whose constitutions are least likely to have been shaken by much child-bearing. And in the third place, there is abstracted from the class of widows by remarriage, those who are possessors of considerable means, and whose property affords a temptation to second husbands—that is to say, those who would be the most able to survive notwithstanding the loss of their husbands: leaving behind those who, being poor, are the least able to struggle with their difficulties. Even supposing, then, that no widows were remarried, their mortality in consequence of their difficulties in maintaining themselves and their children, would be above the average; and this extra mortality is made greater by the remarriage of those among them who are most likely to be long-lived. Hence, then, the enormous mortality of widows, as shown by the statistics, is amply accounted for, apart from any intrinsic effect which marriage produces."

demands for individual evolution are diminishing, and there is arising a surplus of energy which makes possible the rearing of offspring as well as the preservation of self; and, speaking generally, these instincts and emotions are strong in proportion as this surplus vital energy is great. But to have a large surplus of vital energy implies a good organization—an organization likely to last long. So that, in fact, the superiority of *physique* which is accompanied by strength of the instincts and emotions causing marriage, is a superiority of *physique* also conducive to longevity.

One further influence tells in the same direction. Marriage is not altogether determined by the desires of men; it is determined in part by the preferences of women. Other things equal, women are attracted towards men of power—physical, emotional, intellectual; and obviously their freedom of choice leads them in many cases to refuse inferior samples of men—especially the malformed, the diseased, and those who are ill-developed, physically and mentally. So that in so far as marriage is determined by female selection, the average result on men is that while the best easily get wives, a certain proportion of the worst are left without wives. This influence, therefore, joins in bringing into the ranks of married men those most likely to be long-lived, and keeping in bachelorhood those least likely to be long-lived.

In three ways, then, does that superiority of organization which conduces to long life, also conduce to marriage. It is normally accompanied by a predominance of the instincts and emotions prompting marriage; there goes along with it that power which can secure the means of making marriage practicable; and it increases the probability of success in courtship. The figures given afford no proof that marriage and longevity are cause and consequence; but they simply verify the inference which might be drawn *à priori*, that marriage and longevity are concomitant results of the same cause.

Extract (slightly abridged) from "Light Science for Leisure Hours", by R. A. PROCTOR, *pp.* 240–246*: being an Article contributed to the "Daily News"*, 17 *Oct.* 1868.

The Royal Commission on the Law of Marriage has attracted attention to many singular and instructive results of modern statistical inquiry. Not the least important of these is the apparent influence of marriage on the death-rate. For several years, it has been noticed by statisticians that the death-rate of unmarried men is considerably higher than the death-rate of married men and widowers. We believe that Dr. Stark was one of the first to call attention to this peculiarity, as evidenced by the results of two years' returns for Scotland. But the law has since been confirmed by a far wider range of statistical inquiry. The relative proportion between the death-rates of the married and of the unmarried is not absolutely uniform in different countries, but it is fairly enough represented by the following table, which exhibits the mortality per thousand of married and unmarried men in Scotland:—

Ages.	Husbands and Widowers.	Unmarried.	Ages.	Husbands and Widowers.	Unmarried.
20 to 25	6·26	12·31	55 to 60	26·14	28·54
25 „ 30	8·23	14·94	60 „ 65	35·63	44·54
30 „ 35	8·65	15·94	65 „ 70	52·93	60·21
35 „ 40	11·67	16·02	70 „ 75	81·56	102·71
40 „ 45	14·07	18·35	75 „ 80	117·85	143·94
45 „ 50	17·04	21·18	80 „ 85	173·88	195·40
50 „ 55	19·54	26·34			

From this table we are to understand that out of 100,000 married men (including widowers), from 20 to 25 years old, 626 die in the course of each year; while out of a similar number of unmarried men, between the same ages, no less than 1,231 die in each year. And in like manner all the other lines of the table are to be interpreted.

Commenting on the evidence supplied by the above figures, Dr. Stark stated that "bachelorhood is more destructive to life than the most unwholesome trades, or than residence in an unwholesome house or district, where there has never been the most distant attempt at sanitary improvement of any kind." And this view has been very generally accepted, not only by the public, but by professed statisticians. Yet, as a matter of fact, we believe that no such inferences can legitimately be drawn from the above table. Dr. Stark appears to us to have fallen into the mistake, which M. Quetelet tells us is so common, of trying to make his statistics carry more weight than they are capable of bearing. It is important that the matter should be put in a just light, for the Royal Commission on the Law of Marriage has revealed no more striking fact than that of the prevalence of immature marriages, and such reasoning as Dr. Stark's certainly cannot tend to discourage these unwise alliances. If death strikes down in five years only half as many of those who are married as of those who are unmarried between the age of 20 and 25 (as appears from the above table), and if the proportion of deaths between the two classes goes on continually diminishing in each successive lustre (as is also shown by the above table), it seems reasonable to infer that the death-rate would be even more strikingly disproportionate in the case of persons between the ages of 15 and 20 than in the case of persons between the ages of 20 and 25. We believe, indeed, that if Dr. Stark had extended his table to include the former ages, the result would have been such as we have indicated. Yet few will suppose that such very youthful marriages can exercise so singularly beneficial an effect.

To many Dr. Stark's conclusion may appear to be a natural and obvious *sequitur* from the evidence upon which it is founded. Admitting the facts,—and we see no reason for doubting them,—it may appear at first sight that we are bound to accept the conclusion that matrimony is favourable to longevity. Yet the consideration of a few parallel cases will suffice to show how small a foundation the figures we have quoted supply for such a conclusion. What would be thought, for example, of any of the following inferences?—Among hothouse plants there is observed a greater variety and brilliancy of colour than among those which are kept in the open air, therefore the housing of plants conduces to the splendour of their colouring. Or, again—The average height of life-guardsmen is greater than that of

the rest of the male population, therefore to be a life-guardsman conduces to tallness of stature. Or, to take an example still more closely illustrative of Dr. Stark's reasoning—The average longevity of noblemen exceeds that of untitled persons, therefore, to have a title is conducive to longevity.

We know that the inference is absurd in each of the above instances, and we are able at once to show where the flaw in the reasoning lies. We know that splendid flowers are more commonly selected for housing, and that life-guardsmen are chosen for their tallness, so that we are prevented from falling into the mistake of ascribing splendour of colour in the one instance, or tallness in the other, to the influence of causes which have nothing whatever to do with those attributes; nor is any one likely to ascribe the longevity of our nobility to the possession of a title. Yet there is nothing in any one of the above inferences which is in reality more unsound than Dr. Stark's inference from the mortality bills, when the latter are considered with due reference to the principles of interpretation which statisticians are bound to follow.

The fact is, that in dealing with statistics, the utmost care is required in order that our inferences may not be pushed beyond the evidence afforded by our facts. In the present instance, we have simply to deal with the fact that the death-rate of unmarried men is higher than the death-rate of married men and widowers. From this fact we cannot reason, as Dr. Stark has done, to a *simple* conclusion. All that we can do is to show that one of *three* conclusions must be adopted. Either matrimony is favourable (directly or indirectly) to longevity, in a degree sufficient wholly to account for the observed peculiarity; or a principle of selection—the effect of which is such as, on the whole, to fill the ranks of married men from among the healthier and stronger portion of the community—operates in a sufficient degree to account wholly for the observed death-rates; or, lastly, the observed death-rates are due to the combination, in some unknown proportion, of the two causes just mentioned.

No reasonable doubt can exist, as it seems to us, that the third is the true conclusion to be drawn from the evidence supplied by the mortality bills. Unfortunately, the conclusion thus deduced is almost valueless, because we are left wholly in doubt as to the proportion which subsists between the effects to be ascribed to the two causes thus shown to be in operation. It scarcely required the evidence of statistics to prove that each cause must operate to some extent. It is perfectly obvious, on the one hand, that although hundreds of men who would be held by insurance companies to be "bad lives" may contract marriage, yet on the whole a principle of selection is in operation which must tend to bring the healthier portion of the male community into the ranks of the married, and to leave the unhealthier in the state of bachelorhood. A little consideration will show also that, on the whole, the members of the less healthy trades, very poor persons, habitual drunkards, and others whose prospects of long life are unfavourable, must (on the average of a large number) be more likely to remain unmarried than those more favourably situated. Another fact drawn from the Registrar-General's returns suffices to prove the influence of poverty on the marriage-rate. We refer to the

fact that marriages are invariably more numerous in seasons of prosperity than at other times. Improvident marriages are undoubtedly numerous, but prosperity and adversity *have* their influence, and that influence not unimportant, on the marriage returns. On the other hand, it is perfectly obvious that the life of a married man is likely to be more favourable to longevity than that of a bachelor. The mere fact that a man has a wife and family depending upon him will suffice to render him more careful of his health, less ready to undertake dangerous employments, and so on; and there are other reasons which will occur to everyone for considering the life of a married man better (in the sense of the insurance companies) than that of a bachelor. In fact, while we are compelled to reject Dr. Stark's statement above quoted, we may safely accept his opinion that statistics "prove the truth of one of the first natural laws revealed to man—'It is not good that man should live alone.'"*

Dr. William Robertson, Superintendent of Statistics in Registrar-General's Department, Scotland, has favored us with the following remarks on the subject of Dr. Stark's papers:—

It seems certain that during the two quinquenniads 30–35 and 35–40, the force of mortality is greater among the *un*married than among the married women of Scotland. At these periods of life the number of "primiperæ" is comparatively small, and therefore the risk from child-bearing causes is perhaps at a minimum among the married females. But a great many illegitimate children are borne at these periods by the unmarried, as may be seen by reference to Table XXV at page lxix of the second volume of the Scottish Census of 1871. I have an impression that these illegitimate births must be followed by a higher proportion of deaths among the mothers than would follow a like number of legitimate deliveries.

In explanation of the admittedly higher rate of mortality among married females who have arrived at the period of life when pregnancy is comparatively rare, I may refer to the doctrine, which I first learned from my late preceptor, Professor James Hamilton, of Edinburgh, and which I have likewise heard from Sir James Simpson, the late Dr. Samuel Pagan, and Dr. M. Duncan—namely, that such women as have borne a large number of children encounter considerable danger towards the end of their child-bearing life. Many of them are affected with "prolapsus uteri", with "pendulous belly", or with other local complaints that may interfere with the process of delivery. Not a few must be presumed to have suffered in strength in the course of repeated pregnancies and nursings, and to have become likely subjects for some of the diseases apt to prove fatal after parturition, as if precipitated by the process of

* It seems to us that the statistics give no more support to this unquestionably true statement, than they do to the very questionable statement as to the destructive tendencies of bachelorhood; in fact, they seem to give precisely the same support to the one as to the other.—Ed. *J.I.A.*

delivery. I have even heard it said, and certainly with some show of probability, that females who have had very many children are more apt than others to suffer from uterine hæmorrhage; because a large part of the whole internal surface of the uterus has at various times had placenta attached to it, and is likely to have suffered in consequence, giving rise to such serious conditions as "placenta prævia", "adherent placenta", &c., in after-pregnancies.

Dr. Stark's contributions to the statistics of the relative mortalities of married and of unmarried *men* are quite original, and certainly very curious. Of course, it is quite plain that a vast proportion of all males who in early life are crippled, maimed, infirm, lunatic, drunken, filthy, dissipated, extravagant, criminal, paupers or miserably poor, hideously unattractive, or who follow occupations hostile to matrimony,—such as the military profession, with peculiar risks to life,—must necessarily belong, for the most part, to the *un*-married class. And those with whom the business of life has been a failure, or whose physical or mental powers have suffered from laborious occupations during their early bachelor days, are not likely to become Benedicts in their later years, even were they so fortunate as to fall in with complaisant-enough Beatrices. Such "malheureux", of course, die at all ages in higher ratio than the married of corresponding ages.

Whether these and similar considerations suffice to explain the statistically-suggested advantages of matrimony, it would be impertinent in a bachelor dogmatically to assert. I have, however, a strong *impression* that they do not afford a sufficient explanation,—that the married life is conducive in a more direct way to longevity, were it only by the circumstance that it encourages and forms habits of regularity, providence, and striving after respectability—matters about which too many poor bachelors are not over-solicitous. Beyond all question (although there may be Lady Macbeths), the influence of the wife upon the husband is, as a general rule, *for his good;* and the affectionate care of a wife and family often supports a husband in sickness, or averts its invasion.

This tribute to the fair sex will be read with the greater interest as coming from a confirmed old bachelor.—Ed. *J.I.A.*

Married Women's Policies of Assurance (*Scotland*) *Act,* 1880.

43 & 44 Vict. Chapter 26.

An Act to extend to Scotland the Facilities for effecting Policies of Assurance for the Benefit of Married Women and Children now in force in England and Ireland. [26th August 1880.]

33 & 34 Vict. c. 93.

Whereas by the Married Women's Property Act, 1870, increased facilities are given for effecting policies of assurance for the benefit of married women and children in England and Ireland:

And whereas it is expedient that such increased facilities for effecting policies of assurance for the benefit of married women and children should be extended to Scotland:

Be it therefore enacted by the Queen's most Excellent Majesty, by and with the advice and consent of the Lords Spiritual and Temporal, and Commons, in this present Parliament assembled, and by the authority of the same, as follows:

Married woman may effect policy of assurance for her separate use.

1. A married woman may effect a policy of assurance, on her own life or on the life of her husband, for her separate use; and the same and all benefit thereof, if expressed to be for her separate use, shall, immediately on being so effected, vest in her, and shall be payable to her, and her heirs, executors, and assignees, excluding the jus mariti and right of administration of her husband, and shall be assignable by her either inter vivos or mortis causa without consent of her husband; and the contract in such policy shall be as valid and effectual as if made with an unmarried woman.

Policy of assurance may be effected in trust for wife and children.

2. A policy of assurance effected by any married man on his own life, and expressed upon the face of it to be for the benefit of his wife, or of his children, or of his wife and children, shall, together with all benefit thereof, be deemed a trust for the benefit of his wife for her separate use, or for the benefit of his children, or for the benefit of his wife and children; and such policy, immediately on its being so effected, shall vest in him and his legal representatives in trust for the purpose or purposes so expressed, or in any trustee nominated in the policy, or appointed by separate writing duly intimated to the assurance office, but in trust always as aforesaid, and shall not otherwise be subject to his control, or form part of his estate, or be liable to the diligence of his creditors, or be revocable as a donation, or reducible on any ground of excess or insolvency: And the receipt of such trustee for the sums secured by the policy, or for the value thereof, in whole or in part, shall be a sufficient and effectual discharge to the assurance office: Provided always, that if it shall be proved that the policy was effected and premiums thereon paid with intent to defraud creditors, or if the person upon whose life the policy is effected shall be made bankrupt within two years from the date of such policy, it shall be competent to the creditors to claim repayment of the premiums so paid from the trustee of the policy out of the proceeds thereof.

Application and short title of Act.

3. This Act shall apply only to Scotland, and may be cited as the Married Women's Policies of Assurance (Scotland) Act, 1880.

By this short Act, which was prepared and promoted by the Association of Managers of the Scottish Life Offices, and the

passing of which, in a peculiarly busy session of Parliament, is mainly due to the tact and perseverance of Mr. W. Holms, M.P. for Paisley, persons resident in Scotland obtain the facilities for effecting settlement policies which were granted by the Married Women's Property Act, 1870, to persons under English law.

We have been favored with the following notes by Mr. Wm. C. Smith, Advocate, Edinburgh, who was of counsel for the Offices in revising and settling the terms of the Act:—

The tendency of recent legislation has been to introduce to Scotland the whole provisions of the English Act (as in the Scotch Act of 1877 giving married women a right to personal earnings), but the present Act is very judiciously confined to the question of life assurance.

The first section is the less important. It professes to enable a married woman to effect a policy for her separate use. It seems probable that she could do this without the help of the Act in cases where she had separate estate from which she could pay the premiums. But this was at least doubtful, and it is now made certain by the Act, not only that she is capable of making such a contract without consent of her husband, but that the sum due under a policy for separate use, belongs to the married woman or her estate. In other cases, where the married woman has no separate estate, it is unlikely that the first section of the Act will frequently be made use of, because the husband will more probably avail himself of the provisions of the second section. If a policy were effected in the name of the wife under the first section, the husband supplying the premiums, it may be that, as under the second section, the husband would have no power to revoke; but this is not certain.

The second section is the really important one, and will probably be largely used. It provides for such a form of policy as will constitute an irrevocable trust in favour of the family of the assured. The wife ought to be named and the children named or described in the policy; and if that is done, no question can rise, as has been suggested, with regard to the right of a second wife or a second family to take the proceeds of the policy. Of course, if wife and children pre-decease the assured, the right will return to him, and he may assign it or dispose of it by will. Where a trustee is not appointed, the money is held in trust by the legal representative free from the claims of creditors. This will no doubt prevent in future the occurrence of many

serious cases of disappointment, in which parties have trusted to the common law relating to reasonable provisions given by husbands to wives and families, and in which a sequestration has left the wife and family quite unprovided for. The appointment of a trustee, if not in the policy, requires to be intimated to the assurance office. The Act does not state what "due" intimation means, but it would have been impossible, by laying down an invariable rule on this subject, to prevent questions arising as to whether or not the legal representative should join in the discharge by the trustee. The section is framed so as to protect the policy against the Scotch law of bankruptcy and of donations between spouses. In the excepted cases, namely, (1) where there has been intent to defraud creditors, and (2) where bankruptcy occurs within two years from the date of the policy, the proceeds of the policy do not belong to the creditors of the bankrupt or fraudulent insurer; but these are merely entitled to claim the repayment of the premiums.

The Act makes no provision for the substitution of new policies for those already existing, the holders of which desire to bring them under the provisions of the Act. The most obvious and safest method of such substitution would no doubt be the issue of new policies, though it is quite possible that an indorsement or memorandum on an existing policy would have the same legal effect. But it would of course be impossible, by any such endorsement, to prevent questions arising with regard to rights of creditors accrued before the passing of the Act.

The following remarks on the subject appeared in the *Scotsman* of 30 August last.

The really important feature of the new legislation is, that the assurances made in favour of the family will not be liable to be taken back by the caprice of the husband, and should he become bankrupt or insolvent, his creditors will be unable to claim any interest in the policy, unless, indeed, it should appear that the assurance has been made for the purpose of defrauding creditors. Formerly, it too often happened that a sequestration carried off the little fund to which a man was trusting for the protection of his wife and children in evil times; and even where the whole fund was not swept away, a Court of Justice might be asked to say how much of it was a reasonable provision to the family, and how much was in excess or irrational, and must therefore go to the creditors. The Act will make it possible to escape these unpleasant contingencies

by very simple means, and it will thus give peace and security to domestic arrangements.

The question of the exclusion of the husband's right to recall a gift which he has made in this way to his wife is a more delicate one. The existing law of Scotland on the subject is, that all gifts by one spouse to the other may be recalled. The reasons given for this law are of a strange and contradictory character. The first reason is romantic—"lest either of the two spouses should, by ill-judged testimonies of their affection, undo themselves or their families," so that, apparently, it is necessary to discourage such gifts. The second reason is cynical—namely, that if one spouse asked the other for a gift and were refused, this refusal would "stop and quench mutual love between them;" so that such gifts are not prohibited, but permitted in a worthless form, or under conditions that may destroy their value. There is not much danger now-a-days of ruinous profusion between spouses, and it is, perhaps, better that a man should make up his mind once for all what he is going to do with his money. Whether or not the general principles of the old law will stand the test of modern criticism, the new Act seems to provide one simple means by which, if used, a man desiring to make a safe provision for his family may so far protect himself and them against his own caprice. No doubt, from the peculiar nature of an assurance policy, the provision may be destroyed if the husband ceases to pay the premium. But even here, if the Act is effectual to any extent whatever, the surrender or purchase value of the policy would belong to the wife and children.

HOME AND FOREIGN INTELLIGENCE.

The Madras Equitable Assurance Society.

HAVING received from Mr. A. H. Bailey some of the publications of this Society, we extract therefrom the following information, which we believe will be interesting to our readers.

The Society is mutual, the members being all persons who hold policies of assurance, either on their own lives or on the lives of other parties, *for the duration of life.*

Claims are paid immediately on the casualty being proved to the satisfaction of the directors. An official notice of death in any of Her Majesty's Government Gazettes is a sufficient proof of death.

Life policies can be surrendered when three years' premiums have been paid.

Premiums can be paid in London at a fixed exchange of 2*s.* per rupee, and payment of death claims received there at the

current exchange on Madras. Renewal premiums are payable half-yearly, on 1 January and 1 July.

The funds of the Society not required for current purposes must be invested in securities of the Government of India, or in securities the interest on which is guaranteed by the Government of India or by the Secretary of State for India in Council.

Assurances may be granted on the lives of Europeans, East Indians, Eurasians, and Parsees.

No policy is issued for a less sum than £50,* or for a larger sum than £3,000, on any one life.

The following reduced scale of whole-term premiums came into force on 1 January 1871. No provision appears to be made for reduction of the premium in the event of the life assured residing permanently in a temperate climate.

Table of Yearly Premium of Assurance of One Thousand Rupees, payable Half-yearly, on 1*st January and* 1*st July, for Life, with participation in Surpluses.*

Age next Birthday.	Civil.	Military and Naval.	Age next Birthday.	Civil.	Military and Naval.	Age next Birthday.	Civil.	Military and Naval.
	Rs.	Rs.		Rs.	Rs.		Rs.	Rs.
18	38	43	33	45	50	47	59	64
19	38	43	34	45	50	48	60	65
20	38	43	35	46	51	49	61	66
21	39	44	36	47	52	50	63	68
22	39	44	37	48	53	51	64	69
23	39	44	38	48	53	52	66	71
24	40	45	39	49	54	53	68	73
25	40	45	40	50	55	54	70	75
26	41	46	41	51	56	55	72	77
27	41	46	42	52	57	56	73	78
28	42	47	43	54	59	57	76	81
29	42	47	44	55	60	58	79	84
30	43	48	45	56	61	59	82	87
31	43	48	46	57	62	60	85	90
32	44	49						

Policies are also granted for terms of years without participation in surpluses.

The following table is taken from the report for 1880:—

* In this reprint rupees have been converted into pounds sterling at the rate of 2*s*. per rupee.

Abstract of the Business of 1880.

Class of Risks.	Policies subsisting 31 December 1879.			Granted.			Revived.			Voided.			Policies subsisting 31 December 1880.		
	Policies.	Lives.	£	Policies.	Lives.	£	Policies.	Lives.	£	Policies.	Lives.	£	Policies.	Lives.	£
Life, Fixed Rates of Premiums .	637	...	340,840	29	...	17,300	2	...	900	35	...	14,960	633	...	344,080
Life, Annually increasing Rates .	45	...	16,017	...	...	None	...	...	None	1	...	320	44	...	15,697
Terms of Years .	15	...	10,300	...	...	...	...	...	None	5	...	4,750	10	...	5,550
	697	490	367,157	29	24	17,300	2	2	900	41	25	20,030	687	491	365,327

It further appears from the report that the voided policies included—

Claims, £7,080, assured by 20 policies on 11 lives; surrenders, £4,600; lapses, £4,850; term expired, £3,500.

From the same report it appears that the total expenses are 10·66 per-cent of the gross premiums, the principal item being a payment of 5 per-cent on the gross premiums to the secretaries and treasurers for office allowance and commission.

Quinquennial valuations are to be made by the actuary of an assurance office in London, having a duration of not less than 20 years.

The two last valuations, for the quinquennial periods ending 31 December 1875 and 1880, have been made by Mr. Bailey.

On 10 May 1876 he reported as follows:—

Since the last quinquennial valuation of your Society was made, much discussion has taken place on the subject of the mortality of Europeans in India; indeed it may be said that a controversy has arisen on the question. On the one hand it has been asserted with some confidence, but with no evidence in support of the assertion, that there is no material difference between the mortality of Europe and India. To this it was answered that the carefully compiled statistics of the Funds of the East India Company, prove the contrary; showing, as they do, that the rate of mortality among Indian civilians has been much greater than that experienced by the Life Assurance Companies of Great Britain, and that the excess is still more marked among the Military Officers. To this it was rejoined that the experience of these Funds is spread over a long period of time, going back, in some cases, to last century; that a great change in the habits of Europeans in India has taken place of

late years; and that therefore the experience of past generations is inapplicable at the present time. It cannot be denied that these considerations are entitled to due weight; the evidence certainly goes to prove that an improvement has taken place in the mortality of Europeans in India. I think it is well worth your while to consider whether the experience of your own Society should not be ascertained. Your records must contain a number of facts which would throw some light upon the question.

Under these circumstances it seemed to me that at all events the time had arrived for dispensing with the table of mortality used in the three last valuations; which was based on a combination of the experience of the Bengal and Madras Military Funds, and substituting more recent intelligence. After a good deal of enquiry and consideration, I came to the conclusion that the mortality experience of the adult male subscribers to the Uncovenanted Service Fund of Bengal, which was published by Mr. A. J. Finlaison in 1874, would afford the best available basis for the valuation of the liabilities of your Society. The period embraced in these observations extends from 1837 to 1872; the number of subscribers was 1,964, all of whom underwent medical examination before admission. Of these 436 died, 537 quitted the Society, and 991 were alive at the close of the observations. It will be observed that this is the experience of recent times; the numbers are, I think, sufficient for the purpose; and it may be reasonably considered that the mortality among such a body as this would fairly represent that likely to prevail in your Society; and therefore, with some exceptions at particular periods of life where, the numbers being small, other statistics were made available, this experience has been made the basis of the present valuation.

The next question is as to the rate of interest to be adopted, your remarks on which have been well considered. The laws of your Society require that its funds should be invested in Indian Government Securities exclusively, and these, as you point out, yield at the present time a rate of interest slightly under 4 per-cent. The Life Assurance Companies of this country for many years past have succeeded in obtaining a higher rate of interest on their funds than 4 per-cent; $4\frac{1}{2}$ per-cent is probably about the average, and it is not easy to understand why the rate of interest should be lower in India than in England. The explanation is, that here a small portion only of the assets of life assurance companies is invested in Government securities. It is well understood that one great advantage of such securities, their ready convertibility, the life assurance companies do not want. Unlike banks and other commercial undertakings, they are never exposed to sudden demands upon their resources; and therefore the greater part of their assets may be safely invested for long periods of time; and in life assurance finance the importance of improving the funds to the best advantage is so great, that I cannot help suggesting for your consideration the expediency of modifying this rule regarding investments. Meantime it may be pointed out that your Society derives, and for some time will continue to derive, a special advantage in the matter of interest, inasmuch as the General Fund has credit for the interest

not only of its own investments, but also for those of the Guarantee Fund. The effect of this was last year that the interest exceeded 5 per-cent of the amount of the General Fund. So that, taking into account your past experience, present circumstances, and future prospects, it seems to me that 4 per-cent is the proper rate of interest at which the liabilities should be valued; and that to adopt a lower rate would be both unnecessary and undesirable. I have therefore prepared for the present valuation an entirely new set of monetary tables, based on the mortality experience already described and at 4 per-cent interest.

With this preliminary explanation I proceed to the actual affairs of the Society.

On 31 December 1870, the date of the last valuation, the amount of the General Fund was		£88,359·4
Add receipts during the quinquennium from gross premiums	£97,701·8	
Interest less reduction in value of securities .	25,790·7	
Guarantee Fund	440·8	
		123,933·3
		212,292·7
Deduct for—		
Claims	49,905·0	
Surrenders	1,180·6	
Reductions of premium	34,928·3	
Expenses	11,171·7	
		97,185·7
General Fund 31 December 1875		£115,107·0

The liabilities under policies in force at the same date were as follows:—

Number.	Class of Risk.	Sum Assured.	Premium.
		£	£
579	Life, fixed rate . .	314,090·0	16,731·6
59	Life, annually increasing	22,837·0	2,321·8
28	Terms of years . .	16,990·0	646·9
666	Total . . .	353,917·0	19,700·3

For each of these policies a separate calculation was first made, of the premium which, according to the rates of mortality and interest assumed for the valuation, would be just sufficient for the risk.

The difference between the premiums payable and the premiums thus computed, sometimes called the "loading", was then ascertained, the present value computed, and the amount thus determined (which is equal to 33·7 per-cent of the gross premiums) reserved to provide for future expenses, profits, and contingencies. In this way care has been taken not to anticipate future profits.

Particulars of the valuation can be given in any detail that may

be desired; the following summary of the general result will probably however be sufficient :—

Value of sums assured whole life fixed premium . . .		£175,026·5
Value of sums assured annually increasing premium . .		15,411·0
		190,437·5
Deduct—		
Value of future premiums whole life fixed premium	£188,104·8	
Value of future premiums annually increasing	20,960·3	
	209,065·1	
Less value of loading	73,767·7	
		135,297·4
		55,140·1
Reserve for short term policies		323·5
Total estimated liability		£55,463·6

and the General Fund being £115,107·0, the amount of the apparent surplus is £59,643·4.

Of this sum I recommend that £51,052·2 should be appropriated for division on the present occasion. This would be equivalent to 65 per-cent on £78,540·8, which has been the amount paid in premium during the quinquennium on the participating policies in force at its close, and would yield an abatement of premium for the current five years at the rate of 65 per-cent for policies of five years' standing, with a lesser proportionate abatement for the more recent policies. In making this recommendation I do not overlook the circumstance that a balance remains in hand out of the sum appropriated for division five years ago. This is a necessary consequence of the method of dividing the surplus prescribed by the Regulating Act, and which was deliberately adopted as a measure of precaution. It must not be forgotten that some portion of the surplus arises from the Guarantee Fund, a source of profit which is exceptional and gradually wearing out. And in a Mutual Society it is prudent to leave some profit unappropriated, there being no proprietary capital to fall back upon in case of need.

And on 20 April 1881 he reported :—

Not much addition has been made during the last five years to the general stock of information on the subject of the mortality of Europeans in India; but the question is by no means free from difficulty. The results of different investigations, all based upon apparently trustworthy statistics, exhibit some inconsistencies which are not easy to explain. And while no one now contends that the rates of mortality in India and England are identical, the general feeling seems to be among those who pay attention to these subjects that the time has not yet arrived for forming an entirely trustworthy table of Anglo-Indian mortality. Under these circumstances I turned to the facts derived in 1877 from the experience of your own Society. Valuable and interesting as these materials are, the facts in my judg-

ment are too few for the formation of monetary tables to be used for a valuation. But taking the interval between the ages of 20 and 50, probably the chief period of life in which Europeans reside in India, I find that the general rate of mortality experienced in your Society, coincides almost exactly with that of the Uncovenanted Service Fund, which formed the basis of the table of mortality adopted for the last valuation. I have therefore decided to use the same table on the present occasion.

The rate of interest realized by the General Fund during the last five years has been as follows:—

In 1876	. . .	5·308	or very nearly	$5\frac{5}{16}$ per-cent
„ 1877	. . .	5·089	„	$5\frac{1}{16}$ „
„ 1878	. . .	5·537	„	$5\frac{1}{2}$ „
„ 1879	. . .	4·780	„	$4\frac{3}{4}$ „
„ 1880	. . .	4·896	„	$4\frac{7}{8}$ „

Part of this arises from the interest of the Guarantee Fund, which is slowly but steadily diminishing, for which the General Fund has credit. On this account therefore the rate must be expected to diminish somewhat in future. But although opinions differ as to the general rate of interest that may be expected to prevail in future, it seems to me that in the circumstances of your Society, there can be no reason for anticipating that the rate will fall below 4 per-cent in any period of time to which it is needful to look forward. And accordingly 4 per-cent is the rate that has been assumed in the valuation.

By the abstract of the accounts of the General Fund for the five years under review with which you have furnished me, it appears that during that period the annual premiums have averaged £20,502·2, and the annual claims £11,190. The percentage of the expenses of management on the premium income has been 11·2; and the General Fund has increased from £115,107 on 31 December 1875 to £138,889·4 on 31 December 1880.

The liabilities at the latter date were as follows:—

Number of Policies.	Class.	Sum assured.	Annual premium.
633	Life, fixed rate	£344,080	£18,155·7
44	Life, annually increasing	15,697	1,806·6
10	Terms of years	5,550	218·3
687		£365,327	£20,180·6

A valuation of these liabilities has been made in the manner explained in the Report of 1876.

For the whole life policies at a fixed rate of premium, which comprise upwards of 94 per-cent of the whole amount of the risks, a separate calculation has been made for each policy of the amount of premium which would suffice to provide for the risks according to the rates of mortality and interest adopted. The total of these amounts is £11,987·5. Deducting this from the premiums actually payable, a margin is left of £6,168·2, very nearly 34 per-cent, the present value of which has been reserved for future expenses, contingencies, and surplus.

The following is the result of the valuation :—

Whole life fixed premium—		
Value of sums assured		£196,790·4
Value of future premiums . . .	£197,311·0	
Deduct value of margin . . .	70,590·6	
		126,720·4
Amount to be reserved		70,070·0
Reserve for annually increasing policies . . .		903·3
„ „ Policies for terms of years . . .		109·1
Total		£71,082·4

and the amount of the General Fund being £138,889·4, there is an available surplus of £67,807·0.

I recommend that the amount appropriated on the present occasion should be £60,424·2. This will provide an abatement of premium for the current quinquennium of 70 per-cent for policies of five years' standing, with corresponding reductions for policies of more recent date. Provision will thus be made for any lapsed policies that may be reinstated and for other contingencies; but the chief reason for the recommendation is on account of the exceptional item of £5,214·0 by which the surplus is increased, arising from the increase in the quinquennium in the value of Government securities. You have resolved that the whole of the Society's holdings in Government paper shall in future be in 4 per-cents; that the balance at the debit of the account shall be brought forward without alteration from year to year; and that at the end of each quinquennium the securities shall be valued at a price based on the current market rates. These arrangements seem convenient; but in fixing the price, some caution is, I think, necessary. It must not be forgotten that in the business of life assurance, the securities have to be realized in various states of the money market, the transactions extending over a considerable period of time. It is worth considering, therefore, whether a price representing the average of a few years is not a better exponent of value than the accidental price of the particular day which closed the quinquennium. And at all events, as this particular item of £5,214·0 may have to be written back again on a future occasion, I think it undesirable to regard it as divisible surplus.

It may be interesting for you to know that the ages of the lives assured are thus distributed:

Age.	Sum assured.
24 to 29	£13,900
30 „ 39	78,140
40 „ 49	119,820
50 „ 59	82,240
60 „ 69	50,217
70 „ 79	20,210
80	800
	£365,327

Life Insurance in New York in 1880.

From the chart issued by the New York Insurance Department, under date of April 15, the *Chronicle* (New York) of 21 April 1881 condenses the figures, which we have converted into English money at the rate of 5 dollars to the £.

Name of Company.	Assets.	Liabilities.	Income.	Expenditures. To Policy-holders.	Expenditures. Dividends to Stockhold'rs.	Expenditures. All Other Payments.	Expenditures. Total.
NEW YORK COMPANIES.							
Brooklyn	£326,028	£277,645	£59,646	£59,121	£2,500	£19,367	£80,988
Equitable	8,141,280	6,356,821	1,747,140	958,588	1,400	301,383	1,261,371
Germania	1,800,631	1,553,819	337,789	197,638	4,800	64,044	266,482
Home	984,227	706,837	134,956	101,428	3,000	24,510	128,938
Homœopathic	129,891	107,293	29,530	18,840	...	6,199	25,039
Manhattan	2,030,258	1,624,259	324,784	252,193	8,000	50,446	310,639
Metropolitan	389,564	317,677	138,111	77,471	1,400	71,321	150,192
Mutual	18,305,931	15,939,696	3,428,139	2,632,138	...	523,455	3,155,593
New York	8,606,228	6,755,715	1,764,834	899,979	...	243,596	1,143,575
Prov. Sav. L. A. Soc.	30,691	10,402	13,970	4,893	...	8,047	12,940
United States	1,021,590	845,216	180,551	98,549	5,925	51,370	155,844
Washington	1,163,196	976,186	268,298	186,293	1,785	48,226	236,304
Totals	£42,929,515	£35,471,566	£8,427,748	£5,487,131	£28,810	£1,411,964	£6,927,905
Totals 1879 (12 companies)	40,512,008	33,918,260	8,231,682	5,791,832	27,213	1,198,197	7,017,242
COMPANIES OF OTHER STATES.							
Etna, Conn.	£5,265,454	£4,187,899	£790,777	£535,639	£15,000	£94,591	£645,230
Berkshire, Mass.	702,236	602,829	116,410	74,875	357	21,488	96,720
Connecticut General, Conn.	262,707	194,957	41,086	21,579	3,198	9,310	34,087
Connecticut Mutual, Conn.	9,886,239	8,599,831	1,630,012	1,282,739	...	203,796	1,486,535
John Hancock, Mass.	514,219	454,796	99,771	76,726	...	42,869	119,595
Massachusetts Mutual, Mass.	1,398,349	1,146,754	231,739	140,069	...	50,249	190,318
Mutual Benefit, N.J.	7,145,363	5,949,152	1,160,296	957,477	...	168,921	1,126,398
National, Vt.	477,347	328,380	76,281	37,985	...	10,624	48,609
National of U.S. of A., Ill.	748,711	588,481	115,777	101,329	14,000	25,202	140,531
New England Mutual, Mass.	3,184,586	2,487,947	489,345	380,161	...	63,009	443,170
North-Western Mutual, Wis.	3,659,066	2,846,619	628,992	436,102	...	100,243	536,345
Penn. Mutual, Penn.	1,486,250	1,162,725	291,952	179,415	...	47,493	226,908
Phœnix Mutual, Conn.	2,122,228	1,891,570	320,557	264,802	1,200	74,616	340,618
Provident Life and Trust, Pa.	1,074,685	850,985	232,521	94,437	...	36,600	131,037
State Mutual, Mass.	582,896	416,599	85,415	42,504	...	10,939	53,443
Travelers', Conn.	808,962	642,110	135,842	50,904	...	22,744	73,648
Union Central, Ohio	317,693	257,312	89,649	44,285	2,000	24,724	71,009
Union Mutual, Me.	1,324,167	1,197,356	222,050	213,431	...	58,746	272,177
Totals other-state companies	£40,961,158	£33,806,302	£6,758,472	£4,934,459	£35,755	£1,066,164	£6,036,378
Totals 1879 (19 companies)	39,793,404	33,316,581	7,002,509	5,358,033	28,439	976,355	6,362,827

Exhibit of Policies.

Name of Company.	Policies in force 31 Dec. 1879.	Policies in force 31 Dec. 1880.	Insurance in force 31 Dec. 1879.	Insurance in force 31 Dec. 1880.
NEW YORK COMPANIES.				
Brooklyn	3,258	3,259	£1,263,293	£1,223,137
Equitable	48,048	52,272	32,471,543	35,519,541
Germania	19,657	20,189	6,539,199	6,777,104
Home	7,424	7,467	2,861,693	2,869,613
Homœopathic	2,261	2,331	657,853	630,119
Manhattan	11,416	11,351	6,666,524	6,521,625
Metropolitan	7,680	6,895	2,230,070	2,012,696
Mutual	95,423	97,978	59,752,173	61,200,433
New York	45,705	48,548	25,483,552	27,145,383
Provident Savings	1,077	2,243	441,406	585,542
United States	9,711	9,428	3,472,541	3,371,353
Washington	10,139	11,076	4,289,455	4,690,354
Totals	261,799	273,037	£146,129,302	£152,546,900
Totals 1879 (12 companies)	258,706	261,799	144,348,978	146,129,700
COMPANIES OF OTHER STATES.				
Etna, Conn.	56,252	56,651	£15,547,608	£15,590,364
Berkshire, Mass.	5,373	5,582	2,291,727	2,386,006
Connecticut General, Conn.	3,417	3,530	1,067,350	1,079,989
Connecticut Mutual, Conn.	64,504	64,343	32,917,025	32,421,073
John Hancock Mutual, Mass.	6,365	6,039	2,384,543	2,195,717
Massachusetts Mutual, Mass.	13,065	13,175	5,755,429	5,855,138
Mutual Benefit, N.J.	42,286	44,350	23,544,049	24,293,396
National, Vt.	4,032	4,295	1,661,210	1,743,489
National (U.S.A.) Ill.	7,931	6,994	2,854,430	2,348,985
New England Mutual, Mass.	18,750	18,967	11,007,897	11,229,725
North-Western Mutual, Wis.	33,066	34,172	12,389,778	12,993,416
Penn. Mutual, Penn.	11,189	12,234	5,935,607	6,321,713
Phœnix Mutual, Conn.	22,672	21,544	7,017,710	6,523,217
Provident Life and Trust, Penn.	7,357	8,202	4,497,259	5,151,090
State Mutual, Mass.	4,650	4,731	1,947,302	2,022,587
Travelers', Conn.	11,352	11,914	3,636,426	3,819,728
Union Central, Ohio	4,276	4,801	1,493,224	1,669,130
Union Mutual, Me.	14,915	14,120	5,339,474	5,007,372
Totals other-state companies	331,452	335,644	£141,288,048	£142,652,135
Totals 1879 (19 companies)	337,042	333,687	145,391,503	141,862,533

INDUSTRIAL POLICIES IN FORCE NOT INCLUDED ABOVE.

	Number.	Amount.
Metropolitan, New York	110,193	£1,820,774
Prudential, Newark	87,462	1,469,578
John Hancock, Boston	30,702	627,804
Germania, New York	7,841	178,099
Provident Savings, New York	476	10,439
Totals	236,674	£4,106,694

CORRESPONDENCE.

ON THE RELATION BETWEEN THE HEIGHT AND WEIGHT OF MEN.

To the Editor of the Journal of the Institute of Actuaries.

Sir,—The following statistics regarding the weight of the human body, and the effect which various influences have on it, may be interesting to your readers. They are founded on 2,000 cases taken from the records of a Canadian life company. This number was chosen as sufficient to give reliable results, without increasing needlessly the amount of labour.

Although I am acquainted with several tables purporting to show the average weight corresponding to various heights, I have been unable to find on what basis any one of them rests. Some differ very greatly from others. It thus became a question by what table should a company be guided. To solve the matter I made an investigation, and give herewith the results, with very slight adjustment. Only healthy English-speaking lives were included.

Average Weight at various Heights.

Height.	Weight.			Height.	Weight.		
	lbs.	st.	lbs.		lbs.	st.	lbs.
5 ft. 1 in.	125·	= 8	13·	5 ft. 7 in.	145·5	=10	5·5
5 „ 2 „	128·	= 9	2·	5 „ 8 „	151·	=10	11·
5 „ 3 „	131·	= 9	5·	5 „ 9 „	156·5	=11	2·5
5 „ 4 „	134·	= 9	8·	5 „ 10 „	161·5	=11	7·5
5 „ 5 „	137·5	= 9	11·5	5 „ 11 „	167·	=11	13·
5 „ 6 „	141·	=10	1·	6 „ 0 „	173·	=12	5·

The average height of adult Canadians of British extraction was found to be 5 feet 8·6 inches, and their average weight 155·0 lbs. The French cases were kept separate, and their average height was found to be 5 feet 7·3 inches, and their weight 149·9 lbs. French Canadians are, therefore, as a rule, about one and one-third inches under the English standard in height, and five pounds under it in weight. They are, however, generally heavier than English people of the same height.

The declined lives were also kept separate. Taking into consideration only those cases in which the cause of rejection was a tendency to lung disease of any kind, whether manifested in the applicant personally or in his family history, it was seen that, although their average height was exactly the same as that of the healthy cases, their average weight was only 147·6 lbs.—about seven and one-half pounds under the healthy standard. Some such result was expected, and it but confirms the rule that light weight generally accompanies a consumptive tendency. The other rejected cases exhibited nothing remarkable.

The influence of age is considerable, as may be seen from the following table:—

Weight at various Ages.

Ages.	Weight.	Ages.	Weight.
	lbs. st. lbs.		lbs. st. lbs.
16 to 20	142·5 = 10 2·5	41 to 45	159·2 = 11 5·2
21 „ 25	149·6 = 10 9·6	46 „ 50	163·5 = 11 9·5
26 „ 30	151·3 = 10 11·3	51 „ 55	167·7 = 11 13·7
31 „ 35	157·3 = 11 3·3	56 „ 60	172·4 = 12 4·4
36 „ 40	158·2 = 11 4·2		

The weight of healthy persons thus increases about thirty pounds in forty years, or about three-fourths of a pound for each year. By far the largest increase is at the younger ages. Whether this rule of increase holds good much after sixty, I have no means of knowing. It is, however, evident that a young man may be considerably under the tabular weight for his height and still be perfectly healthy, while the same variation, in an elderly man, would be very suspicious.

We have already seen that a consumptive tendency and a spare habit of body, generally go hand in hand. As weight is found to increase with age, we may, perhaps, infer that the liability to consumption is less among lives selected late in life, than among those entering at an early age, although the deaths from that disease are pretty evenly distributed over the ages, among assured lives as a class.

The influence of occupation is next to be considered. The following table shows the main results on this score:—

Average Weight of Persons engaged in various Occupations.

Occupation.	Weight.	Occupation.	Weight.
	lbs. st. lbs.		lbs. st. lbs.
Agents, Brokers, &c. .	156·2 = 11 2·2	Founders and Moulders .	151·3 = 10 11·3
Butchers . . .	169·3 = 12 1·3	Hotel-keepers, &c. . .	166·7 = 11 12·7
Barristers . . .	160·1 = 11 6·1	Labourers . . .	148·3 = 10 8·3
Bankers, Ins. Managers, &c.	154·2 = 11 0·2	Merchants . . .	153·7 = 10 13·7
Blacksmiths . . .	155·4 = 11 1·4	Machinists . . .	153·5 = 10 13·5
Clerks and Salesmen .	147·1 = 10 7·1	Masons, Bricklayers, &c.	162·2 = 11 8·2
Carpenters, Coopers, &c. .	152·6 = 10 12·6	Manufacturers . .	151·1 = 10 11·1
Commercial Travellers .	158·1 = 11 4·1	Shoemakers and Saddlers	148·0 = 10 8·
Clergymen . . .	163·7 = 11 9·7	Teachers	151·4 = 10 11·4
Druggists . . .	148·1 = 10 8·1	Tailors	145·1 = 10 5·1
Doctors	160·6 = 11 6·6		
Farmers	160·9 = 11 6·9	TOTAL . . .	155·0 = 11 1·

As these statistics are based exclusively on Canadian data, they may be compared with those of other countries, and I will be much

pleased to see this done. It must be remembered, however, that the average age at entry should be taken into consideration. In the company referred to it is 28 years.

Yours truly,

Montreal, T. B. MACAULAY.
21 *October* 1880.

P.S.—Since writing the above, I have noticed in Dr. Allen's *Examinations in Life Insurance*, a statement to the effect that the average height of Continental Europeans is a little less than 5 feet 6 inches; that of emigrants from Great Britain, about 5 feet 7 inches; and that of Americans, about 5 feet 8 inches. We have seen that the average of English-speaking Canadians is over 5 feet $8\frac{1}{2}$ inches. If Dr. Allen is correct, he strongly confirms the opinion I have always held, that Canadians are a hardier, better developed, and more muscular race of men than can be found in almost any other country.

A valued correspondent writes to us on the subject of Mr. Macaulay's letter:—

The circumstances of age, occupation, race, season, &c., are so various, that tables constructed like Macaulay's, Quetelet's, Hutchinson's, and others, on the basis of observations made on a particular class of men, must be expected to give discordant results.

The table [printed below] which I am accustomed to regard as *the model* for men at the age of (say) 30 years, is constructed on the principles laid down by Mr. Brent, and explained many years ago, in a work on "Corpulence", by the late Dr. Chambers, of London. It assumes that the proportions of an ancient statue (say the Fighting Gladiator) are the correct thing, that the specific gravity of the human body is the same as that of water, and that the cubes of homologous dimensions are in the proportion of the weights; and six per-cent for clothes is added to the weight calculated for the displacement of water by a body built on such a model. The resulting formula is $W=\frac{h^3}{2{,}000}$, where h is the height in inches and W the weight in pounds.

When a man's weight exceeds or falls short of the standard according to the table, by (say) 15 per-cent, I consider he is not a well-proportioned man, and in case of his life being proposed for assurance, such divergence from the standard of ideal perfection should be seriously considered as a possible ground for the application of extra premium.

Table of Proportions between Height and Weight of Healthy Men.

Height.	Weight. Model.		−Weight. ⅙th too little.		+Weight. ⅙th too much.		Height.
Inches.	lbs.	st. lbs.	lbs.	st. lbs.	lbs.	st. lbs.	ft. in.
63·	125	8 13	104	7 6	146	10 6	5 3
63·5	128	9 2	107	7 9	149	10 9	5 3·5
64	131	9 5	109	7 11	153	10 13	5 4
64·5	134	9 8	112	8 0	156	11 2	5 4·5
65	137	9 11	114	8 2	160	11 6	5 5
65·5	141	10 1	117	8 5	165	11 11	5 5·5
66	144	10 4	120	8 8	168	12 0	5 6
66·5	147	10 7	122	8 10	172	12 4	5 6·5
67	150	10 10	125	8 13	175	12 7	5 7
67·5	154	11 0	128	9 2	180	12 12	5 7·5
68	157	11 3	131	9 5	183	13 1	5 8
68·5	161	11 7	134	9 8	188	13 6	5 8·5
69	164	11 10	137	9 11	191	13 9	5 9
69·5	168	12 0	140	10 0	196	14 0	5 9·5
70	172	12 4	143	10 3	201	14 5	5 10
70·5	176	12 8	147	10 7	205	14 9	5 10·5
71	179	12 11	149	10 9	209	14 13	5 11
71·5	183	13 1	152	10 12	214	15 4	5 11·5
72	187	13 5	156	11 2	218	15 8	6 0
72·5	191	13 9	159	11 5	223	15 13	6 0·5
73	195	13 13	162	11 8	228	16 4	6 1
73·5	199	14 3	166	11 12	232	16 8	6 1·5
74	203	14 7	169	12 1	237	16 13	6 2
74·5	207	14 11	172	12 4	241	17 3	6 2·5
75	211	15 1	176	12 8	246	17 8	6 3
75·5	215	15 5	179	12 11	251	17 13	6 3·5
76	219	15 9	182	13 0	256	18 4	6 4

We are inclined to question the correctness of the principle adopted in the calculation of this "model" weight, that the bulk and weight of a man increase as the cube of the height, or that in well-built men of all heights, the breadth is proportional to the height. We believe that tall men are generally slighter, and short men stouter, in proportion, than men of middle height; and a comparison of Mr. Macaulay's figures with those in the other table supports this opinion. Mr. Hutchinson's table is printed in vol. i, p. 88, of this *Journal,* in an extract from Dr. Chambers's work on Corpulence.—Ed. *J.I.A.*

INSTITUTE OF ACTUARIES.

PRELIMINARY EXAMINATION, 1881.

Examiners.—Messrs. T. G. ACKLAND, J. HERON DUNCAN, W. T. GRAY, and W. VAUGHAN.

I.

1. Describe a Trial Balance, and explain its uses. Would you take out the gross debit and credit postings, or only the balances, and why?

2. An Assurance Company purchases an annuity-certain of £1,331, payable at the end of each year for 3 years, and pays £3,310 cash for it, thus securing exactly 10 per-cent on the outlay. What Ledger accounts should be debited and credited at the time of purchase, and at the close of each year?

3. State and prove a rule for the division of decimals. When a vulgar fraction is converted into a decimal, in what cases will a terminating decimal be obtained?

4. Find the sum of 2·024 of 7*s.* 4*d.*, ·128 of 12*s.* 9*d.*, $1{\cdot}02\dot{3}$ of £1·28, and $\cdot\dot{4}2857\dot{1}$ of 7 guineas.

5. Find, by "contracted multiplication", the fifth power of ·12345, rejecting millionths and all inferior denominations throughout the solution.

6. Solve *any two* of the following equations:—

$$(\alpha)\quad \left.\begin{aligned} \frac{x}{a}-\frac{y}{b}&=1+\frac{a^2}{b^2} \\ \frac{x}{b}+\frac{y}{a}&=1+\frac{b^2}{a^2} \end{aligned}\right\};\qquad (\gamma)\quad \frac{x+\dfrac{1}{x}-1}{x-\dfrac{1}{x}+1}=1-\left(x-\frac{1}{x}\right);$$

$$(\beta)\quad x+y=\frac{1}{x}+\frac{1}{y}=\frac{5}{2};\qquad (\delta)\quad (a+x)(b+x)-a(b+c)=\frac{a^2c}{b}+x^2.$$

7. A gentleman's property consists of land yielding 3 per-cent, mortgages yielding 4 per-cent, shares yielding 5 per-cent, and certain life annuities amounting to £3,090 per annum and worth 10 years' purchase. The dividends on shares form one-fourth of the rest of his income, the rents one-fifth, and the interest one-sixth. What is he worth?

8. A certain number consisting of two digits is equal to six times the sum of the digits, and if 117 be subtracted from 3 times the number the digits are reversed. Find the number.

9. For what value of m will the equation $2x^2+8x+m=0$ have equal roots? Show that if one root of the equation is impossible, the other is also impossible.

10. The sum of the first and fourth terms of an Arithmetical Progression is -16, and the sum of the third and sixth is $+16$. Determine the series.

11. If the natural numbers be divided into groups in the following manner:—

1 | 2, 3 | 4, 5, 6 | 7, 8, 9, 10 | 11, 12, 13, 14, 15 | &c.,

find the sum of the numbers in the nth group.

12. Define the logarithm of a number to a given base. Show that the characteristic of the logarithm of a number can be determined by inspection when the base is 10, and explain why $\log_{10}\cdot 0356$ and $\log_{10}\cdot 04$ have the same characteristic, and $\log_{10} 3\cdot 56$ and $\log_{10}\cdot 0356$ the same mantissa.

13. A Water Company has 12 collectors, 4 of whom call at the office two days a week each, 4 call three days a week, and 4 call four days a week, the days for calling in no case being fixed. What is the probability that on an assigned day one collector, and only one, of each set of four, will call?

II.

14. How many different signals can be made by a two-masted vessel with five flags of different colours, the flags being used one or more at a time, and those in use being fastened one above another to either or both masts?

15. If $\{1+m+m^2+m^3 \ldots \text{to infinity}\}\ \{1+n+n^2+n^3+ \ldots \text{to infinity}\} = \{1+mn+m^2n^2+m^3n^3+ \ldots \text{to infinity}\}$, then shall the Arithmetic Mean of m and n be equal to the square of their Geometric Mean.

16. An Assurance Office issues 1,000 new policies per annum,—say on the 1st January of each year,—and the policies cancelled from all causes—assumed to be at the end of each year—are in the constant ratio of one-tenth of the number existing at the beginning of the year. Assuming the above law to hold good for a number of years, find the greatest number of policies which can exist on the books of the office at any time.

17. Find the value (using the Table of logarithms before you) of

$$2\cdot 5 \times \frac{1-(1\cdot 0041\dot{6})^{-112}}{\cdot 0041\dot{6}}.$$

18. Calculate the middle term of $(10+a^2bx)^{20}$.

19. If x be small compared with unity, prove that

$$\frac{\sqrt{1+x}+\sqrt[3]{\{(1-x)^2\}}}{1+x+\sqrt{1+x}} = 1-\frac{5}{6}x \text{ nearly.}$$

20. Three persons—A, B, and C—contemplate assuring their lives, and will select either the "Surrey", the "Sussex", or the "Berks" Company. The chances that A will choose the first, second, or third of these companies, are as 3 : 4 : 5; similarly, the chances of B doing so are as 4 : 5 : 6, and of C as 5 : 6 : 7. If eventually all choose the same office, what is the probability that it is the "Surrey"?

21. When $(2n+1)$ dice are thrown, show that the chance that the sum of the numbers turned up is $(7n+3)$ is equal to the chance of the sum being $(7n+4)$, and that this chance is greater than that in favour of any other number.

22. How many triangles can be formed by joining any three of the angular points of a regular plane figure of 20 sides?

23. Find u_{12}, and also u_2, when $u_5=55$, $u_6=126$, $u_7=259$, $u_8=484$, $u_9=837$, and Δ^4 is constant.

24. Show that $u_n=\{u_{n-1}+\Delta^1 u_{n-2}+\Delta^2 u_{n-3}+\Delta^3 u_{n-4}+\ldots\Delta^{n-2}u_1\}+\Delta^{n-1}u_1$, and hence determine a series of such a nature that the terms after the first shall be respectively double the first terms of the successive orders of difference ($u_2=2\Delta^1 u_1$, $u_3=2\Delta^2 u_1$), and so on.

25. The H^M premium at age 40 is at 3 per-cent $=\cdot025891$,

,,	,,	$3\frac{1}{2}$	,,	$=\cdot024654$,
,,	,,	4	,,	$=\cdot023517$,
,,	,,	$4\frac{1}{2}$	,,	$=\cdot022470$,
,,	,,	5	,,	$=\cdot021509$,
,,	,,	6	,,	$=\cdot019811$.

Interpolate the corresponding premium at $5\frac{1}{2}$ per-cent—(I) using *two* of these values, (II) using *four*, and (III) using *six*.

INTERMEDIATE EXAMINATION, 1881.

Examiners.—Messrs. C. D. HIGHAM, G. HUMPHREYS, M.A., J. WHITCHER, and T. E. YOUNG, B.A.

I.

1. Find the probability that three persons, A, B, and C, aged respectively x, $x+a$, and $x+\beta$, will all be alive n years hence.

2. Find the probability that one at least of three lives, aged respectively x, y, and z, will fail between the nth and $(n+m)$th years.

3. Find the probability of a person aged x dying not more than t years after one aged y.

4. A loan of £X is to be discharged by an annuity (made up of principal and interest) of £X : 10 payable at the end of each year, the interest thereon being at i per unit per annum, convertible half-yearly: when will the debt be extinguished?

5. A loan of £P is to be discharged by $p+q+r$ annual instalments compounded of principal and interest. The p instalments are to be of £a each, the q of £β, and the r of £γ. Show how to find the rate of interest i.

6. Show how to construct a table of the principal and interest payments according to the equal half-yearly annuity scheme, stating what independent checks of its correctness occur to you, and by what formula you would verify the amount of principal redeemed in the nth half-yearly payment.

7. What is the precise meaning of the expression "*Force of Mortality*"? Explain its value and use in life assurance investigations; and assuming $\mu_x=\dfrac{d_{x-1}+d_x}{2l_x}$, show under what progression of the decrements $\mu_x \gtrless q_x$.

8. If P be the population of a country at a given period, $1:p$ the ratio of mortality and $2:q$ that of births for the first m years, and $2:p$

and $1:q$ the like ratios for the next n years, what will the population be at the end of $m+n$ years?

9. Mention the leading differences between the data, and the confidence to be attached thereto, in the case of an observation of the mortality amongst assured lives and that of an ordinary Census enumeration.

10. "If β=the present value of a benefit of £1 upon a given life, or the joint existence of any combination of lives,

,, β_1=the present value of a similar benefit, the life or lives being one year older,

,, π=the probability of the benefit of which β is the present value being received in the first year,

and Π=the probability of the life or lives upon which that benefit depends surviving a year,

then $\beta = v\Pi\left(\frac{\pi}{\Pi}+\beta_1\right)$."

What conditions must the benefits fulfil to which the demonstration of the above equation applies, and in regard to which it subsists?

11. Find the value of an annuity on a life aged y, the first payment to be made at the end of the year of death of a person aged x, but the annuity to continue for t years certain, whether the life aged y survive or not.

12. Find the value of an annuity for such portion of a term of t years certain from the present time as will remain after the death of a person aged x, the first payment to be made at the end of the year of his death.

13. State briefly the manner in which you would proceed to construct a table of life annuities.

II.

14. Define the expression, "*Expectation of Life*", and demonstrate the error of basing the computation of a Life Assurance premium upon this function.

15. Assuming the loading to be κ per-cent of the sum assured on the single premium, deduce an office annual premium for a whole-life assurance, with return of total office premiums paid.

16. Explain the nature and use of the D and N columns: in what way does the N of the *English Life Tables* differ from *Davies's* N?

17. Describe at length a method of constructing a table of *last survivor* annuities.

18. Show how to apply Cash allotted as Bonus to a whole-life Policy in making the sum assured payable in the lifetime of the assured.

19. What adjustment in a valuation is necessary in consequence of the payment of claims one month after proof of death, instead of at the end of the year in which death happens; and how would you estimate it?

20. A, aged x, and B, aged y, are entitled to an annuity of £P, payable half-yearly during their joint lives: they wish to exchange it for a half-yearly annuity of the same amount during the life of C,

aged z. What sum must be paid to carry out this plan, reckoning interest at i per unit?

21. A bond bearing interest at rate i per unit per annum is redeemable at par in n years. It is bought at a premium of p per unit. Show how to find approximately the rate of interest realized by the buyer.

22. A whole-life policy is to be subject to a varying annual premium: starting at P', it is afterwards to be increased at the end of n years by q per-cent, and at the end of $2n$ years by r per-cent on the original: it is then to remain constant. Find this premium.

23. A perpetual annuity is to be enjoyed, first by a person aged x for his life; afterwards by a successor to be appointed at his death; and, when the second life fails, by a third to be then appointed, and so on. Determine the present value of the annuity for the first n successive lives, the ages being all different.

24. Taking the Registers of a Life Office, how would you arrange the *data* for ascertaining the expected number of deaths, and the expected amount of claims, and how would you make the estimate?

25. What books should be employed in an Office to enable the Actuary at any time to make a valuation? Sketch the headings of the various columns essential for this purpose.

FINAL EXAMINATION, 1881.

Examiners.—Messrs. G. W. BERRIDGE, T. G. C. BROWNE, R. P. HARDY, and J. MEIKLE.

I.

1. Describe and deduce the working formulas of any two of the following methods of Graduation,—namely, those of Davies, Finlaison, Gompertz, Makeham, Woolhouse.

2. Indicate briefly how a graduation based upon some form of simple averages of numbers living at each age would affect the results.

3. The annual premium charged by an Assurance Office for its survivorship assurance risks have been computed upon the Carlisle Table. Will they, on the whole, be increased or diminished by the substitution of the H^M Table, other things remaining the same?

4. State your opinion of the Reinsurance method of valuation, and what provision is made by it for future expenses.

5. Show under what conditions the reserve obtained by it is greater, equal to, or less than the reserve derived from a corresponding net-premium valuation.

6. Give briefly your reasons for or against the abolition of the restrictions imposed by the Bank Charter Act.

7. State the circumstances under which a married woman can alienate her reversionary interests, and when she cannot.

8. A policy is made payable to A and B, without further descriptive words. Can the survivor give a good discharge? and, if so, why?

9. Suggest the points, for embodiment in a printed form, upon which an Agent should be desired to report in connection with a Life proposal transmitted by him.

10. A holds from B a leasehold property at a ground-rent of £a, determinable on the failure of the joint existence of 3 lives, but renewable for ever when a life drops on payment of a fine of £b; and he desires to surrender his present lease in exchange for one for 999 years certain. What addition should be made to the ground-rent in consideration of the change?

11. A 5 per-cent debenture for £100, repayable in 10 years, is purchased for £106. 10*s*. Give a ready practical method of approximating to the rate of interest yielded to the purchaser.

12. A leasehold house, with 60 years of the lease unexpired, was taken by a railway under compulsory powers 20 years ago, and the price was invested in Consols. The life tenant has received the dividends only, but now it is ordered that it shall be ascertained what annual payment would have exhausted the fund, and that payment be made to the life tenant of the difference between the dividends actually received and such annual payment. How would you treat the case?

II.

13. Had the early graduators of mortality tables any object in view beyond the production of a regular series of values? If so, do you think it was attained in any degree, or is attainable? Give reasons.

14. How would you proceed to obtain the rate of mortality experienced by an Assurance Company?

15. A Mutual Assurance Society separates its profit into two parts, namely:—(a) Profit from interest realized above the rate assumed at the previous valuation; and (b) The remainder of the profit. (a) is divided among the policyholders in proportion to the values of the policies at the date of the previous valuation, and (b) is divided in proportion to the loading received during the valuation period just closed. State under what circumstances this method of division would work unsatisfactorily.

16. Show how you would proceed to ascertain the true profit or loss arising from the claims on a life office in any year.

17. Give a form of book for valuing the Limited Payment and Endowment Assurance policies of an office from year to year; explaining the principles upon which you would determine the annuity for valuing the premium.

18. Explain the various estates that can be had in real property, and the interests of a personal nature that can be derived from it; and why are they personal?

19. What are the practical difficulties connected with assurances issued under the provisions of the Married Women's Property Act, 1870?

20. Two methods of the creation of a public debt have received support,—namely, an issue at par, and a full rate of interest; and an issue at a discount, and a rate of interest somewhat less than would be otherwise accepted. Give instances, and state the circumstances under which it is probable that the first way would be preferable, and the reverse.

21. State what main functions the Bank of England performs besides those of an ordinary Bank possessing a right of issue.

22. A B is entitled, during the life of C D, to certain rectorial tithes producing £500 per annum, and to the rectory house and glebe worth £100 per annum. The Ecclesiastical Commissioners are desirous of acquiring A B's interest in the tithe, making over to him the house and land in fee. On what basis would you calculate the sum to be asked by A B?

23. How would you value a reversion on the death of a single life to leasehold premises on which a mortgage debt had been created?

24. What risks are run by paying a claim by the death of a foreigner upon a receipt given under the local foreign law?

25. Indicate briefly your opinion as to the principles to be employed in determining the surrender value of an ordinary whole-life assurance.

PROCEEDINGS OF THE INSTITUTE.—SESSION 1880–81.

First Ordinary Meeting, 29 *November* 1880.

The President in the Chair.

The following gentlemen were elected members of the Institute, namely—

Fellow.

Bowley, Edwin.

Associates.

Bellamy, George Bumpus.
Chalmers, Alexander Henry.
Douglas, Gordon.
Gamble, Alfred Francis Morgan.
Mills, Joseph.
Morris, Alfred.
Reid, George Alexander.
Reeves, George Merritt, M.A.
Thomson, Ernest Burslem.

The President then delivered his Inaugural Address.

Second Ordinary Meeting, 3 *January* 1881.

The President in the Chair.

The following gentlemen were elected members of the Institute, namely—

Birks, Edmund Alfred.
Doyle, Thomas.
Mitchell, Walter.
Newcome, Frederick Nathaniel.

Mr. F. N. Newcome then read a paper "On the Simultaneous Construction of Compound Interest and Annuity Tables", and the following gentlemen took part in the subsequent discussion:—The President, Messrs. R. P. Hardy, and Cornelius Walford.

Third Ordinary Meeting, 31 *January* 1881.

The President in the Chair.

The following gentlemen were elected members of the Institute, namely—

Donaldson, John.
Dovey, William Roadly.

A paper by Mr. T. B. Sprague, "On the Construction and Use of a Series of Select Mortality Tables to be employed in Combination with the Institute

$H^{M(5)}$ Table—Parts II and III", was read by the Hon. Sec., Mr. G. W. Berridge.

The following gentlemen took part in the discussion on this paper:—The President, Messrs. Baden, Berridge, A. J. Finlaison, G. King, and Walford.

Fourth Ordinary Meeting, 28 *February* 1881.

The President in the Chair.

The following gentlemen were elected members of the Institute, namely—

Davies, John. | McKenzie, Duncan John McGregor.
Pulley, William Pritchard.

Mr. G. F. Hardy then read a paper "On the Mortality observed amongst the various Classes of Bonus Policies in the British Empire Mutual Assurance Company", and the following gentlemen took part in the discussion upon the paper:—The President, Messrs. Ackland, Capern (visitor), R. P. Hardy, Humphreys, Higham, Justican, G. King, Searle, and Whittall.

Fifth Ordinary Meeting, 28 *March* 1881.

The President in the Chair.

The following gentlemen were elected members of the Institute—

Lowndes, Arthur. | Price, William John.

Mr. Cornelius Walford then read a paper "On the Position of the Insurance Press in relation to Insurance Offices and Insurance Interests."

The following gentlemen took part in the ensuing discussion:—The President, Messrs. Adler, Berridge, Baden, Day, Higham, Humphreys, Dr. Leser (visitor), Manly, and W. White (visitor).

Sixth Ordinary Meeting, 25 *April* 1881.

The President in the Chair.

The following gentlemen were elected members of the Institute:—

Ayling, Charles Stephen. | Stancliffe, Frederick.

A paper by Mr. Harald Westergaard, of Copenhagen, entitled, "Notes on the Mortality of the Danish Clergy, from 1650 to 1878", was read by Mr. W. Sutton; and the following gentlemen took part in the discussion:—The President, Messrs. Bumsted, Day, Sutton, and Walford.

The Thirty-second Annual General Meeting, 11 *June* 1881.

The President (Mr. A. H. Bailey) in the Chair.

Mr. Berridge (Hon. Sec.) read the circular convening the meeting, the minutes of the ordinary meeting held in April, which were signed as correct by the President, and the following Report of the Council and Statement of Accounts:—

"In again meeting the members of the Institute at the close of another session, the Council have to report that the number of members on the 31st March 1881 was 355, as against 356 at the corresponding period of 1880. Twenty new members have been elected during the year, and 21 removed from the list by death or resignation.

"By reference to the accounts, which have been duly audited, it will be observed that the income of the year was £975. 7*s*. 6*d*., the expenditure

£934. 9*s*. 8*d*., and that the funds of the Institute now amount to £3,168. 9*s*. 5*d*. The increased expenditure during the year has been due to the cost of printing, compiling, and binding the catalogue of the library, and to the accounts for five numbers of the *Journal* being paid during the year, instead of four, the usual number.

"At the first ordinary meeting of the session, the President (Mr. A. H. Bailey) delivered an address; and at the succeeding meetings the following papers were read and discussed:—

"*First*—'On the Simultaneous Construction of Compound Interest and Annuity Tables'—by Mr. F. N. Newcome.

"*Second*—'On the Construction and Use of a Series of Select Mortality Tables to be employed in combination with the Institute $H^{M(5)}$ Table (Parts II and III)'—by Mr. T. B. Sprague.

"*Third*—'On the Mortality observed amongst the various Classes of Bonus Policies in the British Empire Mutual Assurance Company'—by Mr. G. F. Hardy.

"*Fourth*—'On the Position of the Insurance Press in relation to Insurance Offices and Insurance Interests'—by Mr. Cornelius Walford.

"*Fifth*—'Notes on the Mortality of the Danish Clergy, from 1650 to 1878'—by Mr. Harald Westergaard, of Copenhagen.

"The reports of the examiners for the examinations of the Institute, held on the 22nd and 23rd of April last, give as the result:—

"PRELIMINARY EXAMINATION.

"That out of seventeen gentlemen who presented themselves for this examination, the following have passed:—

1.—H. R. Harding.
2.—P. J. Hawkins.
3.—A. B. Woods.
4.—C. R. Ray, S. G. Warner, } Eq.
6.—J. Thompson.
7.—R. Cross.
8.—W. J. Price.

"INTERMEDIATE EXAMINATION.

"Twelve candidates presented themselves, and the five following passed:—

1.—G. M. Reeves, G. H. Ryan, } Eq.
3.—F. Colenso, J. Graham, } Eq.
5.—R. Wilson.

"FINAL EXAMINATION.

"Four candidates presented themselves, and the three following passed:—

1.—Geo. Todd.
2.—A. F. Burridge.
3.—W. Somerville.

"The Council have arranged for the continuation of the classes during the next session. The subjects treated in the classes are those of the preliminary and intermediate examinations. The usual notice of the time of commencement will be given.

"The catalogue of the library has been printed, and a copy sent to each member. Several additions have been made to the collection, but it is still far from complete, and the Council will gladly welcome any assistance that may be given towards its completion.

INSTITUTE OF ACTUARIES.

Income and Expenditure for the Year ending 31 March 1881.

Dr.

1880-81.	£ s. d.	£ s. d.
Amount of Funds, 31 March 1880—		
Messenger Legacy Fund	268 19 0	
Brown Prize Fund	267 17 9	
General Fund	2,590 14 10	
		3,127 11 7
Annual Subscriptions—		
Town Fellows	252 0 0	
Country ,,	67 4 0	
Town Associates	302 8 0	
Country ,,	107 2 0	
		728 14 0
Mortality Experience Sales		29 13 6
Journal Sales		77 3 0
Examination Fees		46 4 0
Dividends		93 13 0
		£4,102 19 1

Cr.

1880-81.	£ s. d.	£ s. d.
Rent	85 0 0	
Salaries	177 10 0	
Journal Expenses	288 11 6	
Stationery	133 17 7	
Ordinary Meetings	14 13 10	
House Expenses	34 7 0	
Postage	11 18 9	
		745 18 8
Library		132 16 4
Tutor		31 10 0
Examination Expenses		21 15 0
Miscellaneous		2 9 8
Amount of Funds, 31 March 1881—		
Messenger Legacy Fund	275 2 6	
Brown Prize Fund	275 13 11	
General Fund	2,617 13 0	
		3,168 9 5
		£4,102 19 1

Balance Sheet, 31 March 1881.

Dr.

	£ s. d.	£ s. d.
Messenger Legacy Fund (£211. 1*s.* 10*d.* Consols), cost	203 17 8	
Unappropriated Dividends	71 4 10	
		275 2 6
Brown Prize Fund (£200 Metropolitan Railway 4 per-cent Debenture Stock)	200 0 0	
Unappropriated Dividends	75 13 11	
		275 13 11
General Fund		2,617 13 0
		£3,168 9 5

Cr.

	£ s. d.	£ s. d.
Consols, £1,600 Stock, cost		1,498 19 3
Metropolitan Railway 4 per-cent Debenture Stock, £1,200		1,225 17 3
Cash at London and Westminster Bank	439 15 6	
Less Cheque not passed through Bank, £5; due to Petty Cash, 11*s.* 7*d.*	5 11 7	
		434 3 11
Arrears of Subscriptions:—		
Country Fellows	6 6 0	
,, Associates	3 3 0	
		9 9 0
		£3,168 9 5

Signed by

E. A. Colquhoun,
T. J. Searle,
C. Windett.

The adoption of the Report, and of the Accounts for the year, having been proposed by the President and seconded by Mr. G. Cutcliffe, was agreed to unanimously.

The following list was declared to constitute the President, Vice-Presidents, Council, and Officers, for the ensuing year:—

President.

Arthur H. Bailey.

Vice-Presidents.

George Wm. Berridge.
Charles John Bunyon, M.A.
George Cutcliffe.
Ralph Price Hardy.

Council.

Marcus N. Adler, M.A.
Andrew Baden.
Arthur H. Bailey.
George William Berridge.
Thomas G. C. Browne.
Charles John Bunyon, M.A.
*Henry Cockburn.
*Frank A. Curtis.
George Cutcliffe.
Archibald Day.
David Deuchar.
John Ralph Grimes.
*Major-Gen. J. C. Hannyngton.
Ralph Price Hardy.
Stewart Helder.
Augustus Hendriks.
George Humphreys, M.A.
Frank McGedy.
Henry William Manly.
James Meikle.
Arthur Pearson.
Henry William Porter, B.A.
H. Ambrose Smith.
Thomas Bond Sprague, M.A.
William Sutton, B.A.
Spencer Campbell Thomson, B.A.
James Valentine.
*John Whitcher.
John Hill Williams.
Thomas Emley Young, B.A.

Note.—Those marked * are new members.

Treasurer.

John Ralph Grimes.

Honorary Secretaries.

George Humphreys, M.A.
Henry William Manly.

JOURNAL

OF THE

INSTITUTE OF ACTUARIES

AND

ASSURANCE MAGAZINE.

Law Reports.—Dalchosnie Disentail Case.—M'Donald *v.* M'Donalds.

Various important questions having been raised in this case, some of which will interest English actuaries who are not concerned with the intricacies of the Scotch law of entail, we purpose giving a tolerably full report of the proceedings, taken partly from the "Court of Session Cases" (T. & T. Clark, Edinburgh) and partly from the Appellants' Case as printed for the use of the House of Lords.

One of the most interesting points decided in the case may be explained as follows:—

C and D had reversionary interests in an entailed estate, expectant on the death of A and B without male issue; and A was entitled to have the estate disentailed, on paying to C and D the calculated values of their interests. They alleged that B had had serious illnesses, which reduced his expectation of life greatly below the average; and that the value of their reversionary interests was in consequence greatly increased. They maintained, accordingly, that regard should be had to these circumstances in calculating the value of their interests. On the contrary, A maintained that the state of health of B was not to be taken into

account, but that he must be assumed to be an average life, with an expectation of life corresponding to his real age. On this point, the opinions of the Judges were much divided. Lord Adam, the Judge before whom the question first came, was of opinion that the state of health should be taken into account; and this view was acquiesced in by the majority of the Judges of the Inner House, on the assumption that B would consent to a medical examination. It turned out, however, that B would not so consent, and the Judges of the Inner House then decided that the calculation should be made on the assumption that he was an average life. This decision was reversed on appeal by the House of Lords, who decided that B's state of health should be taken into account, but that it was for C and D to substantiate their allegation that his expectation of life was below the average; in other words, that, in the absence of any allegation to the contrary, B must be considered an average life for his age, and that the onus of proving that he was not so lay with C and D. The opinions of the various Judges on these points are well worthy of careful study.

A second point peculiarly interesting to persons engaged in the practical business of life insurance, is the decision of Lord Fraser (see page 122) to the effect that C and D were entitled, for the purpose of proving their case, to have access to the medical reports furnished to an insurance company on the occasion of B insuring his life some time previously.

ALASTAIR M'IAIN M'DONALD, *Petitioner.*—Miss ELIZABETH MOORE MENZIES M'DONALD *and* Miss ADRIANA M'DONALD, *Respondents.*

Major-General Alastair M'Iain M'Donald, being heir of entail in possession of the estates of Dalchosnie, Lochgarry, and Kinloch Rannoch, in Perthshire, presented a petition on 13 Nov. 1877 to the Court of Session, for authority to record an instrument of disentail of these estates. The petition stated that he was desirous of acquiring the estates in fee-simple, and that his petition was for that purpose presented under the statutes 11 & 12 Vic. c. 36, 16 & 17 Vic. c. 94, 31 & 32 Vic. c. 100, and 38 & 39 Vic. c. 100.

The entail was dated in 1837. The petitioner was born on 22 March 1830. The three next heirs entitled to succeed were the petitioner's brother, Captain John Alan M'Donald, born 22 March 1834, and his two sisters, Misses Elizabeth M. M. M'Donald and Adriana M'Donald, born respectively on 10 June

1827 and 30 November 1828. The only other heir of entail in existence was the petitioner's youngest sister, Miss Jemima M'Donald, who was born on 16 June 1832.

The legal conditions, apart from procedure, under which the petitioner was entitled to acquire the estate in fee-simple, were these: (1) that he obtained the consent of Capt. J. A. M'Donald, the next heir; and (2) that he either produced the consents of his sisters, the two respondents, or, in absence of their consents, that, on his motion, the Court ascertained the value in money of their expectancies or interests in the entailed estates, and that the sums so ascertained were paid or provided by him.*

The petitioner produced a consent to the disentail by Captain M'Donald; but the two next heirs, the Misses M'Donald, refused their consent.

The Lord Ordinary (Lord Adam) remitted to Mr. T. B. Sprague, manager of the Scottish Equitable Life Assurance Society, "to enquire and report as to the value in money of the expectancy or interest in the entailed estates with reference to the present application" of Misses Elizabeth and Adriana M'Donald.

Mr. Sprague returned a report, dated 31 May 1878, the import of which, in so far as necessary, is as follows:—

"The net annual value, as obtained from the Valuation Roll of the

* The Entail Amendment (Scotland) Act, 1875, (38 and 39 Vic. c. 61) sec. 5, sub-sec. 2, enacts,—"In the event of any of the foresaid heirs (*i.e.*, the heirs whose consent to a disentail is required by section 3 of the Entail Amendment Act, 1848), except the nearest heir for the time, whether an heir-apparent or not, entitled to succeed, declining or refusing to give or being legally incapable of giving his consent, the Court may dispense with such consent in terms of the provisions following (that is to say):—(*a*) When any of the foresaid heirs entitled to succeed, except the nearest heir for the time, declines or refuses to give, or is legally incapable of giving, his consent, the Court shall, on a motion to that effect by the petitioner in the application, and on a statement by him of the declinature or refusal or incapacity of such heir or heirs aforesaid, and after such intimation to the heir or heirs so declining or refusing, or to the guardians or other persons interested in the heir or heirs incapacitated as aforesaid, as the Court shall think necessary, ascertain the value in money of the expectancy or interest in the entailed estate with reference to such application of such heir or heirs declining or refusing, or incapacitated to give consent as aforesaid: (*b*) Upon such value in money being ascertained to the satisfaction of the Court, the Court shall direct the sum so ascertained to be paid into bank in name of the heir or heirs the value of whose expectancy or interest has been ascertained as aforesaid, or that proper security shall be given over the estate which is the subject of application for the amount so ascertained in favour of the heir or heirs aforesaid: (*c*) Upon such value in money being so paid or received to the satisfaction of the Court, the Court shall dispense with the consent or consents of the heir or heirs the value of whose expectancy or interest has been ascertained as aforesaid, and shall thereupon proceed as if such consent or consents had been obtained: Provided always that nothing herein contained shall render it competent to dispense with the consent of the nearest heir for the time entitled to succeed to any entailed estate sought to be disentailed."

County of Perth, being £1,672, of which £585 arose from the shootings and fishings, he took the capital value of the estate at £48,000, and made his calculations by the Carlisle $3\frac{1}{2}$ per-cent Tables.

"He calculated the expectancy of the Misses M'Donald on the alternative views, that a sum of £5,000 which the petitioner said he had expended on improvements, and was in course of constituting and charging on the estates (by a petition which had been sisted since the disentail petition was presented), was or was not to be allowed as a deduction from the value of the estates.

"The various existing heirs, as stated to the actuary, are—

"*A.* The heir in possession, namely, General M'Donald, was born on 22 March 1830, and is a bachelor. He has been assumed to be an average life, as no evidence has been produced or allegation made to the contrary; and the actuary considers that the fact of his being in the army, and liable to be ordered on active service, is not an element to be taken into account in making the calculations required for the purposes of the remit, and should be disregarded. Should General M'Donald marry and leave issue who shall attain the age of twenty-one, the interests of the subsequent heirs of entail would be entirely defeated; and the probability of his marrying and having issue therefore appears to the actuary to be a very important element in the calculations. The actuary is of opinion, from the examination of such statistics as are known to him, that the probability of General M'Donald marrying may properly be taken as 15 in 100. The probability that if he marries he will have issue who shall attain the age of twenty-one, is a more uncertain element. This the actuary has taken at 80 in 100, so that the compound probability that the General will marry and have issue as above, is 12 in 100. The probability that he will not leave issue is therefore 88 in 100. The values of the expectancies of Miss M'Donald and Miss Adriana M'Donald have, therefore, been first calculated on the assumption that General M'Donald will leave no issue, and have then been reduced in the ratio of 88 to 100, or multiplied by ·88.

"*B.* The next heir is Captain John Alan M'Donald, the only surviving brother and heir presumptive of General M'Donald. He was born on 22 March 1834, and is married, but has no issue. His wife is in good health, and was born on 28 June 1835. It appears from a medical report, produced to the actuary, that Captain M'Donald, although at present in good health, has suffered from ailments which, in the opinion of the actuary, reduce his prospect of life greatly below the average of persons of his age. The actuary has had considerable experience in dealing with proposals for insurance on the lives of persons whose expectation of life is below the average, and, having consulted the medical officer of the Scottish Equitable Life Assurance Society as to the probable effect of the ailments above referred to, he is of opinion that Captain M'Donald's probable duration of life is only equal to about one-half of the expectation of life of healthy persons of his age, and that therefore, his real age being forty-four, it is fair and right that he should be assumed, in the calculations, to be sixty-four years of age. The actuary has, however, at the request of the agent of the petitioner, made his calculations on the two alternative principles, of taking Captain

M'Donald at his real age, and at the advanced age of sixty-four, as above explained. The actuary considers that it may safely be assumed that Captain M'Donald will not now have any issue. He has, however, taken into account that, were Captain M'Donald to succeed to the estate under the entail, he would be in a position to charge the estate with an annuity to his widow to the extent of one-third of the free rental, and thus materially reduce the income of the next heir.

"*C.* The third heir is Miss Elizabeth M'Donald, born 10 June 1827; and

"*D.* The fourth heir is Miss Adriana M'Donald, born 30 November 1828.

"The actuary considers it may be assumed that neither of these ladies will have issue.

"*E.* The only other existing heir under the entail is stated to be Miss Jemima M'Donald, who was born on 16 June 1832.

"It may be assumed that this lady also will leave no issue, even were she now to marry.

"Each of the three ladies would have the power, should she succeed to the estate, of making provision for any husband she might leave surviving her, by way of annuity, to the extent of one-half of the free rental; but the actuary has not thought it necessary to make allowance, in his calculations, for this circumstance, as the probabilities of marriage are small and uncertain. They would also affect the result in opposite directions, and, to a considerable extent, neutralize each other, and therefore probably produce very little effect on the final results.

"The actuary understands that he is to value the expectancies of Miss Elizabeth M'Donald and Miss Adriana M'Donald as at 14 November 1877, being the date of the first interlocutor in the petition; and he considers that the ages of the several heirs to be employed in the calculations should be those at the birthday, whether last or next, which is nearest to the said 14 November 1877. These ages will therefore be—

"*A.* General M'Donald, forty-eight.
"*B.* Captain M'Donald, forty-four (but taken as sixty-four).
(Mrs. M'Donald, his wife, forty-two.)
"*C.* Miss E. M'Donald, fifty.
"*D.* Miss A. M'Donald, forty-nine.
"*E.* Miss Jemima M'Donald, forty-five.

"The expectancies of *C* (Miss M'Donald) and *D* (Miss Adriana M'Donald), the second and third expectant heirs, of which the actuary is to estimate the value, consist each of two parts. The first part is the chance of the heir surviving the previous heirs in the entail, and enjoying the rental for her life; and the second is the chance of the heir acquiring the fee-simple of the estate, by surviving all the other heirs in the entail. For the calculation of the value of the first chance, it is only material to have information as to the heirs before them in the entail; but the value of the second chance depends very much upon the number and ages of the heirs who succeed them under the entail; and the actual selling value of the estate, as a fee-simple property, is obviously a very important element in the calculation of the value of this second chance.

"Before the actuary had been furnished with any information as to the existing heirs who succeed *C* and *D* in the entail, he had calculated the values of their interests in the estate simply as the values of their respective life interests in the income, expectant on the decease of the preceding heirs; but after ascertaining that there was only one other existing heir in the entail, namely, *E* (Miss Jemima M'Donald), he became satisfied that the value of the chance of either *C* or *D* succeeding to the fee-simple of the estate, by surviving all the other heirs, must be considerable, and on making his calculations on a fresh basis, so as to allow for this chance, he found that the resulting value was more than double that previously obtained.

"The results of the actuary's calculations, in which due allowance has been made for all the before-mentioned circumstances, are as follows:—

"Taking Captain M'Donald at the age of sixty-four, and disregarding the £5,000 of improvement outlay as a charge against the estate, the actuary finds that the sum to be paid by General M'Donald to his sisters, Miss M'Donald and Miss Adriana M'Donald, should be £9,114

And, if the £5,000 is allowed, ... £8,122

Divisible thus:—

1. To Miss M'Donald, ...	£5,140	or	£4,564
2. To Miss Adriana M'Donald, . .	3,974	or	3,558
	£9,114	or	£8,122

"Taking Captain M'Donald at his real age of forty-four, the sum to be paid by General M'Donald to his sisters should be £5,724

"And if the £5,000 is allowed, ... £5,114

"Divisible thus:—

"1. To Miss M'Donald, ...	£3,141	or	£2,799
"2. To Miss Adriana M'Donald .	2,583	or	2,315
	£5,724	or	£5,114"

Both parties lodged objections to the report.

Of the petitioner's objections, the following only need be stated here:—"(3) In respect that the actuary has, contrary to the true intent and meaning of the entail statutes, made any deduction whatever in estimating the value of the life of Captain John Alan M'Donald (the consenting heir), and has not made the valuation of his interest and expectancy, enuring to the petitioner by virtue of Captain M'Donald's consent to the disentail, on the footing of his being of his actual age. (4) Alternatively to the last objection, in respect that the actuary has greatly over-estimated the number of years which should, in his calculations, be added

to Captain M'Donald's age in respect of his alleged ill-health, and has dealt therewith as if he were 'dealing with proposals for insurance on the lives of persons whose expectation of life is below the average'. The petitioner avers, and offers to prove, that the expectation of life of Captain M'Donald is not below the average, and that his life has been repeatedly insured at ordinary rates. (5) In respect that the actuary, in estimating the expectancy of the second and third heirs, has assumed that Captain M'Donald will have no issue, and has greatly under-estimated the expectancy of the petitioner having issue. (6) In respect that the actuary has made his calculations on the footing 'that the ages of the several heirs to be employed in the calculations should be those at the birthday, whether last or next, which is nearest to the' 14th November 1877, the date of the first interlocutor in the application. (7) In respect that the actuary has estimated the value of the 'chance' of the second and third heirs 'acquiring the fee-simple of the estate by surviving all the other heirs in the entail', and has thereby valued their interests as expectant fee-simple interests instead of confining the inquiry to the value of their expectant interests as heirs of entail. It is understood that, upon this principle, the actuary has allowed to these heirs for this 'chance' a resulting value more than double what, on a proper interpretation of the entail statutes, and in particular of the 'Entail Amendment Act, 1875', sec. 5, the said heirs are entitled to as the 'value in money of the expectancy or interest in the entailed estate' of such heirs. (8) In respect that the actuary, by estimating the value of the expectancy of Miss Jemima M'Donald, and taking her existence, age, and probability of issue into account, has introduced a fourth heir into the calculations, contrary to the true intent and meaning of the Entail Acts. (10) Generally, and without prejudice to the special objections above set forth, in respect that the actuary has greatly undervalued the interest of the petitioner in the entailed estate as heir in possession thereof, and the value of Captain John Alan M'Donald's expectancy or interest therein enuring to the petitioner in virtue of Captain M'Donald's voluntary consent to the disentail, and has greatly over-estimated the value of the expectancy or interest of the second and third heirs in the entailed estate."

The respondents objected to the report, in so far as it proceeded (1) upon the basis of an assumed annual net rental of £1,672. 7*s.* 1*d.*, and a selling value of £48,000; and (2) upon the assumption, without enquiry, that the life of the heir in possession is an average

life, and that the fact of his being in the army and liable to be ordered on active service is not an element to be taken into account in the reporter's calculations.

The Lord Ordinary on 13 July 1878, after hearing counsel on the report and objections thereto, decided that the sum expended by the petitioner (prior to the presenting of the petition) on permanent improvements on the entailed estate proposed to be disentailed, of the nature contemplated by the Entail Acts, and capable of being constituted a burden on the said estate, ought to be allowed as a deduction from the value of the said estate: Repelled the petitioner's third objection: Sustained the petitioner's sixth objection: and repelled the petitioner's seventh objection: Further, sustained the first objection for the Misses M'Donald, and repelled their second objection; and granted leave to the petitioner to appeal.

In a note appended to his decision (which is technically called an interlocutor), he said:—

"The question involved in the petitioner's third objection is one of great importance, and will be of constant occurrence in this class of cases. The question is whether, under the 5th section of the Entail Amendment Act of 1875, when an heir of entail has declined to consent to a disentail, and the duty falls upon the Court of ascertaining the value in money of the expectancy or interest in the entailed estate of such heir so declining, the *actual* ages of the heirs of entail are, in all cases, to be taken as the basis of the calculations which may require to be made.

"What the Court has to do, is to ascertain the value in money of the expectancy or interest of the heir of entail declining to consent. It is obvious that the most material element to be taken into consideration, in ascertaining this value, is the probable duration of the lives of the other heirs of entail. But it is equally obvious that the probable duration of life of a person, depends upon his state of health, as well as upon his actual age. Were the Court to refuse to take into consideration the state of health of the person, the probable duration of whose life is in question, it appears to the Lord Ordinary that great injustice would be done to the heirs of entail declining to consent to the disentail, and who, by the Act of 1875, are compelled to sell their interests in the entailed estate. Suppose, for example, that the heir in possession, or the nearest heir for the time, was known to be hopelessly ill of an incurable disease, and had only a few months to live, it would be manifestly unjust to calculate the value of the interest or expectancy of the two next heirs on the footing that such heir in possession or nearest heir would attain to the average duration of human life. That, no doubt, is an extreme case, but it illustrates the principle contended for by the petitioner.

"That great difficulties may be encountered in ascertaining the state of health of the person, the probable duration of whose life is

the subject of enquiry, is no doubt true. It is said, for example, that a medical examination is a necessary element in the enquiry, but that it could not have been the intention of the Legislature to compel heirs of entail to submit to a medical examination when, as in this case, they had no interest whatever in the question at issue; so that it would come to depend on the willingness or unwillingness of an heir of entail to submit to examination, whether any reliable evidence could be got as to the state of his health.

"Whether the Court would compel an heir of entail to submit to a medical examination does not require to be determined in this case, because Captain M'Donald, the heir, the state of whose health is in question, does not object to being examined; but the difficulty of the enquiry does not, in the opinion of the Lord Ordinary, absolve the Court from the duty of ascertaining, by the best available means in its power, the value of the interest or expectancy which may be the subject of enquiry.

"The Lord Ordinary has reserved for further consideration the petitioner's 4th objection. He may state that, in his opinion, the question whether or not the expectation of life of Captain M'Donald is below the average, is not one which should be investigated by way of a proof before the Lord Ordinary, as the petitioner suggested. He thinks, however, that the petitioner is entitled to further enquiry. It appears to the Lord Ordinary that a medical man of eminence and experience in such matters should see Captain M'Donald, and communicate with Mr. Sprague, who would then report to the Lord Ordinary how many years, if any, ought to be deducted from the average duration of life in Captain M'Donald's case, or, in other words, how many years, for the purpose of calculation, ought to be added to his life.

"It is obvious, however, that no enquiry at all will be necessary if the petitioner's views are right as to the 3rd objection.

"The Lord Ordinary has communicated with Mr. Sprague with reference to the 5th objection, and has ascertained from him that the state of Captain M'Donald's health does enter into the question of the probability of his having issue. The Lord Ordinary has, therefore, for the same reasons, reserved the consideration of this objection.

"With reference to the 6th objection, in calculating the average of a large number of lives, the method adopted by the reporter no doubt is practically correct; but it was explained to the Lord Ordinary that it happened in this case that the birthdays fell on the same side of the line, and therefore that the method adopted became practically unjust. That being so, it appears to the Lord Ordinary that the petitioner is entitled to have the calculations made on the footing of the ages of the parties, either at their last or next birthdays as may be thought right. The Lord Ordinary has therefore sustained the objection.

"The Lord Ordinary thinks that the 7th objection falls within the principle of the case of *Wilson* v. *De Virte*, 19 Dec. 1877 (Court of Session Cases v. 328), and must therefore be repelled. It appears to the Lord Ordinary that, if an heir of entail who shall succeed to an entailed estate, shall either have the power of acquiring it in fee-simple

by disentailing it, or, as in this case, shall be entitled to hold it in fee-simple as being the only heir of entail in existence, the interest of such heir of entail in the estate is much more valuable than if he had succeeded merely to a life interest in it. In the one case he can at any time sell the estate, if he chooses, for its full value; in the other case his interest may or may not be of great value, as, being merely a life interest, its value will vary according to the time which, from age or otherwise, it is probable he will live to enjoy it. But if an heir of entail is thus in a position to acquire an estate in fee-simple, the Lord Ordinary does not see why, in calculating his chance of succeeding to it, that element should be left out of view. * * * *

"The Lord Ordinary has sustained the first objection for the Misses M'Donald, because he thinks that the actual rental should be taken as the basis of the calculations, and not the rental from the valuation roll. The parties should have no difficulty in adjusting this matter.

"If the Lord Ordinary is right in holding that the chance of the objectors acquiring the estate in fee-simple is an element to be taken into the calculation, it is obvious that the value of the estate is material. The Lord Ordinary thinks that it ought to be ascertained by a remit to persons of skill in such matters.*

"The Lord Ordinary has ascertained from Mr. Sprague that it was not suggested to him, by either party, that the life of General M'Donald was other than an average life, and that he made his calculations accordingly. The Lord Ordinary has also ascertained from Mr. Sprague that the fact that a general officer is liable to be ordered on active service, is not considered as reducing his life below the average, if it be otherwise an average life. The Lord Ordinary has therefore repelled this objection."

The petitioner appealed, and the case was heard before the Second Division of the Inner House.

The argument was mainly in regard to the third and seventh objections for the petitioner.

Argued for the petitioner;—

Third Objection.—The actuary had dealt with Captain M'Donald's life from an insurance point of view, and in insurance offices the lives insured were in general selected lives. But in assessing the value of expectancies under the statute, it was necessary to proceed on a general rule, and the only practicable rule was to take the actual age of the person in question.

Seventh Objection.—It was only the value of the expectancy as heir of entail that was to be valued. The chance of succeeding in fee-simple was outwith the entail, and was no more a legitimate

* It was afterwards remitted to two land surveyors to report as to the value of the estate; and they reported that the clear annual value was £1,700, including about £675 for shootings; and that the selling value of the estate, including the growing wood, was £52,000.

element in the present enquiry than the chance of the Legislature abolishing the law of entail.

Argued for the Misses M'Donald :—

Third Objection.—" The expectancies or interests " which the Court had to value were the reversionary interests of the second and third substitutes. Obviously the value of these interests depended on the probable duration of the life of the intermediate heir. What they wanted was simply such an enquiry as an intelligent man of business would make, before entering into any transaction with regard to the purchase or sale of such a reversionary interest. If there was any insuperable difficulty in the way of ascertaining the value of these interests, the result would be, not that the petitioner would be entitled to have the expectancies valued in a rough and hap-hazard way, but that a condition which the statute had said should be precedent to the disentail—the ascertainment of such value to the satisfaction of the Court—had not been implemented, and the disentail could not go on. But all difficulties in the present case were removed, as Captain M'Donald had no objection to be examined.

Seventh Objection.—The value of the expectancy of a substitute heir was what it would sell for, and in order to arrive at this value the whole chances must be taken into account—the chance of succeeding *plus* the right which possession would give such heir after having succeeded. This was the principle given effect to in *De Virte* v. *Wilson.*

After hearing counsel, the Court made a second remit to Mr. Sprague, to report—" (1) On what footing the chance of marriage in the case of General M'Donald is calculated, and what are the principles on which such chances are calculated in such schemes as the Ministers', Advocates', or Schoolmasters' Widows' Funds, or in similar schemes; and (3) on what principle the chances of acquiring the estate in fee-simple have been valued in the case of Elizabeth and Adriana M'Donald, and to what extent and proportions, if the whole estate be assumed as a capital sum to be divided, according to the interests of the parties, present or prospective, therein, the interest of the heir in possession and the other heirs-substitute would be increased." Mr. Sprague returned a report, from which the following is an extract :—

" As regards the principles upon which the chances of marriage are calculated in such schemes as the Ministers', Advocates', or Schoolmasters' Widows' Funds, the actuary has ascertained that in process of years accurate statistics of their experience have accumu-

lated, from which it has been practicable to calculate the probability of marriage with great exactness. The available information on the subject of the chances of marriage was very conveniently summarized in Mr. W. T. Thomson's 'Report' on the Writers to the Signet Widows' Fund as at Whitsunday 1872; from which the following figures are extracted. Marriages of bachelors above the age of 60 are so few, that the probability of such a marriage taking place may be neglected.

Probability of Marriage of Bachelors.—Annual Marriage Rate per 100.

Ages.	Writers to the Signet Widows' Fund.	General Population.	Peerage.	Schoolmasters' Widows' Fund.	Ministers' Widows' Fund.
44–49	2·781	1·448	1·977	2·297	4·135
49–54	1·254	·705	1·074	1·553	3·533
54–59	2·228	·349	1·046	1·181	2·090

"In estimating the chance of marriage in the case of General M'Donald, the actuary had before him Mr. Huie's tables (published in the year 1868), and also the statistics of the rate of marriage among the families of the peerage, compiled by Mr. A. Day, and given in the 10th volume of the *Journal of the Institute of Actuaries*, p. 186. The former tables indicated that the probability of a bachelor of 48 marrying at some future time was 17 in 100 according to the experience of the schoolmasters, and 33 in 100 according to the experience of the ministers. The actuary considered that the special circumstances of the ministers rendered their experience a very unsafe guide in the present instance, and he accordingly discarded the probability deduced from it. Mr. Day's statistics showed that the probability of marriage, according to the peerage experience, was 11 in 100 for a bachelor of 48, and 13 in 100 for a bachelor of 47. Comparing these figures with the probability of 17 in 100 above mentioned, the actuary came to the conclusion that 15 in 100 would be a very fair estimate of the chance. He has since had the advantage of examining tables of probabilities of marriage given by Mr. James Meikle in his report on the Widows' Fund of the Faculty of Advocates as at 15 May 1877. According to these, the probability of marriage for an unmarried advocate of 48 is 12 in 100. If it should be considered that the probability of a military man of 48 marrying is not greater than that of an advocate of the same age, the actuary's estimate of 15 in 100 is rather too high.

"Assuming the whole estate to be a capital sum of £48,000 (or a perpetuity of £1,680 per annum), to be divided according to the interests of the parties present and prospective therein, the actuary finds that, neglecting all questions as to probability of marriage and issue, also all questions as to the terminable rent charges, and taking Captain M'Donald at his real age of 44 (or, in other words, assuming him to be an average life), the values of the interests of the heir in possession and the other heirs-substitute would be as follows:—

Heir.	Value of the Life Interests, Immediate and Reversionary.	Value of the Probability of succeeding to the Fee-simple by outliving all the other Heirs.	Value of Total Interests.	Value of Total Interests, deducting the £5,000.
General M'Donald . .	£24,217	£2,641	£26,858	£24,060
Captain M'Donald . .	6,070	3,691	9,761	8,744
Miss Elizabeth M'Donald	1,536	2,043	3,579	3,206
Miss Adriana M'Donald .	991	2,354	3,345	2,997
Miss Jemima M'Donald .	1,030	3,427	4,457	3,993
	£33,844	£14,156	£48,000	£43,000

"If the £5,000 of improvement outlay is allowed as a charge on the estate, the values will be rateably diminished, as shown in the last column."

After hearing counsel for the parties further on the cause, and on the second report by Mr. Sprague, their Lordships of the Second Division gave judgement as follows:—

Lord Justice-Clerk.—This case raises some questions of considerable importance and of some difficulty in regard to some of the clauses of the statute of 1875, under which an heir of entail in possession may, on certain conditions, acquire the estate in fee-simple.

The position of the present case is this:—The heir in possession, who petitions, is Major-General M'Donald, and the succeeding heirs whose interest is now in question are, first, his brother, Captain M'Donald, and then two sisters. The question is, on what principles the interests of these succeeding heirs are to be determined. In regard to Captain M'Donald, the younger brother, he has given his consent, and therefore as far as his interest is concerned no question arises. But, of course, in order to ascertain what the value of the expectancy of the two sisters is, it is necessary to estimate, in the first place, certain chances of life and marriage in regard to the heir in possession, and, in the second place, the similar chances of the next heir, Captain M'Donald. Then, again, in regard to the two last heirs, it is necessary, according to one view which has been maintained, to see not only what the value of their own chances or expectancy of succeeding to this entailed estate may be, but also whether the fact that a remaining sister, who is not within this enquiry at all, is or is said to be the last member of the destination, gives an additional value to the expectancy of these three heirs of entail, as well as to that of the heir in possession. * * *

The next question relates to the chance of General M'Donald marrying, in regard to which we asked the actuary to give us some detail. I am quite satisfied to accept Mr. Sprague's view upon that matter; and therefore I have nothing further to say in regard to the report so far as it relates to that question. I might have supposed that, in the case of a military man in possession of a landed estate, the probabilities of marriage were as high as in the case of one in any other social position; but I am quite content with the report.

The next question relates to Captain M'Donald's interest. Now, Captain M'Donald is a consenting heir, and makes no claim under this application. He does not want to saddle the entailed estate or the heir in possession with any payment. But it is said that, in estimating the interest of the two succeeding heirs, the life of Captain M'Donald is not to be taken as an average life, but as one far within the average; and, instead of assuming his chances of life as those of a man of his actual age, which is forty-four, the actuary has so far weighted him as to assume him to be of the age of sixty-four. That is rather an arbitrary proceeding, because it depends upon information not obtained judicially, but furnished *ex parte* to the actuary; and the question comes to be this, whether, in estimating the chances of life for such an enquiry as this, we are to take what is an average or an assumption from the tables, or to ascertain the fact for ourselves. I do not wish to give any opinion on the general question so raised, which, I think, is of very considerable importance; and I do not wish to give any sanction to the notion that in all cases it is either necessary or right that we should have a report or an investigation such as would be required by an insurance company if a life were offered for a policy. I do not say so, and I see very considerable difficulties in laying down any such proposition. But in this case we have been given to understand that, to remove all objection, Captain M'Donald is willing to have his chances of life ascertained as a matter of fact. It may be better, if there are no obstacles in the way, to ascertain the fact according to the truth of it than to strike an average which may not be, and in this case probably is not, consistent with the fact. To assume the life as an average one may be a very proper and right thing in the ordinary case. In the present case, I am far from saying that Captain M'Donald could have been obliged to submit to any such investigation, or that the parties in this case would not have been entitled to assume an average because he would not consent. His consent removes any difficulty I had, and I am disposed on that footing, without expressing any opinion on that question, to allow his chances of life to be ascertained by actual examination.

I do not think it necessary to go in detail into any of the other objections, excepting one, on which I have felt the greatest difficulty in this investigation. It is claimed on the part of the two ladies who are the two last heirs of entail under this application, that their interests are not only to be estimated according to their expectancy to succeed to an estate subject to the fetters of an entail, but that we must take into account the chances of the third sister, who is said to be the last substitute of entail—the last member of the destination—predeceasing them, and so opening the succession to the estate in fee-simple to one or other of them. It is said that this is part of the expectancy; and the result, as brought out by the actuary, is that the amount which the heir in possession will be obliged to pay as the value of the defenders' expectancy, is more than doubled by adding the value of the fee-simple to the value of the entail. I have come to be of opinion that that is not a legitimate element in the enquiry; that what the heir in possession is bound to do, is to satisfy the interests of that limited number of the heirs-substitute in regard to the succession under the entail. They have a contingent succession under

the fetters of the entail. It is only under the fetters of the entail that they have any valuable expectancy at all. I do not think it consistent therefore with that provision, that we should value all the contingencies by which the fetters of the entail can be defeated. It is manifest that there are many that might be suggested, apart altogether from the entail running out and coming to the last member of the destination. Farther, before the sisters can make out that they have any such interest, there must be a proceeding to value the expectation of life of one of the heirs of entail who is not involved in this enquiry at all. The statute implies that the heir in possession is to have the whole fee-simple of the estate, provided he satisfy the chances of a limited number. In regard to the rest, their chances of life are of no moment whatever, nor do these, in my opinion, form any legitimate element of enquiry. The estate is the property of the heir in possession in fee-simple, excepting in so far as he is burdened by the interests of the substitute heirs. Therefore, if the heir of entail in possession is to be considered as a fee-simple proprietor, burdened only by the expectancy of a certain number of the members of the destination, he does enough, I think, when he satisfies those members of the destination for the interest which they might have succeeded to under the fetters of the entail. Beyond that I do not think it is desirable that we should go.

The case of *De Virte*, which was referred to, was a case of a different kind, and there certainly an allowance was made for the chances of one of the heirs, within the limit, disentailing the estate. I must fairly own that I think that question, if it occurred again, would well deserve to be reconsidered; and the Court certainly and the actuaries were very much at a loss to know on what footing such an element or contingency should be valued. But that is a totally different matter from going outside the heirs whose interest is really in question, and going on to one, two, or it may be a dozen heirs-substitute, to ascertain whether the chances of survivance would give them an appreciable interest in the destination running out and the acquisition of the estate in fee-simple. I do not think that that is a legitimate subject of enquiry. I am quite aware that cases might be put where considerable hardship may arise. Supposing the next heir of entail or the second heir of entail was himself the last member of the destination, it would be hard certainly that that interest, which is defeated by the disentail, should not be valued. But still the policy of the statute was to take the heir in possession as the fee-simple proprietor, and relieve him of the fetters on his allowing the substitute heirs the real value of their interest under the fetters of the entail.

On the whole matter, I propose, if your Lordships concur in the views I have expressed, that we should make findings to that effect, and then send the case back to the Lord Ordinary to have these given effect to in figures, and have the rest of the questions between the parties disposed of.

Lord Ormidale.—In this application by the petitioner, General M'Donald, for the disentail of the estates described in the petition, it has become necessary to ascertain, in terms of the 5th section of "The Entail Amendment (Scotland) Act, 1875", the value in money

of the expectancy or interest in the entailed estates of Misses Elizabeth Moore Menzies M'Donald and Adriana M'Donald, the second and third substitute heirs of entail, the terms upon which the expectancy or interest of Captain John Alan M'Donald, the first of the substitute heirs of entail, is to be valued, having been arranged extrajudicially. * * * * *

3. The third objection, which the Lord Ordinary has repelled, involves, as his Lordship remarks, considerations of great importance. The objector has submitted that the actual age of Captain M'Donald should alone be dealt with, irrespective altogether of his state of health, past or present, even were it so bad as to lead to the conclusion that he had only a few months to live. It appears to me that the views of the Lord Ordinary on this point are substantially sound. I can find nothing in the statute requiring that the actual age should alone be taken into view. On the contrary, the statute is quite general in its terms, and does not prescribe, in any respect, the limits of enquiry. So far as this point is concerned, it is undoubted that the state of Captain M'Donald's health may very materially affect the expectancy or interest of the subsequent heirs in the entailed estates. Why, therefore, the state of Captain M'Donald's health as affecting the duration of his life is to be excluded from consideration I fail to see. At the same time, I can quite well understand that cases may occur, in regard to which it might be practically impossible to ascertain the true condition of an heir of entail as regards health and prospect of survivance; and in such cases there might be no alternative but to take the actual age. The present, however, is not a case of that description, seeing that Captain M'Donald has no objection to submit to a medical examination as to his state of health; and that being so, I deem it unnecessary at present to determine whether in what, if in any circumstances, the medical examination of an heir of entail could not be enforced. And in the present case, it will be for the Lord Ordinary to consider in what way the true condition of Captain M'Donald as regards his prospect of survivance, can be best ascertained. For my own part, I should think a direct remit from the Lord Ordinary to one or two medical men of eminence to examine Captain M'Donald and report, would prove satisfactory. I do not think, however, that the medical examination of an heir of entail, as regards his or her capability of having issue, ought to be entertained, and this for obvious reasons.

7 and 8. These objections raise considerations of importance, and, in my view of them, they are attended with not a little difficulty. But, ultimately, I have felt myself unable to resist the conclusion, that the chances or probabilities of the second and third heirs coming to be fee-simple proprietors of the estates, do not form legitimate elements in valuing their interest in the entailed estates. It must be borne in mind that it is not the value of the heirs' "consent" to the disentail, but the value of their expectancy or interest as heirs of entail in the entailed estates, which falls, in terms of the statute, to be estimated. This may make a substantial difference. In valuing the "consent" of the heirs,—if that had been the question,—the chance or probability of their coming to have right to the estates in fee-simple might reasonably enough be urged by them for calculation;

but the question under the statute is not the value of the heirs' "consent", but of their expectancy or interest as heirs of entail in the entailed estates. This distinction is noticed by Lord Rutherfurd Clark in the elaborate note issued by him as Lord Ordinary in remitting the case of *Wilson* v. *De Virte* a second time to the actuary for an amended report; and his Lordship's views seem to have been approved of by the Court. It is true that the precise question as it has here to be determined, was neither raised nor decided in the case referred to; but I think it is to some extent indirectly dealt with in that case, adversely to the views entertained by the present Lord Ordinary. I am therefore, especially as I understand that both your Lordships have come to the same conclusion, for sustaining the 7th and 8th objections, and to that effect altering the Lord Ordinary's interlocutor.

10. I agree with the Lord Ordinary in thinking that this objection cannot be disposed of at present. I may here, however, remark, with reference to the probability of the heir in possession marrying and having issue, that so far as I see, there is no reason for holding that there is not as much chance of this as regards General M'Donald as in regard to any other gentleman of his position and circumstances. In short, it appears to me that such a chance in the case of General M'Donald ought to be calculated according to the highest scale applicable to such a matter.

Lord Gifford.—This petition raises some questions of very great difficulty under the 5th section of the Entail Amendment Act of 1875, and these questions are of great importance, for they will constantly occur in similar applications under the statute, and their determination may constitute a rule for all future cases. * *

The third objection raises a question of very great nicety and difficulty, and I am sorry to have the misfortune to differ from your Lordships. In estimating the value of the interest of Misses Elizabeth and Adriana M'Donald, it is necessary to ascertain the value of the expectancy of Captain M'Donald, their younger brother, but who precedes them in the order of the destination. Captain M'Donald is forty-four years of age, and is at present in good health; but it is stated that he has suffered from certain ailments, which it is said have had the effect of reducing his prospect of life greatly below the average of persons of his age; and the question is, whether Captain M'Donald's prospect of life is to be taken at his actual age—that is, at an ordinary average life—or whether allowance is to be made for the injurious effects of the ailments from which he has suffered, by taking his age to be much greater than it really is. It is proposed to calculate Captain M'Donald's interest as if he were sixty-four years of age, whereas his actual age is only forty-four. There is also a question whether any allowance is to be made for the possibility that Captain M'Donald may yet have issue. It is stated that Captain M'Donald is quite willing to submit to any medical examination.

I have felt the greatest hesitation in coming to a conclusion on the questions thus raised. I feel the full force of the observation that the pecuniary interest of an heir of entail in the entailed estate does not depend upon his age alone, but upon his state of health, and upon everything which affects, however remotely, his prospects of

life or of survivance; and, however recondite or difficult of ascertainment the circumstances may be, however far the enquiry may stretch, and however delicate it may become, it is impossible to deny that everything which affects Captain M'Donald's chances of life or chances of having issue, will and must affect the probabilities of the succession of his elder sisters, who are the heirs next called in the destination after him and his issue. I think it must be admitted that if either Captain M'Donald or his sisters were selling in open market their expectancy or chance of succeeding under the entail, every circumstance, both of health and of habits, of the preceding heirs, every incident of ancestral history, and every peculiarity of personal constitution in mind or in body, in habit and in conduct, would be taken into account by a prudent purchaser, who would form his estimate on the widest possible induction; and there is undoubted force in the observation that what a purchaser would do in the open market in estimating and ascertaining value, the Court must do under the very general words of the statute. I confess this was the impression I formed in the earlier stages of the argument.

But I have come to be of opinion, though still with doubt and hesitation, that however accurate in a philosophical or scientific aspect the view suggested may be, it is not possible to carry it out in practice. The enquiry must be limited somewhere; and the Court cannot be called upon to guess, as a private purchaser might and probably would do, as to the effect of remote circumstances, or as to the probabilities arising from bodily, mental, or moral constitution or habits. Many of these peculiarities it is practically impossible to ascertain—even after the utmost enquiry they can only be guessed at. In other cases delicacy forbids the enquiry—for example, in reference to capacity for procreating issue; and in almost all cases, the uncertainty of the result seems to suggest the extreme inexpediency of entering upon such an investigation.

In the present case it is proposed to enquire into the effect of the ailments from which sometime ago (I understand many years ago) Captain M'Donald suffered; and it is said that in Captain M'Donald's case this enquiry will not be very difficult. It is not stated what were the ailments, or what was their nature. Now, I do not myself see where such an enquiry would stop, either in this or in any other similar case. Captain M'Donald is in good health at present; and the question is not as to his present state of health, but as to the effect of lesions or injuries suffered at a period more or less remote. It is not said that a medical man could form an opinion merely by seeing Captain M'Donald. He must also be informed of bygone facts regarding diseases which mere examination of the person would not disclose, and these facts must depend on testimony. I think that no two independent medical men would agree as to the effect of these bygone injuries—probably not even as to their nature—or on the questions, how far have the effects disappeared? how far may the effects disappear still farther as time advances? will nature not effect a complete cure? or may not the effects be counteracted by constitution or by care? may Captain M'Donald not avoid risk by precautions or by medical aid? may he not survive the risks as he becomes older?—and so on. Such questions might be put almost

indefinitely, and they would, or might probably, all be differently answered by medical or other experts. How is the Court to deal with such a proof? It is said all this will be settled by the opinion of one medical reporter. But this would be just delegating to a medical expert the questions of fact of which the Court are to judge; and would virtually amount to a reference of the whole question of value to an arbitrator whose judgement would be final, and into whose grounds of judgement the Court could not go.

Still further, Captain M'Donald might decline to submit to any medical examination whatever, and I rather think he could not be compelled to do so. It is not the value of his own interest that is in dispute, but only that of his two sisters. Captain M'Donald's interest has been already settled by agreement, and must be so settled under the statute; for he is the next heir to the petitioner—that is, the heir-presumptive; and by the statute he cannot be compelled to consent, and may at his own pleasure stop the disentail altogether, or may name his own price. I do not think he could be compelled to submit to unpleasant and painful examinations, merely to ascertain the interests of third parties with whom he has no contract and no privity, and with whom he might not even have relationship. It is true he is willing to be examined; but I think we must look to principle in such cases, for the next case may be one in which a party situated as Captain M'Donald is may refuse to be examined, and a statutory value must not depend upon the accident of a person being willing or unwilling to undergo examination, and I think it safest to decide this case irrespective of that willingness.

It is urged, and with great force, that all insurance companies have regard not to mere age but also to the actual state of health, and, it may be added, to the state of habits of the person whose life is assured. This is true to a certain extent, but it does not go very far. No doubt insurance companies do enquire into the health and habits of the person whose life they assure; but this is only to estimate the conditions of the applicant for assurance, and even this does not always apply. For example, in selling an annuity, insurance companies are not in the habit of making any enquiry; and, as a rule, they will not give a larger annuity on an allegation that the annuitant is a bad life or has bad habits. And in dealing with reversions dependent on the lives of third parties, insurance companies, unless I am greatly mistaken, take the ages of third parties on which the reversion depends, and whom they have no right to examine, at the actual age, and not at any suppositious age fixed on an enquiry as to special circumstances affecting the health or conduct of such third parties. Now, this is the analogy which I think really applies to the present case, and perhaps affords something like a test for trying the present question.

For what is it that Misses Elizabeth and Adriana M'Donald are to receive? It is the value of their reversionary interest—that is, a sum which would buy for them from an insurance or reversion company, a contingent annuity exactly equal to the net rents of the entailed estate, and only to be enjoyed when and as if they had succeeded to the entailed estate. Now, in purchasing such a contingent and reversionary annuity, an insurance company would not

sell it to the Misses M'Donald for a less sum on the allegation that Captain M'Donald, one of the preceding heirs, was an exceptional life, or that his habits or profession or employment tended either to lengthen or to shorten life. A private speculator might possibly do this, but an ordinary insurance company or the Government so far as it sells annuities would refuse to estimate such remote, uncertain, and incalculable probabilities, and would merely take the age and the ordinary probabilities of life as deduced from the Government or the insurance tables, and I think this is the safe, if not the only practicable course in the present case.

Still further, if, because Captain M'Donald unfortunately years ago suffered from certain ailments, you are to shorten his probabilities of life either by twenty years or by any other number, I think it would follow that if it were alleged that Captain M'Donald were a better life than the average, this also must be allowed for and estimated. If you are to reduce the average by adverse circumstances, I think you must also enlarge it by favourable ones, otherwise it would be a most one-sided rule. Suppose it could be said truly that Captain M'Donald's constitution was far better than the average—that he never had a disease or a trace of a disease affecting life, that he came of a long-lived family, and that his habits and conduct made it certain that he should inherit the years of his fathers, I do not see why all this should not be as relevant for enquiry as the contrary view. Certainly a private purchaser of the Misses M'Donald's reversion would take all this into account, and wisely so; but the question is, can the Court embark upon such an enquiry? or must it, just as insurance companies practically do when they sell a postponed and contingent annuity—contingent upon other lives—ignore such considerations as too remote, and proceed only on the average view?

Without going into further explanatory details, I think, on the whole, that Captain M'Donald's chance of life should be taken at his actual age of forty-four, and not at an assumed age of sixty-four, or at any other age than his actual one. I think any other course would be unsafe, and would lead to tedious, expensive, and uncertain investigations. I do not see, except of consent, how the questions raised, being mainly questions of fact—that is, bygone facts—could be settled otherwise than on a proof. I do not think we could compel the parties to take the result of an expert's enquiries as final, except both parties agreed.

I am also very clearly of opinion that no enquiry or proof can be allowed, as to the probability of Captain M'Donald having issue either of his present or of any future marriage.

I agree with the Lord Ordinary in reference to the sixth objection. It so happens that the mode adopted by the actuary is the most unfavourable for the petitioner, because of the dates on which the birthdays chance to fall, as compared with 14 November 1877. It would be preferable to take either the last or the next birthdays with reference to each of the parties. It will make little difference in this case whether the last birthdays or the next birthdays are taken.

The seventh objection raises a question of great and of general interest, and I must add also of very great difficulty. The question is, whether, in estimating the interest or expectancy of Misses

Elizabeth and Adriana M'Donald, the actuary must take into account the possibility that one or other of these ladies might, by surviving all the other heirs of entail, become the last member of the destination, and so acquire the estate in fee-simple. This raises a question of principle of the broadest kind. It is undoubted that each of the two ladies has a possible chance of this occurring, and there are strong grounds for maintaining that this chance is a valuable one, and must be estimated in fixing the value of the interest.

The words of the statute are very general. The Court is directed to "ascertain the value in money of the expectancy or interest in the entailed estate with reference to such application of such heir or heirs declining or refusing or incapacitated to give consent as aforesaid"; and we must apply these words to the circumstances of each case. I have come to be of opinion that the interest to be valued is the interest in the entailed estate, and not the interest in a possible fee-simple estate which can only emerge after the entail has expired and come to an end. The possibility that the entail may expire while the estate is in the possession of one or other of the ladies, is not a right under the entail, but a right at common law, which will only arise when there is no entail in existence; and I do not think that the statute intended that the chances of the entail failing altogether, and the value of a *spes successionis* at common law, should be estimated. The thing which is to be valued is "the expectancy or interest in the entailed estate", not the expectancy of getting a fee-simple estate as the heir *alioqui successurus* at common law. The chance of being heir at common law the ladies will still have notwithstanding the disentail, only that right will be defeasible as a fee-simple succession, and I incline to think that this is what the statute intended. To give the heirs more than the value of the entailed succession would be to give them the value of a common law *spes successionis*, which they have not as heirs of entail at all, but as heirs-at-law of the granter.

The view I take is this: the petitioner, General M'Donald, is absolute proprietor of the estate, subject only to the deed of entail. He is not, as has sometimes been contended, a mere liferenter of the estate with certain powers, or one of a series of liferenters; on the contrary, he is fiar of the estate—strictly and properly fiar—subject only to the disabilities created by the entail. Accordingly, when the law gives him the power of disentailing, it simply enables him to enlarge his radical right of fee by removing the disabilities; and the whole benefit of the enlargement accrues to him and not to the consenting heirs. They only get the value of their entailed interest. It was on this principle that *De Virte's* case was decided; and this decision altered what it appeared was the practice or understanding of actuaries previous to its date. The actuarial view was that, after deducting the life-interest of the heir in possession and the life-interests and value of the powers of consenting heirs, the surplus value, as it was called, should be divided rateably among the heir in possession and the consenting heirs; but this view was negatived. The whole surplus value, as it was called, goes not to the heirs rateably, but solely to the heir in possession; and the heirs whose interests are to be valued, only get the value of their interests

and no more. The whole value of the estate goes to the heir in possession, subject only to deduction, in the present case, of what he has agreed to pay to Captain M'Donald, and the value of the interests of Misses Elizabeth and Adriana M'Donald. There is a fourth heir, Miss Jemima M'Donald; but then under the statute she is to get nothing, and I think it is not right to take her into account at all in order to enhance the shares of Elizabeth and Adriana. If Miss Jemima's interest required to be calculated—and admittedly this must be done if the fee-simple is to be taken into account—I do not see where the process would stop. There might have been several heirs named after Miss Jemima, and although this would diminish the prospect of the fee-simple estate, it would still remain a very valuable prospect and quite capable of being valued in money. I think the statute meant to draw the line at the end of the three next heirs, and to exclude more distant heirs altogether either as to their chance of liferent or of fee. On this point, therefore, though not without diffidence, I am for altering the judgement of the Lord Ordinary.

The only other question argued related, I think, to the probability of General M'Donald, the petitioner and heir in possession, marrying and having issue. General M'Donald is only forty-eight years old, in good health, and in the possession and enjoyment of a very considerable entailed estate. I think the probabilities that he may marry and have issue are in his case at least as high as they would be in the case of any other man of the same age. I should even go further, and say that it is more likely that a bachelor heir of entail in good health, in possession of an entailed estate yielding £1,700 a year, will marry than almost anybody of the same age, unless they also are in possession of an independence. I think, therefore, this probability should be taken on the highest scale shown on any of the tables referred to.

Lord Justice-Clerk.—I have already stated that if I were to judge of this last matter simply by my own lights, I should be inclined very much to the result Lord Gifford points at; but I am not inclined to differ from a professional actuary, like Mr. Sprague, on such a matter.

The Court pronounced this interlocutor, on 16 January 1879:—

"Find in regard to the 4th and alternative objection that the chances of Captain M'Donald's life ought, in respect of Captain M'Donald's consent, to be ascertained in the ordinary way by such professional examination and report as the Lord Ordinary may direct: Sustain the 7th and 8th objections, and to that effect alter the Lord Ordinary's interlocutor: *Quoad ultra* adhere to said interlocutor, and with these findings remit to the Lord Ordinary to give effect to them, and to proceed with the cause; and reserve all questions of expenses."

The cause having accordingly gone back to the Outer House, the Lord Ordinary (Lord Adam) on 13 February 1879 pronounced this interlocutor:—"Remits to Professor W. R. Sanders, M.D., and Dr. Claud Muirhead, to examine Captain M'Donald, and to report as to the chances of Captain M'Donald's

life, and how many years, if any, should be added to his age in estimating the value of the expectancy of the second and third heirs of entail, and recommends them to communicate with Mr. T. B. Sprague before reporting."

Captain M'Donald, on being asked whether he would consent to be examined, replied that he would only do so on condition of the examiner bringing an autograph letter of authority from General M'Donald, the petitioner. General M'Donald declined to grant such a letter, and the Lord Ordinary reported the case to the Second Division for directions how he should proceed.

Parties were again heard: 19 March 1879.

Argued for the Misses M'Donald;—Their contention remained the same as before, that it was the business of the petitioner to make out to the satisfaction of the Court the value of the interests of the second and third heirs, and one factor of that value was the chance of life of the intermediate heir. This chance could not be ascertained without regard to the facts which were now in dispute. The most satisfactory course was a medical examination of Captain M'Donald; but if that was impossible, the Misses M'Donald were entitled to the ordinary right of litigants, namely, a proof of their averments, all the more so because they were not *in petitorio;* they had no wish to sell their interests; the statute obliged them to sell, but only on being paid the value of their interests. There could be no doubt as to the relevancy of their averment, and none as to the materiality. Could the Court possibly declare that the value had, in the words of the statute, been ascertained to their satisfaction, when it was alleged that circumstances existed which would make a difference of £3,000 or £4,000 in the amount? In short, there were only two courses open to the Court. They must find either that there must be an enquiry, or else that a condition which the statute had said should be a condition precedent had not been complied with, and that the disentail could not proceed. The argument on the other side mixed up two different questions—the competency of an enquiry at all, and the expediency of certain lines of investigation. It constantly happened that the best evidence was not available. Any objection to a particular line of investigation would be perfectly open to the petitioner.

Argued for the petitioner;—The Court would not order Captain M'Donald to submit to a medical examination,* and

* Davidson *v.* Davidson, 11 Feb. 1860, 22 D. 749, 32 Scot. Jur. 305.

accordingly the only question was whether a proof *prout de jure* should be allowed as to the state of a living person. The radical difficulty was that he was not a party to the petition, and had no interest in it. The very difficulty of carrying out an enquiry showed that the Legislature did not intend that any such enquiry should take place. The statute contemplated a summary procedure, and not one involving a vast field of enquiry.

At advising,—

Lord Ormidale, after stating the circumstances under which the case came before the Court, said:—For myself I should have been disposed to appoint the petitioner, General M'Donald, to grant the authority asked for by Captain M'Donald, if this had been insisted for by the respondents; but as the petitioner objected to grant the requisite letter of authority, and as the respondents did not ask that he should be ordained to do so, but preferred that the Lord Ordinary should be directed to allow a proof in ordinary terms in regard to the probable duration of Captain M'Donald's life, I am of opinion that this is the course which should be adopted, the petitioner, General M'Donald, leading in the proof. But this course was strenuously opposed on the part of the petitioner; and it was contended by him that now, in consequence of it turning out that Captain M'Donald would not unconditionally submit to a medical examination, there was no alternative but to take his actual age as the basis for calculating the probable duration of his life; and that at any rate, supposing a proof to be allowed, it lay upon the respondents to lead in the proof.

The subordinate question as to which of the parties ought to lead in the proof is comparatively of little moment, although I must say that it appears to me that it lies on the petitioner as the party *in petitorio*, and keeping in view that he is objecting to Mr. Sprague, the actuary's, report, to show, in the first instance, that twenty years ought not to be added to Captain M'Donald's life; but as this somewhat technical point cannot, I think, be of much consequence, I am ready to acquiesce in holding that the lead in the proof should be taken by the respondents if that be the opinion of the rest of the Court.

In regard, however, to the other question, whether any proof at all ought to be allowed, I am very clearly of opinion that it ought. I was of opinion, when the case was first considered by the Court, in disposing of the petitioner's objection to Mr. Sprague's report, and I am so still, that the Lord Ordinary was right in holding the ascertainment of Captain M'Donald's state of health to be essential; and on this point, in place of repeating, or going over the arguments, *pro* and *con*, which were then considered and dealt with by the Court, I have to refer to the note by the Lord Ordinary to his interlocutor of 13 July 1878, and the reported opinions of the Court in disposing of the petitioner's reclaiming note against that interlocutor. It must be kept in view that the petitioner's third objection to Mr. Sprague's report stands repelled by final interlocutor; and that his fourth objection, which relates only to the number of years which must be added to Captain M'Donald's age, alone remains to be disposed of. I

do not see, therefore, how the Court can now refuse to go into an enquiry in some mode and to some extent as to the number of years that ought to be added to Captain M'Donald's age. I certainly cannot yield to the argument of the petitioner, that there is no competent mode of enquiry as to Captain M'Donald's probable duration of life except a medical examination, which he may not be obliged to submit to. At present the Court cannot, by anticipation, deal with the proof that may or may not be tendered by the parties. But supposing that a medical examination of Captain M'Donald—*inspectio corporis*—is the best evidence that could be adduced, it would by no means follow that, failing such evidence, and it being shown that it is not attainable, other, although secondary, evidence may not be competent. It rather appears to me that such evidence would be admissible, provided all has been done to adduce the best evidence; and any objection to such evidence by the petitioner, who refuses to grant the letter of authority required by Captain M'Donald, ought not, I think, to receive much consideration. But, as I think it not unlikely that the respondents will have it in their power to adduce medical gentlemen who have recently examined Captain M'Donald and can speak perfectly well to his condition, in no view would the objection referred to apply. They may, for example, be able to adduce his ordinary medical attendant, and also Dr. Glynn, who subscribes the report No. 31 of process, and which was procured and laid before the actuary by the petitioner himself. They might also adduce Professor Sanders, who may be able to give good and unexceptionable evidence in regard to his, Captain M'Donald's, probable duration of life without again examining him; for it would appear from the correspondence printed in the appendix of 4 March 1879 that the Professor has already, not long since, examined the Captain. And, besides all this, the respondents may be able by unexceptionable evidence, medical as well as non-medical, to establish the important statements made by them in their answers, printed at pp. 9 and 10 of the appendix just referred to, and, in particular, the following statement—"As regards Captain M'Donald himself, it is right to bring under the notice of the actuary the facts that in the year 1858 he fell over the cliffs at Kinsale and injured his head, and that he subsequently suffered from concussion of the brain. Abscesses formed in the wound which was made by the fall, and splinters of bone came away from time to time. In consequence of this fall he was an invalid for many months. He subsequently was obliged to leave the army in consequence of a medical board deciding that after his accident he could not stand a warm climate, and at or about that time he suffered very much from the heat at Gibraltar, where he was stationed. Since his discharge from the army he has been in very bad health, and subject to epileptic fits, and on several occasions his life has been almost despaired of. He had one of those fits so late as April 1877, at which time it was not expected he would survive. These circumstances, it is submitted, go materially to reduce the value of the life interest of Captain M'Donald; and that his life is not considered by himself a good one may be gathered from what is believed to be the arrangement under which his consent has been given, namely, that his wife shall be secured in an annuity after his death."

I am therefore of opinion that the Court ought now to direct the Lord Ordinary to allow the parties an opportunity of proving, *habili modo*, what is the probable duration of Captain M'Donald's life. And, if this course is taken, it will be for the Lord Ordinary to control the proof and direct how it should be taken. It may be assumed that the Lord Ordinary will not allow a proof of remote, indirect, and speculative views, the limit or effect of which could not be appreciated. Without going into any wild, remote, and conjectural or fanciful incidents in the career and habits of Captain M'Donald, and still less of his ancestors, I can very well understand—and indeed in ordinary fairness to the parties must assume—that evidence only of an unexceptionable description will alone be tendered and adduced by them. Professor Sanders, and, it may be, others—the petitioner himself, for example—may be examined before the Lord Ordinary; while the evidence of Dr. Glynn and other witnesses resident out of Scotland may be taken under a commission at no great expense. And it will be for the Lord Ordinary, on the proof being concluded, to determine the point at issue, after again advising, if he thinks it right, with Mr. Sprague, the actuary.

Lord Gifford.—This case is now before us upon a verbal report by the Lord Ordinary (Lord Adam), asking for instructions in consequence of difficulties having arisen in carrying out the interlocutor of 16 January 1879. The chief difficulty is that, contrary to the understanding of both parties when that interlocutor was pronounced, and which is embodied in the interlocutor itself, Captain M'Donald has refused, except under certain conditions, which the Court have no power to enforce, to submit to a medical examination with a view to obtaining a medical report regarding his chances of life; and the question now is, What steps should be taken to ascertain the value of Captain M'Donald's life with a view to fix the value or interest of the two Misses M'Donald, who are next substitutes to him, in the entailed succession? We are now asked to instruct the Lord Ordinary how to proceed.

At last advising I was of opinion, having the misfortune to differ from your Lordships, that, in the special circumstances of the present case, there was no room, even of consent, to remit to a medical man to examine Captain M'Donald, but that Captain M'Donald's life should be taken to be an average one according to his age. I thought that upon the averments of parties it would be inexpedient to allow either a proof at large of all facts and circumstances affecting the probabilities of the endurance of Captain M'Donald's life, or to direct a medical report on these probabilities. Indeed, I thought that an enquiry into all the facts averred by the Misses M'Donald, as tending to shorten the probabilities of Captain M'Donald's survivance, was so highly inexpedient as to be impracticable; and I went so far as to think that, whether Captain M'Donald consented to be medically examined or not, no such enquiry should be entered upon, but that, from the necessity of the case, Captain M'Donald's life should simply be taken as an average one. I still think that a statutory value should not be made to depend upon such an accident as that of a party being willing or unwilling to submit to a medical examination and inspection, but must be the same whether the party consents or not.

To the opinion then expressed I still adhere, but I feel, as I then felt, that the question is one of nicety and of difficulty. Referring to my former opinion, I have only one or two observations to add thereto.

In the first place, if it had been alleged that Captain M'Donald was at the present moment labouring under some known and ascertainable disease attended with danger to life, or was suffering at present from some wound or accident which might or would probably lead to fatal consequences, I by no means say that such an averment would not be relevant or might not be enquired into. If Captain M'Donald had met with and were suffering from a railway or other accident, the consequences of which would certainly or even probably be fatal, or if he were in an advanced stage of consumption, or in the crisis of a fever, I do not say that these circumstances might not affect the value of the Misses M'Donald's interest in the entailed succession. I reserve my opinion in such cases, and I decline to say what would be the appropriate course—whether the proceedings should be sisted till the issue of the injuries or illness should be seen, or whether allowance should be made therefor, or what course should be taken in such circumstances. No such case is before us at present. Captain M'Donald is at present in good and average health. He is not at present labouring under any known or ascertainable disease, and he has sustained no injury from which there is to be dreaded in any reasonable sense a mortal or fatal result. No mere medical examination—even supposing Captain M'Donald submitted thereto—would, apart from the ascertainment of past facts and past events in Captain M'Donald's life and habits, or in his family history, enable any medical man to say, except as the merest guess, that his life was other than an average one, or that his age should be taken, instead of forty-four, his actual age, to be sixty-four, or any other assumed figure. It is in this position of matters that I think the enquiry demanded by the ladies, who are the next heirs of entail, ought not to be gone into.

In the next place, I think the enquiry, if once opened, would in many cases be absolutely interminable. For what is it that the ladies propose to prove? It is, of course, all circumstances tending to impair Captain M'Donald's life, or to render it shorter than it otherwise would be. Every act and event in Captain M'Donald's life and history would in this view be relevant. Youthful indiscretions, if any, might be ruthlessly hunted up. I am not merely supposing; for I see in the papers before me that such are suggested, and, beyond all doubt, unless the enquiry is to be limited, such may be gone into and attempted to be proved. Captain M'Donald's habits, his mode of living, his diet, and, in short, his whole being and surroundings, are to be asked about and sifted to the utmost—where he has lived and what he has done are to be ascertained, and guesses are to be made as to what he will probably do, and what care and precautions he will probably take. Nay, more, his ancestral history for generations must be examined, for this, we know, is always made a material enquiry by insurance companies; and all these are proper facts which cannot be ascertained by an expert by inspection, but must be ascertained by testimony or ordinary proof. I know not where such an enquiry would stop; and I fear that if it were admitted

here it must be admitted in every case where a postponed heir of entail avers that the life of a previous heir is of less than the average value.

Still farther, all this enquiry and evidence is not to be laid before a man of skill whose judgement as arbiter is to be final, but is to be submitted to this Court, that the Judges thereof may guess—for it could be nothing but a guess—how far the life is to be regarded as an inferior or as a deteriorated one. For myself, I am not at all qualified for such an enquiry; and although, of course, if the duty is laid upon me, I must do the best I can, it will be with the utmost reluctance that I will pronounce a verdict saying how long it is probable Captain M'Donald will live.

Of course I admit frankly and fully that in one sense all these enquiries are perfectly relevant. Everything that I have mentioned, and multitudes of other things that might be mentioned, may possibly affect Captain M'Donald's chance of life. Who is, and who probably will be, his medical attendant? has he aversion to taking medicine? is he careless of himself? what care does his wife take of him? where will he probably reside? and so on, indefinitely, are all questions which a speculative purchaser of Captain M'Donald's reversionary interests might fairly enquire into as far as possible, and might quite rightly take into account in forming a guess of what price as purchaser he will give for the reversion. But these matters are not to be made the subject of a proof at large before a court of law; and the statute can never have intended that this Court were to enter into such enquiries, either directly or through a commissioner. But this, I apprehend, is what the Misses M'Donald ask—they ask a proof of all facts and circumstances tending to shew that Captain M'Donald's probabilities of life are less than the average.

And then, as I formerly pointed out, the proof, if competent one way, must be competent the other way also. If the petitioner, General M'Donald, had alleged that Captain M'Donald's life was a better life than an average one, I do not see, on the principles contended for by the respondents, how we could have refused to allow such proof; and such proof may be demanded in the very next case that occurs under the statute. If averages are not to be taken, but each individual case is to stand by itself, then there will be as many special lives above the average as those which are below it. Tables of average will be useless, or almost useless, and the experience of the past will only point to multifold and endless investigation.

And then Captain M'Donald, who is to be the unfortunate subject of all this investigation, is not a party to the present case, or to the present dispute, and has no interest therein to the extent of a single farthing. His consent has been already purchased and paid for as it must be under the statute; and I more than question not only whether Captain M'Donald could possibly be compelled to submit to medical inspection, but whether he could be compelled as a witness to disclose the secrets, not to say the possible peccadilloes, of his whole past life.

But suppose he was a party to the question, or suppose, what is still more pertinent, that the question had arisen, not regarding Captain M'Donald's probability of life, but regarding the probability of life of the respondents the Misses M'Donald themselves; suppose the petitioner, General M'Donald, had averred that his two sisters,

the Misses M'Donald, who are postponed to Captain M'Donald in the tailzie, have bad lives—that they are likely to die very soon—and had therefore proposed to pay them under the statute a much less than the average value of their reversion; suppose he had proposed to state their ages at sixty instead of fifty and forty-nine, and that he had made the very same averments regarding them as they now do regarding their brother,—would your Lordships have allowed the petitioner a proof of such averments? I for one think not, and that although the ladies are necessarily parties to the proceedings in which they claim compensation or value of interest in respect of the disentail.

In my former opinion I pointed out that no injustice is done and no prejudice suffered by taking Captain M'Donald's life as an average one,—for although, when a life offers itself to an insurance company to be assured, the insurance company always makes very extensive enquiries into individual and family history, and even into personal habits of the proposer with whom they are making a contract, yet when selling or purchasing annuities they make no such enquiries; and when the lives of third parties are in question, which very often happens, all such enquiries are almost necessarily excluded. On this point I refer to my former opinion. What the Misses M'Donald ought to receive are sums which would enable them to purchase from ordinary reversion companies, with sufficient security, contingent annuities which they would have had the chance of enjoying if the estate had not been disentailed. Such purchases can always be made without any such enquiry as that now proposed. They are made, and necessarily so, upon averages.

I am therefore of opinion that we should direct the Lord Ordinary to refuse the proof at large which is now demanded by the respondents, and to ascertain the value of the "expectancy or interest" of the respondents, on the footing that Captain M'Donald's life is an average one. I think there can be no other result. I reach this conclusion, however, in the special circumstances of this case, and having regard to the nature of the respondent's averments, which I think would open a quite impracticable field of enquiry.

Lord Justice-Clerk.—I greatly regret that there should be a difference of opinion on the bench upon this question after the long and elaborate procedure that has taken place, and I regret it all the more because I must fairly own that I have not been able to come to so confident a view as your Lordships, or one altogether satisfactory to myself, as to the result. The case has been complicated by some misadventure in the course of the proceedings, and I should not have been ill-pleased to have been able to deal with this on the procedure as a special case, without being put to resolving the more general question which Lord Ormidale and Lord Gifford have discussed, and on which they have differed. The duty which is laid upon us, on a motion to that effect by the petitioner, is to ascertain the value in money of the expectancy or interest in the entailed estate of these two ladies, being the second and third heirs in succession. Now, that is the duty laid upon the Court. It is left to the Court to ascertain the value in any way they think fit; and the first question is, what is the interest of these two expectant heirs of entail? It is

said that General M'Donald is bound to satisfy the Court upon that matter, because he is the petitioner; and to a certain extent that is a sound proposition. And he has satisfied the Court, I think, so far as the burden is laid upon him, by showing us the expectancy of life of Captain M'Donald according to his age, and I am very clearly of opinion that there is no ground whatever for saying that he was bound to go beyond that. But then it is objected by the two heirs who are not consenting heirs, that Captain M'Donald's life is not to be taken as an average life according to his age, but that his is a special case, and that his expectancy of life is below the average; and they say they are entitled to prove that in order to show what the value of their own expectancy and interest is. I must own, my Lords, that I do not look with much favour upon that contention. I do not deny there is a good deal to say for this logically, but in this family contention I am not particularly impressed in its favour; but still it is one which they are entitled to have considered. The actuary to whom Lord Adam remitted this matter—a man of the highest position in his profession—entertained the statement which was made on the part of the heirs of entail, and made apparently for the first time; and he went into that question of whether the life was or was not an average one. I think it would have been better if the Court had, in the first place, had an opportunity of considering whether that was or was not a regular, desirable, or expedient course of inquiry. However, the actuary in his report loaded Captain M'Donald's life to an extent which made it that of a man of sixty-four instead of a man of forty-four, and the question came to be considered before us on objections; and then it was stated that there need be no difficulty at all about the question of compelling Captain M'Donald to submit to a medical examination, because he was quite willing to do so; and on that footing, and that footing alone, the remit to the Lord Ordinary was made, which carried the case back to the Outer House. I wish it to be very clearly understood that, in agreeing to that course, I gave no opinion whatever upon the general question, and I explained that in the clearest and most distinct terms. I said that my inclination of opinion would probably have been the other way, but that since Captain M'Donald was willing I could not say that it was not a material and relevant element in the inquiry. But it now seems that we were all under a mistake—that Captain M'Donald did not consent, and he has not consented; and the result is that on the Lord Ordinary's report we must now decide what course we should follow. In these circumstances, I do not know that I should seriously object to what, if there is to be an enquiry, I should deem by far the most expedient as well as the most competent mode of it, by a remit to the gentleman who had previously examined Captain M'Donald, and who by his previous information might be in a position to give us the information, before answer as to the effect of it, which we stand in need of. I have a very clear opinion that that is the proper course, in any circumstances of the kind, where we are obliged to have an enquiry to our own satisfaction. In any circumstances, I should have objected to a proof at large. What we are to decide is not a question of fact, because the fact of the expectancy of Captain M'Donald's life is not ascertainable. No man can tell us how long

he will live. Scientific men can only say what the average or proportion is of men of his age in regard to the length of time they are likely to live, and that is the expectancy as well as we can reach it. But that is a question of pure medical science, and if I had my way —if I had the power—I should have enquired into the scientific matter in regard to the medical question, just as I should have enquired into the scientific matter in regard to the actuarial question; and, sending the actuarial question to a man of skill out of Court, I should have sent the medical question to a man of skill out of Court, and I should have been inclined to choose a man who had information already on which he could have made his report, with such other enquiry as he might have found Captain M'Donald willing to submit to. I only say these things in explanation, because I understand your Lordships do not concur in that view, and therefore we are driven to decide the general question; and on that general question my opinion is that no sufficient statement has been made by the two heirs of entail, to render it necessary that we should step out of the ordinary course, or ascertain this question of expectancy by any except the ordinary rules.

It was accordingly decided that the cause should proceed on the footing that Captain M'Donald's life, is to be assumed to be an average life and to be estimated according to his present age.

On 28 March the Lord Ordinary remitted it to Mr. Sprague to enquire and report, with the following (in addition to other) instructions: To allow as a deduction from the value of the estate the sum of £3,995. 18*s*. 5½*d*. expended by the petitioner on improvements; to take Captain M'Donald's life as an average one; to proceed on the footing that the probabilities of the petitioner marrying and having issue, are as high as they would be in the case of any other man of the same age; to take the ages of the several heirs as at their last birthdays; and to keep in view that the 7th and 8th objections have been sustained by the Court.

The following is taken from Mr. Sprague's third report in the matter (dated 28 April 1879) :—

"As the annual value, £1,700, is almost exactly 3¼ per-cent on the selling value, £52,000, the reporter has adopted that rate of interest as the basis of his calculations in the present report.

"He has allowed as a deduction from the estimated value of the lands the sum of £3,995. 18*s*. 5½*d*., as directed by your Lordship. Considering this as a charge upon the estate, bearing interest at 4 per-cent, it will give a deduction of £160 from the clear annual value, which will thus be reduced to £1,540. He has taken the ages of the several heirs as at their last birthdays preceding 14 November 1877, namely, General M'Donald 47, Captain M'Donald 43, Miss E. M. M'Donald 50, and Miss A. M'Donald 48. He has assumed Captain M'Donald to be an average life for his age (43), and he has accordingly calculated the probability that he will have issue by a second wife, which probability was so small that it might be neglected

when Captain M'Donald was taken at 20 years' addition to his real age. Taking into consideration the age of his wife (42), and the fact that they have been married for many years, the probability of Captain M'Donald having issue by his present marriage is so small as to be inappreciable. There is, however, the probability that his wife may die before him, and that he may have issue by a second wife. In order to estimate the probability of this, a series of somewhat involved calculations have to be gone through. It is necessary to calculate for each year during which Captain M'Donald may be alive, the various probabilities—(1) that his wife will be dead; (2) that he will be alive; (3) that he will marry a second time; and (4) that the marriage will be fruitful. In this way the actuary arrives at the conclusion that the probability of Captain M'Donald having issue is 5 per-cent, and the probability that he will not have issue is 95 per-cent.

"In consequence of your Lordship's instruction that the reporter should proceed on the footing that the probabilities of General M'Donald marrying and having issue are as high as they would be in the case of any other man of the same age, the reporter has given his very careful attention to these points. In addition to the marriage statistics mentioned in his report of 17 December last, he has found some further statistics in the late Mr. S. Brown's report on the Madras Military Fund, published in 1863. These statistics, which seem to be more appropriate for the present purpose than any of those previously considered, give a higher probability of marriage than all of them, with the exception of those relating to the Ministers' Widows' Fund, the accuracy of which the actuary has seen good reason to doubt. None of the above-mentioned statistics give any information as to the probability that a marriage entered into will be fruitful; and for the purpose of forming an estimate on this point the actuary has extracted from the records of the British peerage the particulars of 339 marriages entered into by men above the age of 40, and from an examination of these statistics he has arrived at the conclusion that the estimate of 80 per-cent, adopted in his report of 31 May last, as the probability that if General M'Donald marries he will have issue, was somewhat too high. As the result of his revised calculations, he now puts the probability that General M'Donald will marry at 19 per-cent (or more exactly ·192), and the probability that he will marry and have issue at $11\frac{1}{2}$ per-cent (more exactly ·114). The probability that he will not have issue is therefore ·886. Thus, while a higher marriage probability has been taken, the effect of combining with it the lower probability of fruitfulness of marriage is, that the resulting probability that General M'Donald will marry and have issue is almost exactly the same as before. The probability that Captain M'Donald will not have issue being, as above stated, ·95, the compound probability that neither he nor General M'Donald will have issue is ·8417. This multiplier has therefore been used in the calculations, instead of the decimal ·88 mentioned in the actuary's report of 31 May last.

"The values of the expectancy or interest in the entailed estates with reference to the present application of Miss E. M. M'Donald and Miss A. M'Donald, have now been estimated simply as liferent interests, to which they will be entitled if the previous heirs in the

entail die before them and leave no issue; and on this footing, having regard to all the various contingencies involved, the actuary finds that the value of Miss E. M'Donald's expectancy or interest is £941, and the value of Miss A. M'Donald's is £720.

"In arriving at the above results, no allowance has been made for the chance of the second or third substitute heir acquiring the fee-simple of the estate by surviving all the heirs in the entail; but the values of their expectancies have been calculated precisely as if there were a long series of heirs in existence in the destination, so as to render it practically impossible that either the second or third heir should survive all the others. The value of the expectancy or interest is therefore not only very much less than it would have been if estimated on the principles adopted in the actuary's report of 31 May last, but also very much less than the sum they might reasonably and properly have required to be paid to them on consenting to a disentail, if the Entail Amendment Act 1875 had not been passed. If, before the passing of that Act, the heir in possession, General M'Donald, had been in treaty with the second and third substitute heirs for purchasing their consent to a disentail of the estates, each of them would have been fairly entitled to say:—'I have a chance of enjoying the estate for my life after the death of the preceding heirs in the entail, and I have a further chance of acquiring the fee-simple of the estate by surviving all the other heirs in the entail. You must, therefore, give me the calculated value of these chances. That sum represents my expectancy or interest in the estate, and is the very least that must be offered to me. But you must give me something further as the price of my consent. After the value of the expectancies of yourself and the three next heirs in the entail are calculated, there will be a surplus representing the value of the expectancy of Miss Jemima in the estate—that is to say, since she is the only remaining heir in the destination, the value of her chance of succeeding to the estate in fee-simple by surviving all the other heirs—and I am fairly entitled to a share of this surplus if I give my consent to the disentail.' In consequence of the provisions of the Act of 1875, it is no longer in the power of the second and third heirs to withhold their consent; and they can therefore no longer ask for any payment to be made to them for the price of their consent, as distinguished from the value of their expectancy or interest. The effect of the Act was therefore to deprive the second and third heirs of all claim to a share of the above-mentioned surplus, and the actuary's calculations, upon which his first report was based, proceeded on this footing. He assumed, however, that, in other respects, the Act left the value of the expectancy or interest of each of them unaltered. The position of an heir of entail in possession, as clearly explained by recent judicial decisions, is that of an absolute proprietor, subject only to the deed of entail; and each of the substitute heirs has a chance of succeeding to that position, that is to say, of becoming absolute proprietor of the estate, subject only to the fetters of the entail; while there is a further chance that those fetters may fall off entirely, through the death of all the other heirs in the destination. The actuary assumed in his first report that, in estimating the value of the expectancy or interest in the entailed estate of the second and third heirs, he was

to take into account the whole of these chances; but, as now directed by your Lordship's interlocutor, he has excluded from his calculations the values of the chances of the second and third heirs succeeding to the fee-simple of the estate. The effect of the Act of 1875 appears therefore to be, not only to deprive the second and third heirs of all claim to the payment of a sum as the price of their consent, but also to reduce considerably the sum to be paid them as the value of their expectancy or interest in the entailed estate."

The respondents having objected that, under the entail, male issue only (and not female issue) of the petitioner would succeed to the estate before them, the Lord Ordinary said it was much to be regretted that the objection was not stated, as it might have been, at an earlier stage of the proceedings. The petitioner maintained that the respondents were barred from now insisting in it; but the Lord Ordinary thought he ought not to allow the calculation of the value of the respondents' interest in the estate to proceed on erroneous data, and he therefore directed Mr. Sprague to amend his calculations accordingly. Having done so, the latter reported:—

"In order to estimate the probability that General or Captain M'Donald will have a son to succeed to the estate, the actuary has made a further examination of the statistics as to the British Peerage families mentioned in his report of 28 April 1879, and has drawn from them the conclusion that the probability that General M'Donald will have a son to succeed to the estate is less than the probability that he will marry and have issue in the ratio of 812 to 1000; the same being the case with regard to Captain M'Donald. He therefore estimates the probabilities (1) that General M'Donald will not have a son to succeed to the estate at ·9074; (2) that Captain M'Donald will not have a son to succeed to the estate at ·9594; and (3) that neither of them will have such a son at ·8706. Using this probability, the actuary finds that the value of the expectancy or interest of Miss Elizabeth M'Donald in the entailed estate is £973, and that of Miss Adriana M'Donald £745."

The sums mentioned in the report, £973 and £745, having been paid into the British Linen Company's Bank, in name of the respondents respectively, the disentailing deed was approved by the Court and ordered to be recorded; and this decision was upheld on appeal by the Second Division of the Court of Session.

The Misses M'Donald appealed to the House of Lords, maintaining that, in the calculation of the value of their interests, the state of health of Capt. M'Donald should have been taken into account, and regard should have been had to their contingent chances of succeeding to the fee-simple of the estate; and that the Court below was wrong on both these points.

It was held by the House of Lords, reversing the decision of the Scotch Court, (1) that the second and third substitute heirs had a right to bring before the Court any facts bearing on the probable duration of life of the first substitute heir, and that averments that the said heir had suffered from ailments which reduced his prospects of life below the average, formed a relevant subject of enquiry; and (2) that the chances of the second or third heir succeeding to the estate in fee-simple, by outliving the fourth and only other heir of entail in existence, were elements to be considered in a valuation of their respective "expectancies or interests".

The Lord Chancellor, in giving judgement (12 March 1880), said—

The question is, whether the expectancy or interest of the appellants has been valued on correct principles.

The first question as to value relates to the life of Captain M'Donald. He was born in 1834, and is married, but has no issue. The appellants offer to prove that Captain M'Donald, though at present in good health, has suffered from ailments which reduce his prospect of life greatly below the average of persons of his age. The respondent, on the other hand, contends that Captain M'Donald's life should be taken as an average one, according to proper life tables; and this has been the opinion of the majority of the learned Judges.

I am sorry that I cannot concur in this opinion. It might have been a convenient and reasonable course for the Legislature to have laid down some rule by which these expectancies or interests should be valued, and the Legislature might have said that in all cases the probable duration of life should be calculated according to some of the well-known tables, but it has not done so. In that state of things, it appears to me that the appellants have a right to bring before the Court, or before any actuary or valuer to whom the case is referred, any facts relevant to the probable duration of Captain M'Donald's life and his state of health; and the ailments from which he has suffered, if calculated to shorten life, must be relevant to such an issue. I see no inconvenience in this course to the respondent. The *onus* of proof will be upon the appellants. The respondent will prove the age of Captain M'Donald, and the probable duration of an average life at that age; and it will be for the appellants to prove that Captain M'Donald's is not an average life, and why.

The other point in dispute arises in this way: A younger sister of the appellants, Jemima M'Donald, is the fourth heir of entail, and she is now the last in the entail. If she should be dead without issue when the appellant Adriana, the third heir of entail, succeeds, if she does succeed, then Adriana will be absolutely entitled to the fee of the estates; whereas, if Jemima or any of her issue are in existence at that time, Adriana will be virtually no more than a liferenter, unless she obtains the consent of Jemima or her issue to disentail. The same would be the case as to the appellant Elizabeth, if she were to survive Adriana.

It is contended by the appellants that, in valuing the interests of

Elizabeth and Adriana, their interests should not be treated as those of liferenters only, but the possibility should be taken into account that, when the survivor of them comes into possession, Jemima may be dead without issue.

The Court of Session were of opinion that this was not an element of value which ought to be considered. My Lords, I am unable to concur in that view. The question, I think, to be asked is this—Supposing the Act of 1875 had not passed, what would have been the interest which the appellants would have enjoyed in specie, supposing they had refused their consent to disentail? It cannot be doubted that what the surviving appellant would have enjoyed would have been an interest equal to a liferent, or equal to a fee, just according as Jemima was or was not at the time dead without issue. This is the amount of interest that would have been enjoyed in specie, and this, as it appears to me, is the interest that must be valued. If a Scotch estate was entailed on A, a bachelor of seventy, then on B, a spinster of seventy, then on C, a spinster of seventy-five, and an Act of Parliament required "the expectancy or interest of B in the entailed estate" to be valued, I cannot doubt that the valuer must have taken into account that when B succeeded, C might be dead, and the estate of B an unshackled fee. The valuation in the case before your Lordships may, and probably will, be a matter of difficulty, depending as it does on contingencies not merely of death but of marriage and of having issue; but this is the task which the Legislature has assigned to the Court, and however difficult it may be, it must, in my opinion, be discharged.

My Lords, I think the interlocutors under appeal should be altered, but I have felt much doubt as to the form in which the alteration ought to be made. The case is one in which the interposition of any delay may lead to serious alteration, by death or otherwise, in the rights of the parties; and, on the other hand, it may be that the respondent will not desire to disentail, if he has to pay a larger sum for consents than the sums already approved by the Court. The actuary to whom the question of valuation was referred by the Court, at a time when he took into account the elements which, I think, ought to enter into the calculation, put the value of the interests of the appellants at £8,122, if I rightly understand the figures; and I understood at your Lordships' bar the counsel for the appellants to say that, if this sum were secured, they would be willing to consent to the immediate disentail, the enquiry into the exact value taking place afterwards. If, therefore, your Lordships concur with me as to the principle of the valuation, I should propose a motion in this form. (His Lordship then read a motion substantially in terms of the judgement of the House (see p. 121).

If I am wrong in the figures which I have used (£8,122), the parties will correct me; and I wish also to add that I have taken these figures merely because they were given by the actuary, and not as adopting either the mode or result of his calculation, which the Court may see fit on the new enquiry not to adhere to. The sum arrived at by the actuary certainly appears to be, under the circumstances, a very large one.

LORD HATHERLEY.—The first question is, whether, in valuing the

life of Captain M'Donald, who is the first heir next to the heir in possession, the actuary should value it according to the tables which are used by actuaries for estimating contingencies of this character, as an expectation of life simply measured by the age of the subject; or whether the state of health and other matters affecting life, should also be taken into consideration in arriving at the value. I think that it is impossible to exclude evidence strictly applicable to the particular circumstances of the case, and tending directly to affect the probable duration of life, from such an enquiry as the Act of 1875 prescribes. Many cases may occur or may be suggested, in which it is all but certain that the ordinary expectation of life of a man of a given age, would not represent the probable value of the life of an individual. The table averages were in fact originally deduced from the total number of men of a given age in all possible states of health, and any one life which is selected may be the lowest of the class as regards health. I think it is right to admit the evidence of the calculated tables applying to any given age as *prima facie* proof; but it should be open to the other side to lead proof of facts, manifestly affecting the alleged average in the particular case. For instance, take the case of a man known to be on his deathbed with consumption; the value of the expectation of life, according to the tables, would not at all represent the true value in that particular case. If you take at the average value any one life out of a class upon which the average is taken, and disregard the circumstances affecting that individual, the effect may be to arrive at a valuation, in some cases far above, and in other cases far below, the actual value of that particular life. I do not see how, under the words of the Act, an enquiry into facts manifestly affecting the alleged average in the particular case, can be shut out.

The second point which has arisen has occasioned me much more doubt and hesitation than the first point. It is this—the first three heirs being the Captain and his two sisters, the fourth heir in the entail is another sister named Jemima. According to the Act of 1875, her interest is not to be taken into account; but if she should die without issue in the lifetime of the survivor of her sisters, then that sister, on coming into possession as heir of entail, will be entitled to deal with the fee-simple, free from the fetters of the entail. Should this interest be valued as part of the interest of either of those sisters, under the directions of the Act of 1875, namely, as "an interest or expectancy in the entailed estate", with reference to this disentailing application?

I have had the opportunity of seeing the opinions of your Lordships, who, with me, heard this matter argued, and I have read with great attention the opinions of the Judges in the Court below, and I have, though with much doubt, come to the conclusion that the view contended for by the appellants is right. As I read the two disentailing Acts of Parliament, they seemed to have proceeded on the principle, that the long periods of entail sanctioned by previous legislation and by the previous state of the law, ought to be abridged; but, at the same time, that such abridgement should take place with a due regard for vested interests, in so far as they would not frustrate the purpose of the Act itself, namely, the intended earlier release of entailed estates from the fetters which virtually excluded them from

commerce. In the Rutherfurd Act a system of consents was established, by which the heir in possession might acquire the unfettered dominion. These consents were to be, in a case such as that before us, those of the first three heirs in succession after the heir in possession. This Act, however, gave no control to the heirs beyond these three, which could prevent the disentailing of the property. The Act of 1875 modified this arrangement, by directing that the immediate heir next to the heir in possession must give his consent, or else the disentailing instrument should not be allowed, but that the next two heirs, if they would not, or from disability could not consent, should be bound by a deed approved of by the Court, provided that the Court satisfied itself of the value of the expectancy (in the words I have mentioned), and that such value was secured to the satisfaction of the Court. This being done, the fourth and subsequent successive heirs were barred by the allowance of the deed. The effect of this enactment was to disregard altogether the interests more distant than that of the third heir in expectancy.

The question now before your Lordships is of a somewhat singular character, regard being had to the circumstances, namely, whether an interest which arises in the third heir only in consequence of the interest of the fourth heir and subsequent heirs being exhausted, namely, the chance of the fourth heir dying before the third heir, and without issue, should be valued? Suppose the fourth heir to be the last, so that there is no heir beyond the fourth who could insist on the fetters imposed on the estate being brought into effect when it comes to the third heir, the question is whether this is an interest or expectancy directed to be valued by the Act of 1875. Is it an "interest or expectancy in the entailed estate"?

The Legislature, in its desire to achieve the object it had in view, namely, the liberation of estates subject to entail, has nevertheless, both in the Rutherfurd Act and in the Act of 1875, intimated the extent to which it thought justice required consent in the one case and compensation in the other, in respect of vested interests, stopping at such a degree of interests as any third substitute heir could acquire by the entail, or, as it is expressed in the Act, "in the entailed estate". It appears to me probable that what the Legislature was aiming at, was the entailed estate only as created by the deed of entail; and the interest now in question is an interest which is beyond the entail, and indeed, after the interest in the estate tail has passed into the second or third heir, and is only arrested at the decease of the third heir by the fact of the next or fourth heir or her issue not being in existence to receive it; and that the interest which the third heir may acquire and dispose of by reason of the termination of the estate tail taking place immediately on the death of the fourth heir without issue, is the original fee-simple, which has been allowed to remain fettered so far as to carry it in its fettered state to the third heir in succession, but which has become free from the fetters after it reached the third heir, who founds her claim not as interested under the entail, but as the unfettered fiar and mistress of the estate from the failure of the entail.

We have the singular result of an interest in the third heir which is to be valued under the Act of 1875, and which results from

the failure not of prior, but of subsequent limitations to the fourth heir, which would not be the subject of any compensation if vested in that heir herself. I feel great difficulty in coming to such a conclusion. The only case cited before us was that of *De Virte* (5 Rettie, 328), which certainly seems to have proceeded on some such view, but we have in the present case the opinions of the majority of the Judges in the Court below against that view, and I feel the words of the Act " expectancy or interest in the entailed estate " very strong; and though I should, from the nature and purport of the two Acts of Parliament, be led to the conclusion that so extended a meaning ought not to be given to them, yet the words used would be sufficient to include all the interest in the estate coming to the third heir by the results of the entail and taken from her by the Act. I have had to consider what, from the nature and the purport of these two Acts of Parliament, would be the reasonable and probable construction of the words I have referred to; and I do feel strongly that the nature and intent of the Acts of Parliament are so very clear, that I should have expected to find a different legislative enactment when we come to the enacting part on which this particular question arises. At the same time, I cannot say that I find words in the Act that would support the meaning which some of the learned Judges in the Court below, thought the words I have referred to would bear. If I came to the other conclusion, I should have to do so upon a very forced construction, because, although it is not the entail itself which causes this reversionary interest (as we should call it in England) to fall into possession, still it is in consequence of the reversionary interest that the estate tail cannot be barred; for the Court is not authorized to allow it to be barred, except upon the condition of having the whole " interest or expectancy " of the person against whom they allow it to be barred valued, according to the position which he or she may occupy at the time when the disentailing deed is proposed to be executed. I think this interest in the estate is part of the " interest or expectancy in the entailed estate " directed to be valued.

Therefore it appears to me that we must hold, as my noble and learned friend on the woolsack has proposed, that the decision of the Court below as to the valuation of this interest was erroneous, in so far as it omitted the valuation of this particular interest, which appears to be not only a substantial interest, but one of very considerable value and importance. Moreover, this is a question of principle, and one probably of not very unfrequent occurrence,—therefore it is well to settle it once by a decision of your Lordships' House upon the subject. I agree, therefore, in the order which my noble and learned friend, the Lord Chancellor, has suggested to your Lordships. I think that the right time for the valuation to begin is the date of the execution of the deed, because as soon as the deed is executed it takes its full effect, upon the conditions being complied with. It will not take its full effect until certain conditions have been complied with, but when they have been complied with, it will immediately take its full effect, and its full effect will begin from the date of the execution of the instrument itself. Subject to the figures being correct, as to which I am not prepared to express a clear opinion, I think that the proposed order is right.

As regards the costs of the appeal, the appellants, having succeeded in the case they have brought to your Lordships' bar, must have their costs. According to the common rule, the unsuccessful party must bear the costs of the appeal.

Lord Blackburn.—My Lords, the respondent is heir of entail in possession of an estate held by virtue of a tailzie dated prior to 1 August 1848. Before the Rutherfurd Act (11 and 12 Vic. c. 36), lands might in Scotland be fettered in perpetuity. That Act was passed on a preamble reciting that "the law of entail in Scotland has been found to be attended with serious evils, both to the heirs of entail and to the community at large". The first section provided that entails made by deeds of tailzie dated after 1 August 1848, might be barred by an heir in possession, whenever that heir was born after the date of the entail, and being of the age of twenty-five, or when he could obtain the consent of an heir-apparent born after that date, and of that age: and the second section gave a precisely similar power as regarded entails made by deeds of tailzie before that date, when the heir in possession or heir-apparent was born after 1 August 1848. But if this had been all that was done, the evils arising from so large a portion of land in Scotland being already under the fetters of tailzies would have continued till 2 August 1873, when first it became possible that an heir of the full age of twenty-five, born after 1 August 1848, could exist.

From that time the estates, bound by the fetters of entail made before 1 August 1848, would continually diminish in number, as heirs either in possession or heirs-apparent born after 1 August 1848 attained the age of twenty-five. Such entails would not necessarily come to an end till the lapse of a lifetime from that date, though the number remaining sixty years after 1848 would probably be very few. It was thought proper to make provision for an earlier unfettering of such estates; and by the third section it was provided that an heir of entail in possession under such a tailzie might disentail, if he could obtain the consents of the three nearest heirs who, at the dates of such consents, were for the time entitled to succeed to such estate: and by section 31 the Court of Session might appoint a guardian to any such heir under incapacity, who should be entitled, "with or without consideration", to consent for that heir.

I think the object of the Legislature was to enable the lands to be disentailed, but to give each of the first three heirs an unfettered discretion as to whether they would protect the entail or not, and to allow him to bargain as he pleased with the heir in possession. Such an heir may, and I believe often does, refuse his assent, from a belief that they ought not to disturb the provisions of the person from whom they derive their interest in the estate, or because he thinks it more for the interest of the family to preserve the entail. Where no such motives for refusing consent existed, an heir probably sometimes drove an extortionate bargain. But the heirs whose consent was asked for, often, and (I suppose) the guardians of an infant heir on his behalf usually, said that they required to be paid the fair value of their consent, but would not ask more; and consequently, actuaries were often asked to put a value on such consent, depending as it necessarily

did in a great degree on the probabilities of the duration of life, marriage, and issue. No such valuation, however, could in any way be brought under the review of the Court. * * *

The Legislature has cast on the Court of Session the task of "ascertaining the value", and when it is "ascertained to the satisfaction of the Court", tells them what to do. But the Legislature has given no directions at all as to how or in what manner the Court is to ascertain the value. In the Succession Act (16 and 17 Vic., c. 51, sec. 21) it is directed that interests during periods depending on lives shall be estimated according to the value given in certain tables annexed to that Act, so as to leave nothing to be ascertained except the age of the parties. Nothing of the sort is done in this Act. But I think the Legislature knew that the value of an expectancy must, in a great degree at least, depend on the probabilities of the duration of life, the chances of marriage, and the chances of such marriages proving fruitful. They must, I think, have known that actuaries had tables, founded on extensive experience, on which they acted, which enabled them to value with considerable accuracy the probabilities of life; and that though the experience on which calculations as to the probabilities of marriage and issue were based was much narrower, and the results more subject to uncertainty, yet that some calculation could be made in that way, and none could be made in any other. I think, therefore, that the Legislature must have contemplated that the Court would call in the assistance of an actuary, to report to them on all those matters which properly come within the province of an actuary. Such was the course taken in *De Virte* v. *Wilson*, the only other case in which, as yet, the Court has had to act; and such was the course taken in the present case. No objection was made on either side, but I do not think any could have been taken on either side. But when questions not properly determinable by an actuary arise, I think they must, unless the parties have agreed to take the actuary as arbitrator, be ascertained in the manner appropriate to the particular question. In this case, when a question as to the value of the lands arose, the Court did not refer that to the actuary, but to a valuer.

The first thing to be ascertained by the actuary in this case was the probability of the heir in possession, the respondent, dying without male issue in the lifetime of each of the appellants, for unless he did so they could get nothing. No point has been raised as to the mode in which this has been finally ascertained. The next thing to do was to ascertain the probability of the nearest heir, Captain M'Donald, so dying. The age of the captain, the fact of his being married, and the age of his wife, were not disputed. But the agent for the appellants brought before the actuary this statement (*see* p. 101).

The contention of the respondent is that the actuary ought to proceed solely on the statistics relating to the value of an ordinary life, and that every other averment is irrelevant. I think the Legislature might have enacted this, but I think they have not, and I think it is impossible to hold that no exceptional case can vary the calculation. An extreme case may test this principle—an heir may be a young man, the expectation of whose life would be forty or fifty years. Suppose it to be averred that, before the application to

disentail the estate is made, he met with an accident occasioning injury to the spine, inducing incurable paralysis, so that he must pass the remainder of his life on a couch, and though he may linger for a time cannot live long, would it be proper to say that no enquiry should be made into such an allegation as this? Lord Gifford shrinks from pushing his argument so far, and I think with reason. I think that the probability of life of an average man of that age is *prima facie* the proper basis of an actuary's calculation. And I think Lord Ormidale seems not to deny that there may be cases in which there may be such insuperable difficulty in ascertaining the effect of exceptional circumstances on the value of the life, as to leave no alternative but to take the actual age.

I agree with Lord Gifford that the Court ought not to attend to every allegation of something on which a guess might be made. I think it must be something on which the Court thinks an estimate may be reasonably made. It is easier to put cases as to the expectancy of marriage or issue, in which it would be proper to reject such averments, than as to the expectancy of life. But I can put one. The probability of life of a man residing in England, is certainly greater than that of the same man residing in the tropics. Insurance companies habitually act on this, and protect themselves by putting in their policies a provision that the policy shall be void if the assured, without their licence, goes into prohibited districts—I think generally between the 32d parallel of north latitude and the 32d parallel of south latitude; and they grant such licences on the terms that an increased premium shall be paid whilst he is in the prohibited space. Yet, I think, an averment that the heir of entail was a keen sportsman who would enjoy elephant shooting, or a man of scientific tastes who would like to explore the course of the Niger or the sources of the Nile, and was therefore likely to go to Central Africa, should be rejected as irrelevant, —not on the ground that his doing so would not affect the probability of his living, but on the ground that there could not be any reasonable estimate of the chance of his so travelling. In this I agree with the Lord Justice-Clerk, and I agree with him in thinking that it would be proper in any other case to reconsider the propriety of the decision of the Lord Ordinary, Rutherfurd Clark, in *De Virte* v. *Wilson*, under which the actuary was permitted to make a guess at the chance of Mrs. Wilson's children being, if she came into possession of the estate, liberal to her. In all other respects, Lord Rutherfurd Clark's note on that case seems to me very sound.

But then I cannot think the averments here made are irrelevant on this ground. Had the only averment been that Captain M'Donald had twenty years ago met with a bad accident, I might have hesitated. But it goes much further. It is said that "he has been in very bad health and subject to epileptic fits, and on several occasions his life has been almost despaired of. He had one of these fits so late as April 1877, at which time it was not expected he would survive". It may be that the facts are greatly exaggerated; but, if correctly stated, it becomes, I think, a question for medical skill to say whether such facts, when ascertained, do not show that the vital energy has been diminished, and if so, how much. I think, if that is found, it is a very convenient form to say, as much has been taken out of his vital energy

by this accident as would have been taken out of an ordinary man's vitality by so many years of ordinary life. And then it becomes a question for an actuary—what difference that makes. In the first report of the actuary in this case, he assumed, without, I think, sufficient grounds or sufficient authority, that it had taken as much out of Captain M'Donald's vital energy as twenty years of ordinary life; and then came to the conclusion, which I suppose was correct, that on that supposition about seventy per-cent should be added to the value of expectancy of the appellants. When this case was before the Lord Ordinary, and when it first came before the Court of Session, it was supposed that Captain M'Donald had no objection to submit to a medical examination. The interlocutor of 16 January 1879 was framed in conformity with the views of the Lord Justice-Clerk and Lord Ormidale, but contrary to the view of Lord Gifford. By it the Court found the chances of Captain M'Donald's life ought, in respect of Captain M'Donald's consent, to be ascertained in the ordinary way, by such professional examination and report as the Lord Ordinary may direct; and remitted to the Lord Ordinary to give effect to their findings, and to proceed with the cause; and reserved all questions of expenses. It seems to me that this was a proper and sensible course to take. It turned out, however, that Captain M'Donald did not choose to be examined, and on that change of facts the Lord Justice-Clerk changed his opinion as to the proper course to be pursued, and this interlocutor was recalled, and another substituted :—" 19 March 1879.—Remit to the Lord Ordinary to proceed with the cause on the footing that Captain M'Donald's life is to be assumed to be an average life, and to be estimated according to his present age, and also remit to the Lord Ordinary to dispose of all questions of expenses."

It was argued at the bar that, without a personal examination of Captain M'Donald, no medical opinion could be formed. In this I cannot agree. I do not doubt that it would be more satisfactory that such a personal examination should take place, and I hope that Captain M'Donald will not persist in his objection; but I do not think it indispensable.

I have no skill in medicine myself, but both in civil causes and in criminal causes I have had many medical men give evidence before me, and I think I am warranted in asserting that they, in their diagnosis, attach much importance to the history of the case,—the antecedent facts. These they must in general ascertain as they best can. But a medical man finding a patient suffering from symptoms, and learning that he had recently been a passenger in a railway train when an accident happened, would not disregard that fact, and might probably on that fact think that the symptoms indicated the effect of a shock to the brain. If there was no such fact, but he learned that just before the illness came on the patient had been working in an ill-ventilated mine, or even sitting for hours in an ill-ventilated court or theatre, he might on that fact think that the symptoms indicated the effect of blood poisoning. In the present case, I should expect that a medical man, either ascertaining for himself, if the Court refers it to him to do so, or learning from the findings of the Court what the facts were, might say, " The illness of Captain M'Donald in April 1877 may either have only indicated that he ought to take care not to

expose himself to the sun, but if he does take such care he may live as long as anyone else; in which case I do not see how any estimate can be put on the diminished value of his life. Or it may have indicated that his constitution was impaired. If I had to judge only from the symptoms as reported to me, I should have inclined, with some doubt, to one of these opinions", stating which it was. "But learning that during the three years that have since elapsed, Captain M'Donald has been in good health", or "has been suffering repeated though slight attacks of the same sort" (as the case may be), "I am confirmed in that opinion", or "I am convinced that opinion was erroneous." I think, therefore, it is important in remitting the case, so as to obtain the assistance of a medical man, to provide how such facts, which I think must be important, are to be ascertained.

The second point is one which may be treated more concisely. The heir in possession of an entailed estate in Scotland is the fiar fettered as far as the provisions of the tailzie fetter him, and no further. If there is no one in existence who can enforce these fetters, he has the absolute fee. From this it follows that, if it can be ascertained that all the heirs in existence will die without issue, the last survivor of the existing heirs will have the fee. A person who is in a tontine has a calculable interest in his expectancy of being the last survivor. Ought the analogous expectancy of one of the three heirs of entail to be the last survivor to be valued in this case, where there are four existing heirs after the heir in possession? The actuary has in this case valued it, and (what I should not have anticipated) found that it more than doubles the sum to be paid to these ladies. On objection the Lord Ordinary thought he had done right. But all three Judges of the Second Division were of a contrary opinion. I have doubt and hesitation on this point; and if it rested solely with me, I am not prepared to reverse their decision, that this should not be valued.

The Rutherfurd Act proceeded on the assumption that the interests of the fourth heir under the fetters of the entail were too remote, to make it proper to consult him as to the propriety of disentailing. This interest of the ladies, only arising after the fourth heir has died without issue, is still more remote. The words used in the Amending Act of 1875 are not such as to make me believe that those who used them had the point present to their minds. But the object of the Act being to facilitate the disentailing of estates, if the words used are such as to be capable of bearing the construction that the only thing to be valued is the expectancy under the fetters of the entail whilst they exist, and not the expectation of the fee that may arise after they have expired, I think we should put on the Act of 1875 that construction which will forward the object of the Act. But I am not prepared to dissent from what I understand to be the opinion of both the noble and learned lords who heard the argument, that the words of the Act are not such as to be capable of bearing that construction.

Lastly, comes the question, what is now to be done? I do not think that the valuation, obtained on a wrong principle, can stand; and if that is set aside simply, the disentail which has been allowed cannot stand. But if one of the heirs should die during the protracted litigation, the object of the petitioner, the respondent here, is

baffled; and I think that should not be allowed. The respondent, though willing to pay the sum at which the Court has valued the interests of the two ladies, may not be willing to pay four times that amount. If so, the Court of Session will have to say what is to be done as to the costs incurred under the petition. If he is willing to pay whatever be ultimately found to be the value, I think it would be the reasonable course that he should pay into the bank, or otherwise secure to the satisfaction of the Court of Session, a sum equal to the utmost amount which it is probable will be added to the value at present come to, and that the litigation should proceed (if the parties wish it, which I hope they may not) as to that sum, the estate remaining disentailed. I think the form of order proposed will be proper, if the sum mentioned is the right figure. I have not investigated that.

The costs of the appeal to this House should, I think, be given to the successful party—that is, the appellants.

This order was pronounced :—

" . . . . The appellants having (by their counsel) consented to an immediate disentail of the estates which form the subject matter of the appeal, on the terms of the respondent paying, within twenty-one days, into the British Linen Company's Bank, such a sum as, with the sum already deposited, will make up the sum of £8,122, as a security to answer the valuation hereinafter directed to be made,—It is declared and adjudged by the Lords Spiritual and Temporal, in the Court of Parliament of Her Majesty the Queen assembled, that, in the event of such payment being made within the time aforesaid, the said interlocutor of the Lord Ordinary in Scotland, of 31 May 1879, and the said interlocutor of the Lords of Session there, of the Second Division, of 7 June 1879, complained of in the said appeal, so far as they approve of the instrument to disentail, No. 22 of process, and interpone authority thereto, and grant warrant for recording same in the Register of Entails, be affirmed; and in that event it is ordered, that it be referred to the Court of Session in Scotland to ascertain anew the value in money, as at the date of the instrument of disentail, of the expectancy or interest of the appellants in the estates, within the meaning of the Entail Amendment (Scotland) Act, 1875: In the event of payment of the said sum not being made within the time aforesaid, it is ordered and adjudged, that the interlocutors complained of in the said appeal be reversed *simpliciter:* And it is ordered that the said cause be, and the same is hereby, remitted back to the Court of Session in Scotland, to do therein as shall be just and consistent with this declaration and judgement: And it is further ordered that, in either case, the respondent do pay or cause to be paid to the said appellants, the costs incurred in respect of the said appeal", &c.

It now remained for the Misses M'Donald to furnish evidence as to the state of health and prospects of longevity of their brother, Captain M'Donald. Having ascertained that his life had recently been proposed for insurance to certain insurance companies, they applied to the Court of Session to order the production of the medical reports made to those offices, and the Court, in legal

phraseology, granted a diligence for the recovery of the reports. One of the companies objected to produce the medical reports in its possession, on the ground that they were confidential and came under the class of privileged communications. The Lord Ordinary (Lord Fraser), after hearing counsel, ordered (22 Feb. 1881) the company to produce the reports, and stated the grounds for his decision in the following note:—

The proof in this case has been fixed for 10 March; but preliminary thereto a diligence was granted for the recovery of certain medical reports upon Captain M'Donald's health, made to insurance companies on the occasion of his applications for insurance on his life. One haver declines to produce these documents, on the ground that they are confidential, and come within the class of privileged communications. This objection the haver is entitled to state, although diligence has been granted for their recovery. (*See* M'Donald *v.* M'Donald, 6 D 954).

The tendency of modern decisions is to increase the class of cases coming under the category of privileged communications. Hitherto the protection has been given to the limited class of communications between a husband and his wife, of the client and his legal adviser, of reports by public officers upon matters of state to their superiors. It has also been extended to communications passing between two defenders to a suit (Rose *v.* Medical Invalid Insurance Society, 10 D. 156); and although it has been decided by the English Courts that a medical man who acquires information from his patient, or a Roman Catholic priest who hears the confession of one of his flock, cannot refuse, in a Court of Justice, to disclose the information they possess, yet these decisions have been regretted by later English judges, and none such have been pronounced hitherto by the Scottish courts. The case of the Roman Catholic priest was discussed and considered in McLauglin *v.* Douglas and Kitson (4 Irvine, 273), but no decision was given upon the general point; and as regards the physician, Mr. Greenleaf mentions that a number of the States in America, in the new codes passed by them, have enacted (as in the revised statutes of New York) that "no person duly authorised to practise physic or surgery shall be allowed to disclose any information which he may have acquired in attending any patient in a professional character, and which information was necessary to enable him to prescribe for such patient as a physician, or to do any act for him as a surgeon."

If the present was a case where a communication made to a physician by his patient was sought to be recovered for a purpose antagonistic to the patient, the Court would have to consider whether the English rule, denying the protection, should be followed, or whether the Scotch Courts, not being controlled by precedents, would create one the other way, by ranking this within the class of privileged communications. But the case is not so. An insurance company, in the course of its business, requires to obtain information as to the health of the person wishing to deal with it, promising, at the same time, to the doctor who gives the report that it will be treated as a confidential com-

munication; this does not possess any of those elements which lie at the root of the protection given to the classes above referred to. It may, no doubt, be inconvenient for the insurance company to be compelled to produce the reports of their medical adviser; and it may hamper the medical adviser, in the expression of his opinion, if it were known that, on the occasion of a dispute, the production of these reports in a court of justice could be enforced. But, notwithstanding this, the mischief would be greater if such important evidence were withheld. A decision of the Common Pleas in 1871 is very much in point upon this subject (Mahony *v.* National Widows' Life Assurance Fund, 5 May 1871, 6 Common Pleas, p. 252). The action there was directed against the insurance company to recover the sum insured upon a life. The company pleaded that the policy had been obtained by fraudulent concealment, and misrepresentation of material facts. The plaintiff applied for inspection of, (1), two reports made to the company by private friends of the assured, to whom the company were referred, with relation to the assured's health and habits; and (2), a report made by a medical man, to whom the assured was referred for examination on behalf of the company. At the head of the printed forms of questions upon which these reports were made, were statements that the company would regard the answers given as strictly private and confidential. The court allowed inspection of the documents on the ground that they were not privileged from inspection, and regarded the statement that the report would be considered strictly private to mean no more than this,—that the company would not needlessly disclose it. It is right, however, to notice that the judges in that case did indicate that there might be circumstances that might warrant the extension of the rule as to privileged communications to such reports. Thus Bovill, C. J., said—"I do not say that in every case the Court would order such " documents as these to be produced. The Court has a discretion, " and is bound to exercise it according to the circumstances of the " particular case. It is easy to see that, in some cases, these documents " may be of importance, and in others not. Here there are no grounds " shown by the affidavits why they should not be produced, except " the mere fact that they are stated to be confidential, as between the " insurance office and the parties who wrote them. This is not any " legal ground of privilege." This remark had reference to what seems to be more confidential than the medical report, namely, the report of private friends as to the state of health of the insured. If the Court does possess such a discretion as is thus claimed, it can only be exercised upon very special grounds indeed. If it were made clear that the document, when produced, would not be competent evidence at the trial, that might be a ground for refusing an order to produce it. But such an objection to the production, though it may be suggested by a haver for the consideration of the Court, is one that can only be competently taken by a party to the suit. The medical reports here sought to be recovered have been given (as was stated to the Lord Ordinary) by living men, whose evidence may be obtained at the trial, and therefore it may be said that there is no necessity for admitting the reports which they made years ago; and consequently, it is argued, if any judicial discretion is to be exercised

in the matter, it should be in favour of the objection to production. Whether the evidence be competent or not, is a question that must be argued by the parties to the cause, when the reports are tendered in evidence at the trial, and cannot be determined now; and therefore there is no specialty in the present case, that would induce the exercise of any discretion against the production.

At this stage of the litigation, the parties came to terms; and the claim was compromised by General M'Donald paying his sisters the sum of £6,000, in addition to the costs of the suit.

On the Valuation in Bankruptcy Proceedings of an Annuity determinable on Death or Marriage.

As already stated, several of the points arising in the preceding case are in no way peculiar to the Scotch law, but may probably have to be considered by English courts of law when the values of complicated interests, depending on the contingencies of life and marriage, require to be judicially determined. It was formerly, we believe, held by English courts of law that interests depending on the contingency of marriage were incapable of valuation; but it will be seen from the following report, which we take from the *Weekly Reporter*, vol. xxv, p. 488, that actuaries must now be prepared to advise the courts as to the value of such interests:—

COURT OF APPEAL.—22 March 1877.

From Court of Bankruptcy.

In re Blakemore.—*Ex parte* Blakemore.

An annuity payable to a woman during life or widowhood is not a debt "incapable of being fairly estimated" under section 31 *of the Bankruptcy Act*, 1869.

In such a case, where the annuitant was sixty-five years of age at the date of the liquidation, it was by consent referred to an actuary to determine what (if any) ought to be the deduction in respect of the contingency from the value of an annuity of the same amount during the life of the annuitant. The actuary was to estimate the value of the annuity at the date of the liquidation on the footing of the debtor being solvent, and proof was to be admitted for the value when assessed.

This was an appeal from Mr. Registrar Murray, sitting as Chief Judge.

John Blakemore, by his will, dated 1 February 1868, bequeathed, in certain events that happened, his trade and business of a cabinet-maker and upholsterer at Leamington Priors, and the capital and

stock engaged therein, to his two sons, John and Alfred Blakemore, in equal shares, subject and charged with the payment unto his wife, Maria Blakemore, during her life and widowhood, of an annuity of £80, to be paid as therein mentioned; and in order the better to secure the payments thereof, the testator declared that before his said sons should be entitled to the benefit of the said bequest they should, at their expense, execute a joint and several bond, in a sufficient penal sum for securing the payment of the said annuity, and should deliver the said bond to his said wife, but such bond should be in addition to, and not in substitution of, any rights and remedies which his said wife might have for the same under his will.

The testator died on 23 August 1868, and his will was duly proved.

In accordance with the provisions of the will, John Blakemore (the son) and Alfred Blakemore duly executed a joint and several bond, dated 8 January 1874, in the penal sum of £2000, the condition being that the bond should be void in case the said annuity should be duly paid.

John and Alfred Blakemore in the year 1876 filed a petition for liquidation by arrangement, and Ebenezer Charles Foreman was appointed trustee.

At the time of the liquidation, Maria Blakemore was sixty-five years of age. She claimed to prove in the liquidation for the sum of £657 as the value, at the date of the liquidation, of her annuity of £80, such being the sum given as the value of an annuity of that amount on her life in the schedule to the Succession Duty Act, 1853.

On 13 January 1877, the trustee rejected the proof, on the ground of the annuity being determinable upon a contingency incapable of valuation.

Maria Blakemore applied to the Court to admit the proof, but Mr. Registrar Murray, sitting as Chief Judge, refused the application.

She then appealed to this Court.

On the appeal,

Winslow, *Q.C.*, and *Finlay Knight*, for the appellant.—This is a debt capable of being fairly estimated. The 31st section of the Bankruptcy Act, 1869, was expressly passed to meet such a case as the present, and the fifth sub-section shows that the mode of valuation may be the opinion of a person only. The *onus* is on the trustee to set a value on the contingency, and if we object to such value we can come to the Court.

Roxburgh, *Q.C.*, and *Romer*, for the trustee.—This is not a debt proveable in the liquidation; the only debts proveable under the section are those capable of being judicially estimated, and this is clearly not such a case. As to a valuation being assessed as a mere "matter of opinion", that does not mean mere guess work, but a value founded on some basis of judicial decision. Your lordships must look at this particular case, and not at any general law of average governing such a case. How can your lordships say what is the chance of this widow marrying again?

James, L.J.—I am of opinion that we cannot judicially declare that this contingency is incapable of being fairly estimated. Like every other contingency affecting human life or human conduct it is, I think, capable of estimation. No doubt it is uncertain whether the

appellant will marry again, just as the duration of any particular life is uncertain. But though the duration of a particular life is uncertain, the ordinary duration of life at a given age is reduced to a certainty when you have regard to a million of lives. The value of the expectation of life is arrived at by an average deduced from practical experience. The intention of the Act was to get rid of the difficulties which had arisen in former cases, in which very great hardships had been inflicted both upon creditors who were deprived of any share of the bankrupt's assets, and upon the bankrupt who remained liable to the creditors' claims. I think that section 31 was intended to apply to all cases of this kind, and pre-eminently to the case of an annuity determinable on a contingency. We cannot say that the contingency is incapable of valuation.

MELLISH, L.J., and BAGGALLAY, J.A., concurred.

It was then arranged that it should be referred to the actuary of the *Law Life Assurance Society* to ascertain the value of the annuity as a simple life annuity, and then deduct from such value the sum which he should think to be the proper deduction for the contingency. The valuation to be made as at the commencement of the liquidation, and upon the supposition that the debtors remained solvent. Proof was to be admitted for the value thus ascertained, and the appellant was to have her costs of the appeal, and in the court below.

Appeal allowed.

The correspondent to whom we are indebted for drawing our attention to the above report, states that, with due deference to the Lords Justices, he thinks the Court below was right in deciding that an annuity payable to a woman during her life or widowhood, is incapable of valuation. He says that, for many years past, he has had the same question constantly arising at the Legacy and Succession Duty Offices, and has found that the authorities there consider that there are no tables or statistics upon which to value an annuity of the above kind; and therefore, in passing the account of an estate charged with such an annuity, they will not allow the *value* of the annuity to be deducted, and the account at once closed, but require actual capital to be set apart and deducted on the account, and the duty paid on such capital when the annuity ceases by death or marriage. It is quite true, he admits, that the whole course of recent legislation in bankruptcy has been in the direction of having the whole liabilities of a bankrupt valued, so that the creditor may prove his debt and receive a dividend upon it, and the bankrupt may be thenceforward free from the liability; but he cannot believe that the intention of the Legislature in enacting section 31 of the Bankruptcy Act was to go so far as to require the valuation of a liability for the making of which valuation there exist no sufficient materials. The question therefore is: Are there any sufficient statistics whereon to

found such a valuation as the one in question? He has been told repeatedly that there are not, and he believes that the Legacy and Succession Duty Offices will not act on the decision *re* Blakemore, and will not accept any actuarial valuation of an annuity determinable on marriage,—at all events, not until a trustworthy table has been published.

It seems to us, however, that it is matter of comparatively little consequence whether there are at present in existence tables of the values of annuities terminable on marriage as well as death; for the probabilities of marriage can be ascertained in the same manner, and probably with the same accuracy, as the probabilities of death; and these being ascertained, the values of annuities terminable upon marriage or death can be calculated at any desired rate of interest, in the manner explained in Mr. Sprague's paper *On the Construction of a Combined Marriage and Mortality Table, J.I.A.*, xxi, 406. There is, therefore, no theoretical difficulty in the determination of the value of an annuity terminable upon marriage or death; and it seems to us that it may be properly held that the Legislature intended that, when it became necessary for a court to determine judicially the value of such an annuity, the responsibility should be thrown upon the Court of having the necessary tables constructed. We are quite satisfied that, when the Courts shall require the assistance of actuaries for this purpose, they will not be found wanting. In addition to the published statistics on the probability of marriage, among which we may mention the tables in Mr. Sprague's paper above referred to, and his paper *On the Rates of Marriage among Widows and Spinsters, J.I.A.*, xxii, 352, a considerable mass of statistics has been accumulated from the experience of the various Widows' Funds that exist in Scotland and elsewhere; and it has been necessary for the actuaries who have from time to time reported on the financial position of those funds, to estimate the value of annuities terminable on remarriage; and we believe they have done so with very considerable accuracy. If it became common for the courts of law to refer questions of this sort to the opinions of actuaries, we have no doubt that, although differences of opinion might at first be found to exist, in process of years, as more accurate and more extensive statistics were accumulated, these differences would become less, until the value of an annuity terminable on remarriage would be calculated with as great accuracy as an ordinary annuity. In the present case, it appears not to have

been thought necessary to have a table of values constructed, but the actuary of the *Law Life Assurance Society* was directed to deduct from the value of the simple life annuity what he considered a proper allowance for the contingency of marriage. We presume that, as in Scotland, the actuary's report in such a case would not be final, but would be open to review if any party interested was dissatisfied and thought it worth his while to challenge the accuracy of the opinion; if, for instance, the widow thought too large a deduction was made, or the other creditors considered the deduction too small; but questions of this sort do not seem to have occurred in England to the same extent as in Scotland.

Does Life-Insurance Insure? *By* Titus Munson Coan.

[From *Harper's Monthly Magazine*, January 1881.]

I.—The Amount of the American Business.

CHANCELLOR KENT said in 1828: "Nothing can appear to an English or American lawyer more idle than the alarm of the French jurist, or more harmless than an insurance upon life, which operates kindly and charitably in favour of dependent families."

Chancellor Kent had in mind the idea of a life-insurance business which should be strictly confined to life-insurance; which should be built upon just arithmetic, and managed honestly, wisely, and economically, by upright and able men. "Life-insurance", wrote a Massachusetts commissioner in 1873, "consists mainly in receiving premiums, investing them at compound interest, and out of the accumulation paying the sums when deaths occur." Surely nothing could be more harmless, or more beneficial in the way of a public trust, than life-insurance; and the trust has grown to proportions which indeed call for upright and able men in its management. During the fifty years since Chancellor Kent wrote as above, Great Britain and Ireland have come to assure £387,000,000 on 810,000 lives; the United States assures £541,000,000 on 1,100,000 lives; while the French, who save more money annually than any other country, assure but £78,000,000 on only 198,000 lives. Is the Frenchman's caution or the American's confidence the better justified by experience? "There are in this country more than 500,000 families who have voluntarily subjected themselves to a tax, amounting in the aggregate to

about £20,000,000 a year, and are under bonds, more or less, in the aggregate amount of about £80,000,000 to continue to pay this tax for life or for a long period."*

Let us look at some of the facts in their experience. I will keep mostly to recent dates, in order to avoid the objection which may be made that life-insurance is an essentially different thing to-day from what it was a few years ago. How far it has been bettered, and how far it still needs improving, I will try to show.

Since 1861 thirty-six companies have started in New York State alone: in March of this year only four of them remained. From 1859 to 1878, fifty-two companies ceased doing business in this State: the most of them failed. Of all American life-insurance companies, two have failed, thus far, to one that survives; while not one of our surviving large companies has yet reached the critical period of its career—the age when heavy pressure from death claims might be expected. But that pressure, it must be added, is not likely to be put upon any of our companies very soon, for the sufficient reason that our companies confiscate the vast majority of the policies for non-payment of premiums. This is done generally at an early period in the so-called investment. *The average duration of an American policy is only about seven years.* Of the multitude of policies which terminate yearly in our companies, only "one in ten matures by death; the other nine mature by causes other than death."† Or, as a searching critic of the subject, Professor Van Amringe, of Columbia College, has put it: "Of every ten policies which cease, but one will cease by death and expiry. One and a-half will be given up for a slight compensation, and seven and a-half will be absolutely thrown away by the holders."

The amount now at risk in the American companies—£541,000,000—though less than it was a few years ago, is more than one-twelfth of the entire capital wealth of the Union. Their yearly income is more than half the yearly accumulation of wealth in the German Empire. In New York State thirty-four companies were doing business at the end of 1878. They had over 600,000 policies outstanding, assuring £296,200,000. Their assets were £80,800,000—more than the value of the entire cotton crop of

* *Traps Baited with Orphan.* By ELIZUR WRIGHT, Ex-insurance Commissioner. Boston, 1877.

† Testimony of Sheppard Homans. Assembly Documents for 1877, No. 103, pp. 348–350. Mr. Homans adds, "The odium attached to the forfeiture of so many policies has made it very difficult to get new business." One would hope so.

the world. Their income for the year was £16,000,000—a sum equal to twice the American tobacco crop of the year, and to more than the entire potato crop; or equal, again, to the entire silk crop of India, China, and Japan.*

II.—The Getting of the Money.

How have the companies come into the possession of this money? By inducing the public to pay it to them in premiums, and by the interest from investments. Most of their bargains with the public are made in the shape of life policies; and endowment assurance policies constitute about a fifth of the business. The first question is, Do the insurance companies make fair bargains with the public? Our annual accumulation of wealth is greater than that of any other nation, and we are paying about one-ninth of it to the life-insurance people. Is that too much or too little? Have the companies charged an equitable price, say within £8,000,000 per year, for the insurance that they have promised?

1. They make a large profit on receiving interest at a higher, and paying it (when they pay it) at a lower rate. They calculate the interest which they promise to pay at four per-cent; they receive six or seven per-cent. This is legitimate enough, but the policyholders are commonly led to think that they are to get much more than four per-cent.

2. In computing the risks of their business, the companies use "a table which gives a death rate, on the whole, considerably larger than that which it expects in practice." Having computed the chances below the average, they then pick out the lives that are above the average, and most of the companies refuse to insure any other. A man cannot go in from the street and claim life-insurance at the average mortuary rates: he will not be accepted unless he can satisfy the medical examiners that his health and strength are better than the average. This selection of risks is another source of great profit. One New York company reported its gain from this source, for eleven months of 1869, as £129,800. Did that company charge too little or too much in making its bargains?

* * * * *

* Mulhall's *Progress of the World.* London, 1880. In this article I give the nearest round numbers, whenever round numbers will help to make the case clearer.

3. [This is based on Mr. E. Wright's theory of self-insurance, and would therefore be unintelligible to our readers. Accordingly, we print only a portion of it.] The loading is the charge for expenses. But how great is this charge? The companies make a charge amounting to one-third of the gross or office premium—say thirty-three per-cent of the many millions per annum which they receive directly from the public. This would seem, indeed, enough to conduct the expenses of the business. An expert estimate gives 18 per-cent as sufficient, even under the present lavish system, to pay all legitimate expenses of management.

4. Another source of great profit to the companies has been the buying-up of policies. This, happily, is not so common as it was. How it was managed we may learn from the instructive testimony of Mr. Stephen English.*

"*Q.* [Mr. Moak]. What were the irregularities which you complained of in regard to the Continental Life?

"*A.* Robbery and plunder.

"*Q.* In what way?

"*A.* They sent agents out all through the West; they would call upon a poor unsuspecting policyholder, and by telling him the company was insolvent, induce him to give up the policy for a small amount, and then pocket the reserve.

"*Q.* In other words, if the reserve was £600, they would get the policy surrendered for a small sum?

"*A.* Yes, for £40; and then pocket the difference.

"*Q.* Who would?

"*A.* The president and vice-president: they have run away."

5. The profit from all these sources is many millions per year. Yet even these profits do not account for half of the enormous wealth of the surviving companies. They have a fifth source of profit, which is more abundant than any or all of these four; namely, the confiscation, as already said, of the vast majority of the policies. In spite of the law of 1879, to be mentioned presently, the forfeiture of the policy and the total confiscation of the reserve is still the fate of all but a small minority of the insured. Here are some of the facts: In 1871 lapse and surrender swallowed up 93 per-cent of the number of policies that were terminated in the New York business (Van Amringe). In 1876 one company confiscated about 2,500 policies—nearly the same number that it issued during the year; 1,254 of them were absolutely forfeited, the holders getting nothing for them. In

* New York Assembly Documents, 1877, No. 103, p. 250.

another company, during the same year, about 3,000 policies were terminated, only about 300 of these by death. Of the balance, about 150 were reinstated; the rest, over 2,000 in number, were absolutely forfeited. Still another company, in the same year, 1876, issued 8,000 policies, and confiscated (always legally) no less than 7,500. Of these, however, it bought up, according to its actuary's testimony, a large number "simply as a gratuity", being "not legally bound to pay anything" to the policyholder who is behindhand with his premiums.* During the year 1879 the same company reports 8,615 of its policies as terminated; the company is 37 years old, and yet only 1,156 of these policies were terminated by death! How many lapses and surrenders there were in its business does not appear in its annual report. The latest State returns are for 1878. During that year the number of policies terminated in the New York business was 87,222. Of these, 11,357 terminated by death and expiry; while 57,895 were terminated by lapse and surrender, representing the failure of £30,000,000 of insurance. The premiums paid upon those policies, and the profits accruing thereon, remain for the most part as profits to the companies. Do their officers lessen the charges for outstanding insurance in view of such experience as this? By no means; "'tis not in the bond". But some of them have announced lessened charges for the future.

The upshot of this slaughtering of policies, which forms the leading feature of our life-insurance thus far is, as we have seen, that the average duration, or "life", of a policy in an American company is but seven years. On the so-called "life sentences" in our State prisons the actual average term of imprisonment is computed at the same number of years. This is more than a curious coincidence: it illustrates another of those great illusions which rule our much-believing, soon-forgetting, long-suffering community.

I am speaking of the present as well as the past condition of the case as regards most of the policies heretofore issued and now issuing. Their usual fate is forfeiture. But, for the policies of the future, this principal source of gain to the companies is checked by a recent law, the New York State law of 1879. That law provides that no policy issued after 1 January 1880, and kept up for three years, shall be forfeited by subsequent non-payment of premiums; but that the reserve, computed at 4½ per-cent,

* Assembly Documents, 1877, No. 93, p. 23.

which shall have accumulated on the policy, shall be applied, according to its amount, to the purchase of further insurance. This further insurance may be either a continuance of the original policy as long as the reserve will pay for it; or paid-up insurance under similar conditions to those of the original policy—of course for a smaller amount, and generally for a very much smaller amount. In one case the law allows a cash payment. In the case of failure to keep up one's endowment policy, the excess of the reserve, if there be any excess at the end of the term of years, shall be paid to the policyholder if he be still living. The new Massachusetts law (Act of 23 April 1880) provides that on policies issued after 1 January 1881, the holder who shall have paid his premiums for two years shall be credited, if he fail to make further payments, with either life or endowment insurance for such an amount as his reserve, less eight per-cent charges, will purchase. If he have no wife or dependent child, he may draw this reserve in cash. A fair surrender value on ordinary life policies is one of the chief things needful in future insurance. But these laws will be a great deliverance to those who shall entrust themselves hereafter to life-insurance under the present system. They would have restored hundreds of millions of dollars to American families had they been enacted and enforced thirty years ago.

These are the chief sources of the profits of the companies, and we see that they are vastly in excess of equity, and that the cost of life-insurance is proportionately too great. This matter is summarized in a few words of expert testimony which I will quote from the State investigation of 1877. The case supposed is that of a policy of £1,000, payable at death, the holder being assured at the age of 25, and paying £20 annually:

"*Q.* [Mr. Moak]. Of an annual premium of £20, £6 would be for loading, £5 for mortality, and £9 for reserve; to secure for himself such an insurance at his age he pays £20, and gets just £5 worth?

"*A.* [Mr. Sheppard Homans]. Of insurance, yes.

"*Q.* In other words, on the mutual plan, without expense [of agents and management, &c.], £5 would pay for just as much insurance as he gets now by paying £20?

"*A.* Certainly.

"*Q.* You say £5 actually pays for the benefits which the man actually gets in insurance for which he now has to pay £20, or, in other words, which the present system requires him to pay?

"*A.* Certainly."

—Considering, I should add, that at 25 the insurer's expectation of life is 39 years, while that of his policy is but 7 years!

Of these excessive profits the agents promise to return a great part in dividends; a small part of them is actually so returned. But the whole system of dividends to policyholders is a vicious one. The promised dividends are a bait to the public, and the occasion of an irresistible temptation to the officers of the companies: the excessive premiums which make them possible demoralize the business. The more nearly a premium approaches the minimum that is consistent with security, the better both for the companies and for the insured.

III.—The Spending of the Money.

It may be said, Grant the fact of these abuses: do not these very overcharges make the companies stronger, and so accrue to the benefit of the policyholders? They would if the companies used the money in that way. But we have seen already that the companies, even the oldest ones, let but a small part of the policyholders' money go back to them. The agents of the companies distribute little tracts, pleading with the public for their salvation by life-insurance, and pleading with more zeal and at greater expense than any tract society. One company admits paying in one year (1876) the sum of £12,000 for "printing and stationery." And in these tracts they announce, among many other good things, the actual payment of large sums upon their policies. But let us see what large sums, to which attention is not called in their tracts, they spend on other objects than the payment of insurance to the beneficiaries of their policies.

The New York companies reported £16,000,000 income for 1878, and £14,400,000 expenditures. A considerable part of this sum was doubtless paid to widows and orphans—how much, the report does not make clear. Mr. Wright estimates that of £20,000,000 paid annually in premiums to all American companies, about one-fourth is annually returned to the beneficiaries. In the New York business of 1878, as reported by the companies themselves, £600,000 were paid to agents; £600,000 more went for salaries, medical fees, "and other charges of employees"; and a lump sum of £1,000,000 is reported, without any explanation, under the heading, "All other expenditures". This makes £2,200,000 per annum for running the business in New York State.

Nothing is harder than to analyse accounts in which there are concealments and evasions. But we may gather from more sources than one some interesting details respecting the ways in which our insurance companies scatter the policyholders' money. They are, principally, payments to agents; payments in salaries, fees, and bonuses; cost of buildings; loss in speculation and in bad investments; and loss by legal plunder and wrecking. A glance at each of these great drains will be enough.

1. Payments to agents, called the "cost of getting the business". During the years 1876, 1877, and 1879, a single New York company paid nearly £400,000 to agents. If we look back to the times of inflation in the business, we shall find still higher figures. During five years, 1867–1871, the authorized companies of New York State paid £8,000,000 to agents, and but £15,000,000 during the same period for losses and claims. These are the reported figures; but the usual practice has been to pay the agents 30 per-cent of the first premium paid, and seven and a-half per-cent on from six to ten annual renewals. Thus the agent gets £75 out of £700 paid in £100 premiums for seven years; but seven years, as we have seen, is the average limit of the existence of an American policy. As the great majority of the policies last less than seven years, the agent receives at least 15 per-cent of the whole premium payments. The agents get less now than formerly, because the business is less; but the community is still paying to them a tax of probably £2,000,000 per annum. Millions past our counting have been won from the public by their solicitations. The agents persuade people into insurance that does not insure. The reformed system will insure, but *it will conduct the business without agents.* It should be known to those of us who are interested, that the strongest English company, the Equitable Assurance of London, built up its great business and its reserve of £11,000,000 without employing agents; it has "never paid any commission at all".* "Good wine needs no bush", and there will be no need to tease people into insurance that does insure.

2. Salaries and bonuses form an important item in the "cost of conducting the business". Salaries, indeed, there must be, and under the present system not a few of them, in order to get men enough, and capable ones, to manage a business that is injuriously enlarged and complicated by questions of dividends

* Assembly Documents, 1877, No. 103, p. 351.

and profits. But even under this system the salaries should be kept within bounds. *Est modus in rebus*; there is, or should be, a limit to everything, even to life-insurance salaries. They should be paid only to competent persons; and they should not be supplemented by bonuses. In 1879 ten New York companies paid for salaries and "other compensation", to employees in their home offices only, £191,600; this does not include medical fees. And a single president admitted, at the State examination of 1877, that he had received £105,181 from the funds of his company during eighteen years, in salaries and "extra compensation".*

In one great company, it was testified at the same examination, "There were bonuses paid to the officers for a series of years, and put down as dividends to policyholders".† The vice-president of that company testified as follows about them:

"*Q.* [Mr. Moak]. How much did the aggregate of the bonuses amount to?

"*A.* I have no recollection, sir.

"*Q.* Can't you give us within £100,000?

"*A.* Hardly."‡

He adds that after continuing this practice two or three years, the officers of the company "discovered it gave rise to comments", and stopped the bonuses. The practice, one would say, might profitably "give rise to comments" both on the part of policyholders and others. So also might the conduct of the president of the same company. The editor of the *Insurance Times* testified that this president's "son had a policy, and he allowed it to lapse, and at his death the policy was revived." In that company, out of 8,595 policies terminated in 1878, 6,300 were terminated by lapse and surrender. None of these policies were "revived". In 1879 it terminated 8,615 risks, but only 1,156 of these were terminated by death.

3. Miscellaneous extravagance. Under the head of "all other expenditures", the New York companies report an item for 1877 of £1,300,000, leaving the public to guess how they spent the money. A part of it goes for the costly buildings which the insurance companies think it necessary to put up. The president of one New York company, after stating the amount of his salary to be £7,500 per annum, went on to say that his company had

* Testimony of H. B. Hyde. Assembly Documents, 1877, No. 93, p. 35.

† Testimony of Stephen English. Document 103, p. 246.

‡ Testimony of Richard A. McCurdy. Document 103, p. 98.

paid about £800,000 for their building in New York, and more than £200,000 for their building in Boston:* for the two, more than half the estimated total cost of the Cologne Cathedral (£2,000,000). The company's officers claim that their buildings are good investments, but whether they are or not is a hard question for an outsider. But another extravagance which I will mention is clearly not a good investment. A vice-president of a leading New York company testified in 1877 that the cost of "luncheons" given to 117 officers and clerks in the New York office, was about £1,200 per year; and he "could not say" that the cost of luncheons and wine dinners (the wine dinners were stopped in 1876) did not exceed £2,000 per year.†

4. Bad investments and speculation. Of course, there is no absolute safeguard in any business against making bad investments. But as the insurance people have got the people's money simply by asking for it, and in vastly greater amount than is needed for fair and honest insurance, the result is that on the one hand the companies indulge in unnecessary and extravagant expenses of many kinds, while on the other they take risks wantonly, and many of them have been ruined in consequence. But I have not space to give details on this point, to which I may return another time; nor can I give more than a glance at the facts respecting the last way I will here mention in which the policyholders' trust is scattered to the winds.

5. The wrecking of the companies. "The State", says a writer in the *International Review*, "has never saved a company. It has connived at the ruin of many, and has itself ruined some which needed only patience to be cured. . . . The State is like a physician who, finding a patient with symptoms of a grave disease, should kill him at once lest he die of the complaint hereafter." In the "transfers" of eight companies (1871 to 1877) to the Universal, the same writer estimates that £5,000,000 worth of policies disappeared, and that in the Universal nearly £15,000,000 finally disappeared within that period of seven years.

IV.—Remedies.

1. The great majority of our policyholders are in the grasp of the old system. What shall they do? Mr. Wright holds out this

* Testimony of H. B. Hyde. Document 93, p. 46; 103, p. 218.

† This is the same vice-president who could not recollect within £100,000 the amount of the bonuses given to officers in his company.

hope, and it is their principal hope. He says: "The present policyholders are bound by contracts which they never would have made if they had fully understood the subject. If they will unite, they can oblige the Supreme Court of the United States to decide whether their rights, under these contracts, are not as good as if they had been rebels." (In the seceding States policyholders were allowed their reserve by just decision of the Supreme Court.) Without such a decision, the great majority of the policies now in force will in the future lapse, as they have lapsed in the past.

2. For the future, the individual may secure himself to a certain degree. "Insist upon having inserted in the policy how much *cash* the company will pay at the end of each and every policy year in case of surrender. There are at least twenty solvent companies in this country that will do it when this demand is loud enough and general enough." Remember that "whether a man will be able to pay in some future year is often as much a matter of uncertainty as whether he will die in that year."

3. Take an endowment policy for a long term. Never take a whole-life policy, to embarrass the declining and unproductive years of life. A life policy, if it is kept up, becomes "a trap which screws up tighter and tighter, till liberation comes by death." A friend of mine has already paid on a life policy for £2,000 the sum of £2,600 in premiums. Counting interest at the legal rates on the payments, he is already some £3,000 out of pocket; and though he is no longer young, he bids fair to be called upon to pay premiums for more than a few years to come. I will not call his case a common one, for very few policyholders keep up their insurance as long as he has done. But for those who do, it is a losing investment, even where dividends have been paid; it is a game in which one cannot win even by dying.

Our present system of life-insurance, in a word, needs radical reform, or it will perish. The public has entrusted to the companies a money interest that is far too great for the honesty, ability, and prudence that have been brought to its management. The public has entrusted this money to the companies in the absence of legal safeguards; these are, at last, slowly and painfully, and by an unequal struggle, being raised around this great trust. Never in history or mythology has such a rain of gold as this descended upon the heads of common men. The way in which they have misused it forms the strongest of comments upon the danger to our community, which thinks itself a free one, from

the overmastering power of great moneyed corporations—the danger long ago pointed out by Tocqueville.

For the companies of the future a greatly decreased cost of insurance, a greatly decreased cost of management, and summary punishment of peculation, must be the rules. The companies should be strictly mutual; there should be no baiting of traps with dividends; the companies must stick to life-insurance, and leave dividends to the banks. For the policyholders the aid of wise legislation is to be sought. The hasty abandonment of policies chiefly benefits the companies. Mr. Elizur Wright, Mr. Homans, and other able and honest actuaries, have their plans, which I cannot now describe, for making insurance safer and cheaper; and the experiment of ready-money insurance (like that of the New York Stock Exchange) may be tried and elaborated according to need, the principle being that each person assured shall contribute a stipulated sum on the occurrence of a death in the society. Under this system (extended to include endowment policies) there are no accumulations to be squandered, and no great expenses of management. It is worth trying, until such time at least as the instituted life-insurance shall be radically reformed, and extend its promised and needed blessings to the country that hitherto it has plundered.

Life-Assurance does Assure. *By* Rev. Stephen H. Tyng, *Jun.*

[Abridged from *Harper's Monthly Magazine*, April 1881.]

As a policyholder, I am impelled by recent strictures upon life-insurance to re-examine the reasons of my confidence. Does life-assurance assure? The question in debate is the trustworthiness and economy of the science and the system. The examination that I now make is in the interest of no company, and in controversy with no critic.

What Life-Insurance has done.

The life-insurance system has been for two centuries a positive force in the progress of modern civilization and the accumulation of national wealth. It has been an important educational factor of every community which it has influenced, in habits of economy, prudence, and providence. And it stands to-day side by side with the savings-bank and the trust company, sharing the con-

fidence with which men who seek the welfare of their fellows crown all three.

Its special plea is a provision against the unequal risks of life, and its peculiar feature is an interest-bearing fund, to which its prospective participants contribute each his share.

More than £4,200,000 was divided, in sums averaging about £500, among widows and orphans by the companies which reported to the New York Insurance Department in 1879. Nearly £1,800,000 in addition was paid on the maturity of endowment policies. Above £2,400,000 was returned to twenty thousand persons who have voluntarily withdrawn.

In all forms more than £11,200,000 was thus distributed in 1879. Since 1 January 1859, nearly £152,000,000 has been distributed by this system among policyholders for death claims, endowments, surrender values, and dividends, by the companies doing business in New York.

And yet, in the face of such facts, we are told that life-insurance has outlived its usefulness, and that a newer and better plan has been evolved from it.

The Life-Insurance Bargain.

In judging any company, the order of criticism should be, first, the character of the men managing its affairs; second, the safeguards of the system accepted by them; third, the fact that it is firmly established, having successfully surmounted the diseases incident to youth; and fourth, the relation of its resources to its liabilities. These are the facts with which life-insurance agents deal. They have a merchandise for sale. In this aspect of their dual office they are "drummers" for trade.* The policy is a contract enforceable by law, but dependent,

* In so widely diffused a population as ours, agents are an essential element of any successful system of commerce. The Equitable Company of London has never employed agents with their proper name, but it has district secretaries, with fixed salaries, instead of contingent commissions, to whom are entrusted local interests, and from whom is expected the increase of its business. To all intents they are the same as our solicitors. The real result of the policy of ignoring agents, and refusing to pay commissions, which has been adduced as the feature of this traditional champion and exponent of a wise economy, is easily shown. Originated in 1756, the company's funds had accumulated in 1839 to £10,689,932. But from that time the decadence and depletion have been rapid, without even a spasm of recovery. In 1849 the fund had fallen to £8,858,047; in 1859, to £6,564,671; in 1869, to £4,609,736; and in 1879, to £4,246,474. Meanwhile its policy issues have dropped to a minimum; for in 1879 the company issued only 136 policies, insuring £185,050, the new premium income of that year being but £6,357, while the management expenses were

perhaps more than any other into which we enter, upon the integrity of the men with whom we covenant. The bargain is fair if we make it. Dry-goods, food, and insurance, are worth their market price to the purchaser. What avails it to tell the buyer that the seller is either making or losing money by the operation? If he want the fabric or provision, he must and he will pay the price. It is equally foreign to the argument to urge upon him, after he has paid his money, that the seller has paid large commissions to his drummers, clerks, porters, besides an enormous salary to himself, and that he has such facilities to loan money that he has been able to command a higher rate of interest than is ruling generally. In the face of all such counsels, purchasers and policyholders will, year after year, return to the old stand, if they are satisfied with the commodity, until they come to be called *customers*. The law of supply and demand will control in this as in all other forms of barter or traffic.

The Scientific Security of the System.

The next question of interest to the buyer is the ability of the principal to deliver the goods sold by the agent. This is answered by the good repute of the house with which he deals, and the certified resources at its command. The examinations of the State, year after year, give him in assurance a more certain basis for his confidence than any mercantile agency can possibly furnish to the trade. He has not only a sworn certificate of the financial condition of the company with which he contracts, but the impartial testimony of experts deputed for the purpose of audit by the Insurance Department. What are the facts which the applicant in this year, 1881, may compare before he commits himself to this covenant? In all the companies reporting to the State in 1879, there was £65,644,962 of re-insurance reserve. This is the amount necessary to cover all liabilities under contract. But besides this there was £13,055,544 of surplus as regards policyholders. All this is the property of nearly 600,000 outstanding policies, or, besides security for the amount pledged to each, the companies hold as trustees an average sum of £22 for

£8,307, of which directors' fees alone absorbed £2,937, or 35½ per-cent. So much for the vaunted value and economy of the non-agency, non-commission policy of the Equitable of England.

[We presume that few of our readers will require to be told that this statement is not correct, and that the *Equitable* of London has no district secretaries.—Ed. *J.I.A.*]

every policy at risk. The case is still stronger if the three largest New York companies are considered in their separate responsibility. The wider the scope of business, the better do the averages work, and the greater the security of the assured. These companies report £27,225,878 of re-insurance reserve, and £5,215,093 of surplus applicable on policies, or absolute security for all risks, and £27 held as an average additional property of each policy. The moral integrity of the managers being assumed, there can be no reasonable doubt of their financial ability to fulfil to the letter every maturing contract.

An additional and most impressive enforcement of both these factors of confidence, is gained by a long look through the past. The figures make an argument for the reliability of the system, which no rhetoric can overturn. From the New York life-insurance reports, since the organization of the department to its last report, as of 31 December 1879, I glean my statistical compend. The 31 companies of this and other States, whose figures appear in the tables of the twenty-first annual report, received from policy-holders in premiums, cash and note, during the twenty-one years, or such portions of that period as are covered by their reports, £179,675,206.

During the same term these companies paid to policyholders, for death claims, matured endowments, annuities, for lapsed policies, surrendered and purchased policies, and for dividends, £120,614,623. The assets of the same companies on 31 December 1879, amounted to £80,303,159, less amount of capital stock, £861,380, or a net sum of £79,441,779. Let this be added to the amount distributed, and we have £200,056,402 as the total paid to, or now held in trust for, policyholders. But if from this latter amount be deducted the receipts from policyholders, there remains, as the past and prospective gain over the payments by policyholders, £20,381,196.

This statement, which is incontrovertible, shows the positive side of security as no figures of speech could pourtray it. But to this let me add a similar comparison bearing upon the relative advantages of large or small companies to the policyholder. The three largest New York companies, during the period covered by the reports from which I quote, received as premiums £72,707,683, and paid to policyholders £48,225,280. Their assets, less capital, were £32,794,483, making the past and prospective net profits to policyholders, derived from interest, over and above premiums paid by them, £8,312,080—or more than 41 per-cent of the gain to

policyholders of all the 31 companies. These enormous assets and gains make the contract of life-insurance "double sure", unless rogues are in the direction of the companies.

Life-Insurance Failures.

As a matter of history, I think it will be admitted by all critics that in no series of facts has "the law of the survival of the fittest" been more undeniably illustrated than in the record of life-insurance companies. During war times and in the after "boom", there were many abortive attempts on the part of adventurers to embark in this business, and by men who were ignorant of the principles of the science and inexperienced in the details of the system, and who, moreover, had been unsuccessful in other fields. But competition with the old and sound companies compelled such moribund institutions to close their doors and books.* Not all of them were dishonest. Not a few voluntarily retired from business after making adequate provision for all their liabilities. The failure of so many life companies suggests another fact which sustains my argument for the strength of surviving companies. During the seventeen years of depreciated currency (1862–1878), when the mean average premium on gold was 30 per-cent, the life companies doing business in New York received in cash for premiums £225,145,200, and expended £149,324,990. Excess of cash income over cash expenditure, £75,820,210, or an average of £4,460,000 per year. Upon this excess the companies had to bear the burden of a 30 per-cent depreciation, or £22,746,063 (equal to £1,338,000 per annum from 1862 to 1878), up to such time as resumption brought currency to par.

A careful examination has convinced me that, with hardly an exception, these failures have occurred in the life of companies which have not attained half the age of legal majority. Hardly one company of fifteen years' standing can be found in the black

* There were, in 1859, only 14 life-insurance companies authorized to do business in this State, 8 of them being New York companies. In 1871 this number had increased to 41 New York companies, and 30 from other States—a total of 71 companies. Of these 71 companies, only 31 (12 of which are New York companies) are now actively engaged in business in the State. Meanwhile 51 companies (33 New York companies and 18 other State companies) have lapsed by failure, withdrawal, amalgamation, or, as in the case of three companies, by the department's estoppel upon their taking new business. It is safe to say that, of more than 170 life companies which have had a name to live all over the United States, only some 45 survive. In fact, a standard insurance authority publishes the names of 100 life companies which have retired during only the last twenty years. Nor do we know of more than one or two life companies organized since 1868 which have survived to this time.

list. The marvel is not that so many companies have closed their doors, but that so large a surplus has been successfully carried over from a period of inflation, by companies which all the assured recognize as firmly established.

The scrutiny of the public press, and the increased vigilance of the insurance departments, have put life-insurance companies to a test beyond that to which any other similar institutions have ever been subjected; and they have come out from the trial with a reputation that other financial corporations might well envy. Taught by experience, the public have become more discriminating in the selection of the company to whose fidelity they intrust the interests of their dependents. There may have been companies constructed for the purposes of fraud. But they have disappeared, and the fittest survive. "Things refuse to be mismanaged long. Though no checks to a new evil appear, the checks exist and will appear."* It is grossly unfair to discriminate against insurance companies in the application of this law. Banks have failed. Railroad companies have collapsed. Merchants have become bankrupt. We have just emerged from an era of fraud. Who condemns the bank system, the railroad enterprises, and mercantile ventures, because dishonesty, taking advantage of inflation, has tainted their past?† But taking all companies—good, bad, and indifferent—into account, it is claimed that "*less than one per-cent* of all the money ever invested in life-insurance in the United States, has been lost through mismanagement, dishonesty, failure, or other cause."‡

How is our conviction strengthened by the wise use made of the money which is the collateral of the contracts? Investments in buildings have been criticized. But the receipts for rent substantially off-set the charge. Other applications of the assets come under the direct inspection of the department. In 1870,

* Emerson.

† According to a railroad journal, during 1880, thirty-one railroads, with a mileage of 3,375, with £33,200,000 in stock, were sold under foreclosure. In five years, 228 roads, with a mileage of 20,000—nearly 23 per-cent of the present total mileage—and representing a nominal investment of £247,200,000, became bankrupt.

‡ The Losses by Failures not One per-cent!—The life companies of this country have received, from the time the business was first commenced down to the present date, the enormous sum of £247,637,000. The losses by all the companies that have ever failed here, will reach between £2,000,000 and £2,400,000, on a very liberal estimate. It is thus shown that 99 per-cent of all the money that has been intrusted to the life-insurance companies has been faithfully administered. There have been failures—large failures, and scandalous ones—but the interest *as a whole* has been, and is to-day, as secure as any human institution the sun ever shone on."—*The Insurance Monitor.*

when 71 companies were contending in New York, at least 11 per-cent of the whole assets was reported as unproductive. The case is reversed in 1880. Every company now doing business in this State has not only all its reserve drawing interest, but all of them together have £8,920,000 in excess of their reserve productively invested. Of the gross assets, at least 93 per-cent is interest-bearing, the small balance being chiefly the moneys in course of transmission from agents, in process of investment, or in bank awaiting the payment of claims.

The last element of uncertainty in life-insurance is "the critical period of heaviest pressure". Have the companies that have survived the deluge of inflation, made their way through this wilderness? The definition of such a period is a matter of pure speculation. It is, according to actuarial subtleties, not very far off for our oldest companies. But if it exist in fact, how are these companies prepared to meet it? If they should cease business to-day, their assets, if administered as heretofore, would more than suffice to meet every liability as it may mature. This ought to be enough, for "sufficient unto the day", &c. But practically there is no such thing as "a critical period" in a well-ordered company, after it has passed its infancy. During the first five years of its existence, the demand for £30,000 annually in death claims might have tested its resources; whereas the payment by one of the largest companies of £1,400,000 in death and endowment claims during 1879, passed as a mere incident of the business.

The recognized Difficulties in process of Solution.

As one of the assured, I find the system begirt by objections just now. So far as they are based on real evils, there seems to be, on the part of the companies, an honest determination to grapple with, and, if possible, to overcome them. Some of the criticisms relate to features which have been already corrected, whilst others anticipate the crystallization of questions still in solution. Let us look at some of these from the point of view of the assured person.

Is it a Plutocracy?

There is really danger to the people in the creation of a plutocracy. The corporations of the land, and their vast accumulations of capital, are a standing menace to the personal and political rights of the community. The power that they are capable of wielding, and the monopolies of their franchise, may

well excite anxiety. The opposition of the Grangers to our railroad system, is the most prominent illustration of this popular unrest. Is life-insurance rightly placed in the same category? There are now only one or two strictly proprietary companies doing business in these States; and influential as they are, there is no reasonable ground to fear their oppressive growth. Others are formally constituted on the stock basis, but are practically mutual in their methods. Nor is the comparative business of all these combined, a very large factor in the system. We have not as yet reached a basis for this allegation. But a chamber of life-insurance, made up of representatives from all the leading companies, may prove the dominating force. This would be possible, if competition were not so commanding an element in the conduct of the business. As a matter of fact, the centrifugal far outmeasures the centripetal force of the system. The attempt to unite all companies in such a representative chamber was long since a demonstrated failure. We are brought down, then, to the system as a simple whole. Notwithstanding its distracting and diverse influences, making combination almost impossible, we are asked to believe that the mutual principle is the source of danger to our interests. This is the *reductio ad absurdum.* For the policyholders are the constituents. They are voters, and have a standing in court. From them proceeds the authority of administration, and they have swift methods for vindicating their jeoparded interests. The people are the Pluto. They are asked to dread a spectre; but, like that of the Brocken, it is only the shadow of themselves cast upon a cloud. The mutual life-insurance system is a democracy. It can never become other than this without a *coup d'état.*

The facts bear out this theory.

Losses and claims paid in 1879	£5,994,627
Paid on lapsed, surrendered, and purchased policies .	2,540,637
Paid dividends to policyholders	2,666,165
Total paid policyholders, 1879	£11,201,429
But the receipts from premiums in 1879 were . .	10,544,344
Excess of payments over receipts by the companies .	£657,085

From this table it appears that the assured have actually received £657,085 more than they contributed in the same year. This does not look like oppression.

The thing for which each assured person bargains is the assurance of his life. This he receives; and besides this, so long

as his policy is in force, he secures as additional advantage certain dividends which the critic calls "profits". But the word "profits" is a misnomer, and misleading when applied to life-insurance. Strictly speaking, there are and can be no "profits" to either the company or the policyholder. On the contrary, the whole business is based on the certainty of loss. This is its unique and anomalous position. The premium paid on a life policy is a loss of money, which the assured does not expect personally to recover. The dividends, so called, are, in marine insurance, named return premiums. At the end of the year the account with each life policy is balanced. Whatever amount of the premium is not needed for security is returned to the assured. This rebate can scarcely be called a profit. If death should intervene, and the policy be made good, its adjustment is a loss of life to the assured, and a loss of so much money to the other members of the society. Fire and marine insurance anticipate but little loss among the number of their risks.

The savings-banks are not thought to endanger the rights of the community, and yet they lack the mutual element of organization. They are administered by self-perpetuating boards. There is this fundamental difference in favour of life-insurance. But who fears that the millions of assets held by savings-banks will be made a power to grind the faces of their poor depositors? The only ground for such demagogism as that which decries the plutocracy of life-insurance, is the practical disuse of the suffrage by the policyholders. It is in the power of the assured in mutual life-insurance companies to dethrone Mammon, if he have seized the highest place and call himself president. That they do not often do this, is the best evidence of popular unbelief in this aspersion, and the best testimony that the society can give to their satisfaction with their trustees.

Surrender Values.

The next inequity which demands examination, is the surrender value of policies whose holders wish to retire from their contracts. This is admitted on all sides to be a vexed question. But as in England, so here, competition will be its cure. When an assured person determines to discontinue his risk, he is liable to lose sight of the fact that he is one of a community, and has no right to press the equity of the one against that of all. He is tempted to look at the matter too selfishly, and is surprised that so small a percentage of his payments is found in the offered price for surrender.

If he be troubled about his health, he will not think of withdrawal. If his chances for life be below the average, he thinks it no inequity to resort to any expedient which will keep his policy in force, and thus add to the risks of other persons assured with him. What right, then, has he, on the other hand, if his chances of life be far better than the average, to leave them with all their associated burdens, and withdraw the whole of his investments?

The position of the company is in this matter mediatorial. It must deal fairly with both sides. The amount that it pays must be reduced by the ratio of accumulated risk which the withdrawing person leaves to the remaining members of the association. Whilst in common law the retirant has no claim, his rights have been sufficiently guarded by the recent legislation of the State of New York. He has, in the policy which he has read and accepted, voluntarily waived all surrender values. The commonwealth have secured to him an amount of paid-up insurance represented by a certain proportion of his reserve, or a cash compensation to be determined by the relative interests of those with whom he is associated in the company. The equity must be determined by circumstances. The company in following equity must not forget justice. The comparison between the rights of one and all, is the important element in its decision; for it is the interest of the society of the assured to have the payments continued to the end of all lives or terms. They can never consent to a premium upon breach of contract. The duty of a company to its faithful constituents often compels an appearance of close dealing with those who withdraw.

Lapsed Policies.

Cognate to the question of surrender values is the difficulty connected with the "lapsing of policies". A life-insurance policy is a contract to which there are two covenanting parties. On the one side there is a solemn agreement to pay periodically a certain sum, and on the other an engagement to pay a much larger sum, on the death of the assured, or after a limited number of years. The ability of the company to fulfil its plighted agreement depends largely upon the faithful fulfilment of his promise by the assured. Here are all the conditions which are needful to bring the transaction under the law of contracts. So far, then, from clamoring for clemency, the assured who faults in his part of the agreement, or seeks release from his obligations, is liable to a claim for compensation from the company. His failure, in so far as he is able, strikes at the solvency of the company, entails actual loss upon it,

and endangers the interests of all who share with him the advantages of the system. His breach of contract is a civil crime, which only desperate circumstances can condone. Moreover, there is an element of moral obliquity in it. He has expressly consented to certain conditions written in his policy, and now he falsifies the confidence which the company have placed in his promise. They have all this while held his "word to be as good as his bond"; but he evades his pledged obligation, and poses himself as a martyr, while in law, equity, and morals, he is really a sinner.

Whatever may have been the history of fraudulent companies in the past, it is the generally accepted theory of educated and honest life-insurance men everywhere, that lapses are a loss, and not a profit. Companies, in the interest of those who abide by their contracts at the cost of much self-sacrifice and self-denial, have a just cause of complaint against persons who for trivial reasons thus forswear themselves. For whom are the officers running the companies?—the benefit of those who stay in, or of those who abandon their contracts and go out? Unquestionably for the former. Does not the outcry of policyholders in fraudulent companies which have failed, tell the story? If the company break its contract, "all the little dogs, Tray, Blanche, and Sweetheart, do bark" at it. No assured person, however many insurance agreements he may have shirked, is base enough to do it reverence. This is the other side to this matter, which intelligent and conscientious men should consider before they complain.

But such strictures against the number of lapsing policies are both disingenuous and untimely. The year 1871 has been chosen as the extreme illustration of this evil. I follow the objector in the path of examination indicated by his references. New York reports for that year show the termination of policies:—

	By Surrender.	By Lapse.	By all Ways.
New York companies . .	15,732	53,658	104,760
Companies of other States .	10,774	35,048	74,717
Total . . .	26,506	88,706	179,477

The ratio of policies terminated by lapse and surrender was in all reporting companies 66·14 per-cent of all policies terminated, and not 93 per-cent, as charged. This is a difference which is not only considerable, but which carries with it other and important explanatory suggestions. In years when notes were protested by

the basketful, when nearly two hundred railroads had made default in the payment of interest on their bonds, when banks closed their doors and suspended payment, when wages were at their lowest point, the percentage of failure in paying life-insurance premiums was about 67 per-cent of policies terminated in all ways. But now the tide has turned. The surviving institutions have been strengthened by the strain to which they have been subjected. Public confidence has been re-established in their integrity and resources. All industries are developing. Real estate is rising in value. Wages are at almost their highest mark. Of all this the diminution in the ratio of lapses in life-insurance is a significant exposition.

But even this admitted percentage is not net. For it is the common experience of all companies that a large number of lapsed policies are revived and restored, sometimes after years have intervened. And even the resulting number, after deducting such, should be lessened by the subtraction of those for which some pecuniary consideration has been paid after the lapse. Indeed, it is the fault of his own carelessness if any policyholder suffer his premium-day to pass without securing, in either paid-up insurance or a cash surrender value, some equivalent for his policy. This reduces once more the ratio of absolute loss to the assured. Infant policies are more liable to lapse than those that have gained a stalwart life. The duration in the one case is the parallel of that in the other. More than half the terminations take place during the first or second years of the policy, as it is the nursery which is most often invaded by death. Indeed, a man cannot fairly be said to have started on his insurance career, as far as his obligations are concerned, until he has proved his good faith by paying his second premium. An allowance should be made for this factor when calculating the average duration of all policies issued. The net number of policies terminated in 1879 was 60,503. The net number of policies lapsed, after deducting restorations, was 18,679.

The case is made much stronger if the amount in risk be the basis of ratio. For manifest reasons, the business of 1878-80 must form the elements of this computation. The amount lapsed, after deducting old policies revived, number of those entered as new risks, purchases, and extensions, is £10,014,897.

But the amount in force 31 December 1877 was .		£311,221,065
Issued during 1878	£31,300,226	
Less amount not taken . . .	3,919,513	
		27,380,713
Whole amount at risk		£338,601,778

A comparison of this total amount at risk and the amount lapsed, will show that the latter is *less than three per-cent* (2·96) of the former. The magnitude of the business, and the small number of policies that lapse, are elements of consideration which emphasize the persistency of policyholders, and their general contentment.

Expenses of Administration.

The question of expense is relative. It is emphatically to the policyholder's interest that such vigor and skill in the management shall be employed as to produce a profitable result. Such a policy is oftentimes expensive at the outset. A reduction of expenses is frequently the precursor of depletion. The vital question is, what the policyholder is to get in strength and profit for his money. Unless the percentage of cost be so great as to endanger the company's ability to meet its part of the contract made with him, he has naturally no desire to intrude either his curiosity or his counsel, provided he is satisfied that the company is skilfully conducted. Details he is willing to leave to the wisdom of managers, who, by the suffrages of policyholders, have assumed full responsibility. Should he sometimes think that certain salaries are too high, he recalls the fact that in many other cases small salaries have not saved a company from insolvency. The skill and care required for the management of life-insurance companies, the judicious investment of their accumulated funds, the watchful oversight over contracts covering protracted periods of time—all these, and a multitude of minor matters, call for talent of the very highest order. A man's labor is worth all that he can get for it. Large responsibilities imply large compensations. The men are few in any community who can judiciously manage millions. Whether the fortune be public or private, they are entitled to a fair income for personal needs. In managing his own, a man has more license for expenditure than in a trusteeship. But the doctrine of proportion holds good, after all, as a law. A man who is a fair seaman, or a superior skipper, may safely sail his ship over the Atlantic while summer seas and skies outline his course. But the gales and hurricanes of winter can only be outridden by a staunch vessel, the skill and courage of whose commander are commensurate. Periods of peace in life-insurance may be and have been followed by crises most momentous. It is the reserve force of the man at the head of its affairs, which alone can control the elements in such an emergency. His services for one such year of financial stress, may be worth to the assured the gross amount of his salary during his entire connection with the company.

A life-insurance company is an organization which, in its subdivision of responsibility and office, has no parallel but that of the government of a State. It employs a large corps of agents, scattered over a widely extended territory; skilful medical examiners to discriminate between good and bad risks, lest the mortality rate be ruinously increased; cashiers, book-keepers, and clerks, to receive and account for the premiums paid; scientific actuaries to guard against insufficiency of reserve, and to determine the proportion of surplus that may safely be returned; real estate experts to watch over its loans; a claim bureau to adjust without litigation the rightful demands made upon it; lawyers of wise counsel, to examine the titles of property on which loans are made and otherwise to represent the interests of its policyholders; and men of executive ability in its chief offices, capable of guiding and conserving these manifold departments. How vast is the force needful for its continued success! The total amount paid by all the thirty-one companies reporting to New York in 1879, for the salaries, commissions, advertising, and all other expenditures of the business, including dividends to stockholders, was £2,178,639, while the total amount of money managed was over £80,000,000. What a difference to the policyholders whether this vast sum was skilfully or unskilfully guarded!

Was this too much? Take for ratio the three largest New York companies, in which the princely salaries are paid. It will be found that the salaries, medical fees, and wages of other employees, were less than three per-cent of income. The total expenses of these companies in reference to income, were 12·74 per-cent; while those of all our active companies, young and old, great and small, during the same time and for the same purposes, have been only 14·3 per-cent of income. Let the ratio be either less than 13 or a little more than 14 per-cent, will any practical business man say that this is excessive? Is either amount "an undue share of accruing profits"? These institutions compare more than favorably, as to economy, with any other organizations actively seeking outside patronage. It is almost impossible to collate the reports of the banks to the Comptroller of the Currency, so as to discover their ratio of expenses to income. A prominent officer in one of our largest corporations gives me as a guess 40 per-cent. But the Insurance Report for 1879 gives us the statistics of companies dealing in other than life-insurance. The comparison in the following table is between fire and marine on the one hand and life-insurance companies on the other, as to assets and expenditures, and the percentage of the latter to the former:—

	Assets.	Expenditure.	Per-cent.
	£	£	
Fire and marine insurance companies of N. Y. State	10,822,405	1,572,517	14·53
Life-insurance companies of N. Y. State	40,512,566	1,150,226	2·84

In both cases, the expenses include commissions and every charge except losses under policies and dividends. If the expenses of life-insurance companies bore the same ratio to assets as those of fire, &c., companies, the £1,150,226 cost of administration would be raised to £5,894,574. Lest some one should take exception to this ratio of assets, I add another table, showing the ratio of expenses to income:—

	Income.	Expenditure.	Per-cent.
	£	£	
Fire and marine	4,216,855	1,572,517	37·29
Life-insurance companies of N. Y. State	8,232,482	1,150,226	13·97

The percentage of expense in relation to income in fire and marine insurance is nearly *three times* that of the life companies of this State.* Included in these amounts are the taxes throughout all the various States.

To the above statement I add that 188 insurance companies (other than life) in the State of New York, received during 1879, for premiums, £12,134,147; for interest, £1,583,985; making the gross income £13,718,132. Their expenses of management in the same year were £4,092,495. The ratio of expense to gross income was therefore 29·83 per-cent. Whilst this is the average presented, it is true that in many cases the expenses of fire companies range between 50 and 60 per-cent. Let the critics contrast these numerals with 14·3 per-cent, being the average per-cent of expenses to income of all life-insurance companies.

But the answer made to all this is that life-insurance agents are overpaid. They are made the scapegoats. What are the facts? The commissions of life-insurance agents in all companies, were in 1879, £676,617.† The whole amount of cash premium receipts was £10,104,497. This is collected in average small

* The mortality among insurance companies other than life has been overlooked in the attempt to sow distrust in the minds of life-insurance policyholders. The New York *Daily Bulletin*, in June 1880, published the names of 300 fire-insurance companies, representing assets aggregating £17,400,000, whose active life ended by failure or voluntary retirement, during the period from 1 January 1870 to 31 December 1879—only ten years.

† New York Insurance Report, 1880, p. xxi.

amounts by agents from their patrons all over the United States. The average commission was therefore 6½ per-cent. Is this an outrageous charge for the business of pursuading men to provide for themselves and those dependent upon them, and the other duties that pertain to this office? The agent who effects insurance on your clothes receives 15 per-cent year after year, and the average commission upon the assurance of your life is 6½ per-cent. The wonder is that the payment of commissions can, by life companies, be so graded as to retain in their employ men of high capacity and character, and at the same time hold the average ratio so low.

But the final fact to be recognized in this connection, is that the administration of existing life companies has not cost their policyholders a single dollar. Not only have the 36 companies, whose aggregate premiums and payments have been tabulated, met all maturing claims, accumulated an adequate reserve, returned as dividends the excess of premiums paid, purchased many cancelled policies, and paid all expenses; but they have, over and above all this, saved as surplus more than £15,200,000, which they held on 1 January 1880, in trust for policyholders. Their interest account shows this net excess after meeting all the cost of the business. By the use of money from which we might have derived no income, had we retained it, these companies have not only accumulated a secure reserve for the payment of our policies when they fall due, but have covered every conceivable item of administrative expenditure. Let the fault-finder name any other business making even approximately as good a return for the investment of £200,000,000, being the amount of the premiums received since organization by all companies reporting to the State of New York.

Protective Legislation.

The appeal in ancient times, whenever supposed difficulties were encountered, was to Hercules. And State or national legislation is now, and among life-insurance critics, substituted for the god of force. The panacea of all the ills to which the life-insurance system is heir, must be sought at Albany or Washington—so they in substance say. Three things within the scope of the law-making power, would have satisfied all policyholders. But many others have been added, which work to their detriment. These three things are:—

1. The fullest publicity in all the affairs of this system. This

has been gained by the compulsory deposit of sworn annual statements, which are open for comparison, and form the legal evidence for complaint.

2. The authoritative examination of the assets of each company, that the certificate of the State may endorse the assertion of the filed reports. This has been most needful in the past, and is a shield for the future.

3. The retention, within the reach of the courts of the State where the company is organized, of sufficient securities to meet the judgments which from time to time may be levied by the courts against the company on resisted policies.

Every good measure which has been enforced by State authority was first suggested by the energy of some ambitious company. It was only hastening a natural progress to make it by law a compulsory part of the system. The vagaries of the Legislatures of Massachusetts and other States, in taxing premiums of companies from outside their limits and retaliating for disadvantages which their own companies suffer elsewhere, are bringing this whole system of a paternal government into contempt. Before that class of objectors who seek a national bureau can succeed in their wishes, the people must consent to a constitutional amendment for the purpose. The fathers reserved all such rights. Their children will some day learn that these rights are safer when left to the defence of fundamental laws of political economy than to the shifting will of political parties. In view of all this discussion, and for its "improvement", as the old divines used to say, I add—

Some Friendly Suggestions to Policyholders.

1. *Look before you leap.* Choose for your investment a life-insurance company that is vigorously and honestly conducted in all its departments, and in which the relation of surplus to liabilities justifies the expectation of stability and security.

2. *Do not overleap.* Take no more insurance than your desponding anticipations of income make it more than probable that you can maintain.

3. *Stick.* When you have gained your footing, stand firm. For your family's sake, for your character's sake, for society's sake, for truth's sake, do not break the contract. Let your motto be, "*The Policy—it must and shall be preserved.*"

The two preceding articles are of a more popular character than those which generally appear in this *Journal;* but they give so much information as to the history and present position of life insurance in the United States, that they have a value altogether apart from the opinions expressed by the writers. In this country the criticisms upon life insurance do not range over so wide a field, and even those who write most strongly against what they consider the evils of the present system of life insurance, admit frankly the beneficent character of the work which is being done by the life offices. Both in this country and in America, the questions of fixed surrender values for all policies, and the greatest possible protection of policyholders from loss by the accidental lapsing of their policies, appear to be growing in importance; and it seems to us that they will have to be fairly faced by every life office that wishes to hold its ground in the future.

As our readers may like to have the opportunity of comparing the figures relating to British life offices with those contained in the preceding articles, we have extracted the following figures from White's *Insurance Register* for 1881:—

Amount of Insurances in force (including Bonus Additions), estimated at	£420,000,000
Total Funds	143,813,793
Annual Premiums	13,174,848
Claims paid in the year	11,149,730
Surrenders " "	720,406
Commission	1,030,446
Other Expenses	1,321,228

It is not possible to give any trustworthy estimate of the number of lives assured, or of the number of policies lapsed. The following table shows the progress of the business during the ten years that have elapsed since the Life Assurance Companies Act came into operation:—

Year.	No. of Companies reporting.	Total Income.	Total Outgo.	Excess of Income over Outgo.
1871	110	£14,456,261	£11,815,363	£2,640,898
1872	118	15,001,631	11,791,580	3,210,051
1873	120	15,535,581	12,125,292	3,410,289
1874	120	15,819,053	12,410,036	3,409,017
1875	114	16,604,649	12,888,188	3,716,461
1876	108	16,978,115	13,600,491	3,377,624
1877	109	17,610,655	13,756,862	3,853,793
1878	107	18,048,355	14,184,486	3,863,869
1879	108	18,961,018	14,830,359	4,130,659
1880	107	19,354,054	16,064,460	3,289,594

JOURNAL

OF THE

INSTITUTE OF ACTUARIES

AND

ASSURANCE MAGAZINE.

Opening Address by the President, A. H. BAILEY, Esq.

[Delivered 28 November 1881.]

IN opening the proceedings of the session a year ago, it occurred to me that it might be useful to place on record a short narrative of the circumstances under which the Institute of Actuaries came into existence, and of the efforts of its founders to have the profession of actuary recognized. On this occasion, before proceeding to the main business of the evening, I propose for a few moments to ask you to consider what that profession is, and what are its position and prospects at the present day. What is an actuary? This is a question sometimes asked, and to which it is not easy to supply a ready answer. If we refer to dictionaries, we find Johnson's definition to be, "the registrar who compiles the minutes of the proceedings of a court; a term of the civil law". The following example is given from Ayliffe: "Suppose " the judge should say that he would have the keeping of the acts " of court remain with him, and the notary will have the custody " of them with himself, certainly, in this case the actuary or " writer of them ought to be preferred." Webster gives two meanings—(1) A registrar or clerk—a term of the civil law, and

used originally in courts of civil jurisdiction, but in Europe used for a clerk or registrar generally. (2) The manager of a joint-stock company, particularly of an insurance company; in America, chiefly applied to the manager of a life insurance company. Andrews's Latin Dictionary gives, for the meaning of the word *actuarius*—(1) A shorthand writer, with a reference to a passage in Seneca; and (2) one who writes out the accounts of his master.

Hence it seems clear enough that the original conception of an actuary was that of an officer of a court of law, and that, therefore, the gentlemen who occupy the posts of Chancery registrars might, with strict propriety, although probably to their own surprise, be termed actuaries. An officer with this designation has always been and still is attached to the Lower House of Convocation of the Church of England; and it is worthy of notice that the chief officer of the first life assurance society established in this country, the now extinct Amicable, was called registrar. In the two oldest now existing, the Royal Exchange and London Assurance Corporations, there was no officer with the designation of actuary until comparatively modern times. The first mention of such an officer in connection with a life office is to be found in the deed of settlement of the Equitable Society, which bears date 7 September 1762. The duties of the actuary are there set out in some detail, and it may not be without interest to learn what the prevailing idea of those duties then was. By clause 32 the actuary was to be chosen in a general meeting out of the members assured for not less than £300; it will be observed that he was not appointed by the directors. He was to give constant attendance at the office in order to receive proposals, and to "execute such other business as shall appertain to the said office". What this other business was is made clear in the following clauses, which set out that the actuary "shall, in a fair and "clerk-like hand, method, and manner, write and keep all such "books as the future occasions of the said society shall require"; and, among several other duties mentioned, that he shall make regular entries each day of sums of money received and paid, and that he shall "make and enter up the minutes of the proceedings "and orders of the court of directors, and of the general courts "or meetings of the society". Clauses 41 and 42 provide that William Mosdell should be the first actuary, and that "in the "name of his said office of actuary" he should receive a yearly salary of one hundred pounds, payable quarterly. In the duties defined, not a word is to be found of what would now be considered

the special work of the actuary; but the 43rd clause is curious,— "And because it is imagined that the said William Mosdell will, "exclusively of his said office of actuary, be otherwise serviceable "to the said society, the said William Mosdell shall receive of "and from the said society after that the said society shall consist "of 300 members, the further yearly salary of one hundred "pounds." Now, as in the account of the Equitable Society in the sixth edition of Dr. Price's work, published in 1803, it is stated that the subject of the rules by which the values of assurances may be computed, had been nearly exhausted by Mr. Morgan, the actuary of the society, I think it may be inferred that this computation of the values of assurances was one of the "services wholly distinct from the office of actuary", for which the second hundred pounds was added to the yearly salary.

Thus it seems to have come to pass that the keeper of the registers of the risks incurred came to be employed to determine the proper rates of premium for those risks; the registrar had to be a scientific computer, and became the actuary in the modern sense of the word. In process of time the services of these skilled registrars were found useful in the many complicated pecuniary interests which the artificial conditions of modern society have created, and in the working of funds established for the benefit of different classes of the community. And because, again to quote Dr. Price, "melancholy experience shows that none but mathe- "maticians are qualified for performing and conducting schemes "of this kind", professors of mathematics have occasionally been tried as actuaries. I hope that no one will for a moment suspect me of disparaging studies, of the value of which I am profoundly convinced, and which in my youth had for me a great fascination, when I say that the attempt to incorporate the professor of mathematics with the business man has not generally succeeded.

An actuary should be a man of general culture, with a knowledge both of books and men, and the more he has of both the better. He comes in contact with various classes of the community, and will be of little use unless he can understand and sympathize with the different objects for which he is consulted, and can adapt himself to the different habits of thought of his clients. He is not unlikely to find that the hard-headed mechanic will insist on understanding the principles on which his benefit society is conducted, while the ordinary man of business will care much for results, but take little or no interest in processes. Again, the

work of the actuary brings him into constant intercourse with the members of the legal and medical professions. Without some smattering of law he will be stranded altogether; and, although this knowledge may be slight, it is very desirable that it should be sound and accurate as far as it goes. The intercourse of the actuary with the physician is of a character to call for a passing remark. The latter studies the individual case, the former deals with averages; the physician, therefore, is disposed to attach to what in life assurance is called the effect of selection, an importance which the actuary, from his different studies, should know is not warranted by reason or experience. *Cuique in arte suá credendum est.*

An actuary then must be a mathematician, but a mere mathematician will be a very incompetent actuary. A simple illustration will suffice. A common case submitted to an actuary is to state the value of a reversion expectant on the failure of a single life. The answer seems easy enough. Refer to a table; there are several such. Or, at all events, the student learns that the formula applicable to the case is $A_x = (1 - i a_x) \div (1 + i)$. The mathematical part is soon disposed of; but the question then arises, what is to be the rate of interest? This can be answered only by watching the market, and the actuary who thinks this beneath his dignity is unfit for his work. It will be found, I think, that no general answer to this question can be given; that the tendency is for the rate of interest to increase as the age of the life tenant is less; and that if he be young, say under thirty years of age, the reversion has no saleable value, the formula notwithstanding. Again, funds in reversion are invested in various ways; another question therefore is, what will be the value of these investments when the reversions fall in? This is by no means an easy question to answer; mistakes in these estimates are not uncommon. I lately heard of a case where property, the reversion to which was offered for sale, which had been steadily increasing in value for more than forty years, and which showed no signs of decay, was valued at 60 percent of the market price. Nor is this all. Law expenses are incurred in all these transactions. Directly or indirectly these must be paid by the vendor; they press heavily in small cases, but as the amounts increase, the expenses do not increase in the same ratio. Thus it comes to pass that although ten times two are twenty, a reversion to £2,000 is more than ten times as valuable as one to £200. If such questions as these arise in so simple a matter as the value of a common reversion, how many others come to the

surface in the more complicated enquiries that engage the attention of the actuary?

I have spoken of the registrar becoming the scientific computer, but the definition of an actuary which I prefer is the scientific financier. And it seems to me that the most fitting, nay, almost the only place for the training of such a financier is the office of a life assurance society. So that when, as not unfrequently happens nowadays, a man announces his intention of bringing up his son (who is of course an intellectual prodigy) to be an actuary, and asks what course he is to pursue, the proper answer, which is not always agreeable to the parental mind, seems to be—endeavor to get him an appointment as a junior clerk; if you succeed in this, tell your son that he must go through the routine of office work; that he should devote his leisure time to acquiring a knowledge of the principles of his profession; that in this, as in most occupations, the blanks are many, the prizes (such as they are) are few; that there is much uphill work, but that for steady perseverance and industry even in this world there is some kind of reward. Men have occasionally attempted to practise as pure actuaries, and have put a brass plate on a door; but whether they have ever taken sufficient fees to pay the rent of their offices is doubtful. Actuarial work seems to be on the increase; but, wisely or unwisely, the public refuse to employ any but those who have a recognized position with a public company. This being the case, it is much to be regretted that the tendency of the present day seems to be, in this country at least, in the direction of diminishing the number of life assurance companies. This tendency is to be regretted, I think, in the interests of the public, as the evidence seems to me to be clear that, speaking generally, life assurance is more costly in the companies doing a very large business than in those of more moderate pretensions. And it is to be regretted in the interests of the profession, because it is unfavourable to the prospects of the younger men. A branch managership of a leviathan company is not a good school for an actuary; but in former times an excellent training was to be found in a chief appointment to a small society at a modest salary.

It is, however, vain to quarrel with the inevitable. The Institute cannot improve the prospects of its younger members; but it can, and I think it has done, much, to help them to acquire a knowledge of the principles of their profession, and, notwithstanding some disappointments, it hopes—not immediately, but still at no distant period—to be able to publish a text-book for their use. In

other ways, too, the Institute may, and I believe will, help to promote the general interests of the profession. It may fairly claim to speak with authority on several subjects that engage the attention of Parliament; and I should not be sorry to see it occasionally endeavoring to initiate legislation on some matters in which we are interested. However this may be, I have reason to know that the disfavor with which it was formerly regarded in some quarters has passed away, that the services it has rendered are more than ever appreciated, and that its usefulness is recognized, not merely at home, but on the Continent, in our Colonies, and in the great Republic. It has been the pioneer of other societies having similar objects, the youngest of which—the Insurance and Actuarial Society of Glasgow—held its opening meeting only a few days ago. Many of you have read with no little interest the thoughtful address delivered on that occasion by its president, Mr. John Graham, who I hoped would have been with us this evening, but who has been prevented from coming here by a pressing engagement. The Glasgow society is, however, well represented on this occasion by our friend and Fellow, Mr. Marr, whose address a year ago to the Actuarial Society of Edinburgh is still fresh in our memory.

On the Transformation of Annuities and Annuity-Values payable Yearly, into the like when payable in Fractional Intervals of a Year, by means of Constant Factors; with Specimens of Tables computed for this purpose, and Examples of their application. By D. J. McG. McKenzie, A.I.A.

[Read before the Institute 28 Nov. 1881.]

It is not to be expected, at this period of time, that anything new remains to be discovered in the Doctrine of Annuities, especially by one not conversant with analysis; yet a useful application may occasionally be found of the results of former investigations, the more so should such have remained in comparative neglect.

The announcement of the relation which subsists between an annuity payable yearly, and when it is payable half-yearly, quarterly, monthly, &c., the rate of interest continuing the same, is due to the illustrious De Moivre, and was made by him in the second edition of his treatise of *Annuities on Lives*, published in 1743. It was reproduced in the subsequent editions of the same work, and is to be found, with some errors of the press corrected,

in that appended to the third edition of his *Doctrine of Chances,* 1756.

Although I have not strictly followed De Moivre, having computed my factors before I found they were described in his work, I here transcribe his rules with the demonstration of them, remarking that, the notation he employs being at variance with that to which we are now accustomed,* and likely, therefore, to cause embarrassment to the student, I have substituted the modern notation throughout.

At p. 297 of the *Doctrine of Chances,* De Moivre writes thus:—
" The values of annuities for lives having been calculated in this " book upon a supposition that the payments are made yearly, " and there being some occasions wherein it is stipulated that the " payments should be made half-yearly. I have thought fit to " add the two following problems, whereby, 1st. It is shown what " the half-yearly payments ought to be if the price of the purchase " is preserved. 2nd. How the price of the purchase ought to " be increased, if the half-yearly payments are required to be the " half of the yearly payments.

" PROBLEM XXXII.

" An annuity being given, to find what half-yearly payments " will be equivalent to it, when interest of money is 4, 5, or 6 " per-cent.

" SOLUTION.

" Take half of the annuity, and from that half subtract its " 100th, or 80th, or 68th part, according as the interest is 4, 5, " or 6 per-cent, and the remainders will be the half-yearly payments " required. Thus, if the annuity was £100, the half-yearly pay- " ments would be respectively £49. 10*s*., £49. 7*s*. 6*d*., £49. 5*s*. 3*d*. " nearly.

" PROBLEM XXXIII.

" The present value of an annuity being given, to find how " much this present value ought to be increased when it is required " that the payments shall be half-yearly, and also one-half of the " yearly payments, when interest is at 4, 5, or 6 per-cent.

" SOLUTION.

" To the present value of the annuity add respectively its " 99th, 79th, or 67th, and the sums will be the values increased.

* NOTE.—He uses r, calling it *the rate of interest,* for that which we now denote by $(1+i)$. It is not to be denied, however, that in this particular instance the expressions deduced are much more compact in De Moivre's notation than in that here employed.

"As there are some persons who may be desirous to see a "general solution of the two last problems, I have thought fit "to add what follows.

"In the first of the two last problems, let A be the yearly "payments agreed on, and B the half-yearly payments required, "$(1+i)$ the yearly rate of interest; then $B=A\{(1+i)^{\frac{1}{2}}-1\}:i$. In "the second, let M be the present value of the yearly payments, "P the present value of those that are to be half-yearly, then "$P=\dfrac{\frac{1}{2}\times i}{(1+i)^{\frac{1}{2}}-1}\times M$."

Turning now to page 320, we find—

"CHAPTER VII.

"Containing the demonstration of what has been asserted "in the 32nd and 33rd problems concerning half-yearly pay-"ments; . . . .

"It is well known that if an annuity A is to continue n years, the "present value of it is $A\{1-(1+i)^{-n}\}:i$; supposing $(1+i)$ to "represent the rate of interest. Now, to make a proper applica-"tion of this theorem to half-yearly payments, I look upon n as "representing indifferently the number of payments and the "number of years. Let us now suppose a half-yearly rent B, of "the same present value as the former, and to continue as long; "then the number of payments in this case will be $2n$, but the "rate of interest, instead of being $(1+i)$, is now $(1+i)^{\frac{1}{2}}$, which "being raised to the power $2n$ will be $(1+i)^n$ as before; for "which reason the present value of the half-yearly payments is "$B\cdot\dfrac{1-(1+i)^{-n}}{(1+i)^{\frac{1}{2}}-1}$. But, by hypothesis, the present values of "the yearly and half-yearly payments are the same; therefore "$A\cdot\dfrac{1-(1+i)^{-n}}{i}=B\cdot\dfrac{1-(1+i)^{-n}}{(1+i)^{\frac{1}{2}}-1}$; and dividing both sides of "the equation by $1-(1+i)^{-n}$, we shall have $\dfrac{A}{i}=\dfrac{B}{(1+i)^{\frac{1}{2}}-1}$, "from whence will be deduced $B=A\{(1+i)^{\frac{1}{2}}-1\}:i$. And in the "same manner, if the payments were to be made quarterly, then "B would be $=A\{(1+i)^{\frac{1}{4}}-1\}:i$, and so on.

"But if we suppose that a rent shall be paid half-yearly, and "that it shall be also one-half of what would be given for an "annual rent, and that the two rents shall be of the same duration, "then the present values of the yearly and half-yearly rents will "be different. For let M and P be the present values of the

"yearly and half-yearly rents, then $M = A\{1-(1+i)^{-n}\} : i$, and "$P = \frac{1}{2}A \cdot \frac{1-(1+i)^{-n}}{(1+i)^{\frac{1}{2}}-1}$; and dividing both values by "$A\{1-(1+i)^{-n}\}$ we shall have $M : P :: \frac{1}{i} : \frac{\frac{1}{2}}{(1+i)^{\frac{1}{2}}-1}$, and con- "sequently $P = M \times \frac{\frac{1}{2} \times i}{(1+i)^{\frac{1}{2}}-1}$."

Factors derived from the second formula of De Moivre were published by Dodson in 1747, and by Fédor Thoman in 1859. Those which I have computed, and which are here submitted to the consideration of the Institute, differ from De Moivre's in that they are not divided by the number of intervals taken as contained in the year (being in fact the reciprocals of the values derived from his first formula), so that they serve to transform annuity-values of unity per annum into annuity-values of unity per interval, while I have at the same time given the corresponding rates of interest for each interval at which the transformations are effected, such rates being the exact equivalents of the true yearly rates.

The benefit of this arrangement is that the ordinary tables of yearly annuity-values are rendered capable of increased power to the extent of the limits of the factors computed for them.

If it be objected that the rates of interest corresponding to these transformed annuity-values are not such as are usual in practice, it is replied that the object I had chiefly in view when computing the factors being to obtain an easy means of valuing correctly loans repayable by constant annuities, the actual rate of interest involved in any terminable annuity is quickly and easily found by means of them, and is generally obtained correct to seven places by one simple interpolation.

It is scarcely necessary to mention that any less number of places may be used, which will effect the particular purpose desired, and for convenience I have pointed off the last two places.

According to the method of De Moivre the amount of £1 is the same whether the interest is assumed as paid once or several times during the year. If we wish to compute annuity-values on this basis, we must first ascertain the interest for each interval, by which to divide the interest or discount of £1 for the time, according as amounts or present values of annuities are wanted; and if we divide the interest for one year by the interest for one interval, we obtain a factor whereby we may multiply yearly annuity-values to change them into annuity-values for the intervals whose factors

have been computed, altogether irrespective of the duration of the annuity.

In computing the factors logarithms are indispensable, which must be to ten places, if we are to have seven true in the factors. I give an example of finding the monthly factor corresponding to 5 per-cent yearly.

Example.

$$\text{Log}\,(1{\cdot}05)^{\frac{1}{12}} = 0{\cdot}00176{,}57749, \qquad (1{\cdot}05)^{\frac{1}{12}} = 1{\cdot}00407{,}41236$$

$$\text{Log}\,i = \text{Log}\,{\cdot}05 = \bar{2}{\cdot}69897{,}00043$$

$$\text{Log}\,[(1+i)^{\frac{1}{12}} - 1] = \bar{3}{\cdot}61003{,}42004$$

Log constant $= 1{\cdot}08893{,}58{,}039$, the number to which is $12{\cdot}27258$, and which is the constant factor required.

The logarithmic factors were found in the above manner for every quarter per-cent of yearly interest from $2\frac{1}{2}$ to $12\frac{1}{2}$, and those for the intermediate eighths were found by interpolation with second differences. The whole were then combined, differenced, and cut down to seven places, the regular progression of the differences being regarded as indicating the correctness of the work. The intervals for which factors have been computed are half-years, quarters, calendar months, fortnights, weeks, and moments, the latter being in De Moivre's form.

Table A (p. 167) is a specimen of the table of factors as it was originally constructed. An abstract of the table, containing the portions which are most likely to be of practical use, is given in the appendix, Tables I and II (pp. 183, 184).

Having computed and tabulated the factors with their accompanying values, the next desideratum was to compute a series of the logarithms of yearly annuity-values to combine with them. As far as regards natural numbers, the tables of Lieut.-Col. Oakes, Rance, Corbaux, and Smart, with those of Mr. Turnbull, appear to leave little to be desired. For facility of combination with the logarithmic factors, and to avoid troublesome multiplications and divisions, logarithmic tables of annuity-values are very convenient. The only tables of this kind accessible are those of M. Thoman, but the rates in his work above 6 per-cent are much too far apart to be of service. I had previously found they contained errors, so that I could not depend upon them without verification for those rates, corresponding to the factors, which they contained. The examination of the logs. of the annuity which £1 will purchase for the 38 rates in Thoman which run parallel with mine, resulted in the discovery of nineteen errors in the English editions, eight of

TABLE A.

Yearly Rates.	$\text{Log}\,(1+i)^{\frac{1}{n}}$	Constants and Differences.		Log constants and Differences.	Rates and Perpetuities.	$\text{Log}\left\{(1+i)^{\frac{1}{n}}-1\right\}$ Cologs. of do.
			FOR WEEKLY VALUES. ($n=52$)			
3	0·00024,68697	52·76116 3156	·000	1·72231,43 25,97	·0005686001 1758·705	6·75480,69 3·24519,31
			FOR MONTHLY VALUES. ($n=12$)			
3	0·00106,97687	12·16412 680	·000	1·08508,07 24,27	·002466270 405·4707	7·39204,06 2·60795,94
			FOR QUARTERLY VALUES. ($n=4$)			
3	0·00320,93062	4·044723 1851	·000	0·60688,88 19,87	·007417072 134·8241	7·87023,25 2·12976,75
7	0·00734,59444	4·103521 1822	·000	0·61315,66 19,28	·01705852 58·62172	8·23194,15 1·76805,85
$7\frac{1}{8}$	0·00747,27087	4·105343 1821	·125	0·61334,94 19,27	·01735543 57·61885	8·23943,54 1·76056,46
$7\frac{1}{4}$	0·00759,93252	4·107164 1822	·250	0·61354,21 19,25	·01765208 56·65054	8·24679,59 1·75320,41
$7\frac{3}{8}$	0·00772,57942	4·108986 1820	·375	0·61373,46 19,23	·01794847 55·71506	8·25402,74 1·74597,26
$7\frac{1}{2}$	0·00785,21161	4·110806 1818	·500	0·61392,69 19,21	·01824460 54·81073	8·26113,44 1·73886,56

which appear also in the French impressions appended to both editions of Hippolyte Charlon's *Thèorie Mathématique des Opérations Financières,* which impressions contain an error not in the English ones. I subjoin a list of these errors.

	Rate.		Value.	Time.		Error.	Correction.
1	$3\frac{1}{8}$	per-cent	log a	36	years	$8{\cdot}\cancel{8}6896{\cdot}42$	8·66896·42
2	$4\frac{1}{4}$	„	„	12	„	$9{\cdot}033\cancel{3}3{\cdot}97$	9·03383·97—H. C.
3	$4\frac{5}{8}$	„	„	27	„	$8{\cdot}81\cancel{7}93{\cdot}11$	8·81693·11—H. C.
4	$4\frac{7}{8}$	„	„	6	„	$9{\cdot}2927\cancel{7}{\cdot}\cancel{8}\cancel{3}$	9·29276·69—H. C.
5	„	„	„	24	„	$8{\cdot}\cancel{6}5486{\cdot}59$	8·85486·59
6	$5\frac{3}{8}$	„	„	3	„	$9{\cdot}567\cancel{4}5{\cdot}73$	9·56795·73
7	„	„	„	8	„	$9{\cdot}196\cancel{0}\cancel{9}{\cdot}89$	9·19610·89—H. C.
8	„	„	log r^n	20	„	$0{\cdot}45\cancel{3}75{\cdot}18$	0·45475·18
9	$5\frac{1}{2}$	„	log a	49	„	$8{\cdot}7730\cancel{6}{\cdot}\cancel{9}6$	8·77307·16
10	$5\frac{5}{8}$	„	„	5	„	$9{\cdot}37\cancel{0}03{\cdot}04$	9·37103·04
11	$5\frac{3}{4}$	„	„	6	„	$9{\cdot}304\cancel{9}5{\cdot}23$	9·30485·23—H. C.
12	$7\frac{1}{2}$	„	„	39	„	$8{\cdot}\cancel{8}\cancel{7}\cancel{7}73{\cdot}74$	8·90173·74
13	8	„	„	39	„	$8{\cdot}\cancel{8}2523{\cdot}54$	8·92523·54
14	9	„	log r^n	5	„	$0{\cdot}187\cancel{2}3{\cdot}25$	0·18713·25
15	„	„	log a	26	„	$9{\cdot}003\cancel{1}9{\cdot}57$	9·00309·57—H. C.
16	10	„	„	32	„	$9{\cdot}02\cancel{0}07{\cdot}23$	9·02107·23—H. C.
17	„	„	„	40	„	$9{\cdot}009\cancel{6}\cancel{4}{\cdot}32$	9·00970·32
18	12	„	„	33	„	$9{\cdot}08962{\cdot}\cancel{4}6$	9·08962·36—H. C.
19	$5\frac{1}{2}$	„	„	3	„	$9{\cdot}\cancel{6}\cancel{3}896{\cdot}88$	9·56896·88

Error No. 19 was found only in the impression appended to the seventeenth edition of Inwood's tables.

Errors Nos. 2, 3, 4, 7, 11, 15, 16 and 18 are common to the French and English impressions, while the French contains an additional error, namely :

Rate.	Value.	Time.	Error.	Correction.
$5\frac{5}{8}$ per-cent	log a	15 years	$\cancel{8}\cdot\cancel{9}0197\cdot10$	9·00197·10

I have only examined Thoman's tables for the above rates, and up to 51 years. These errors have been communicated to the publishers, and will be corrected in future editions, and it is to be hoped the remainder of the tables will undergo revision also.

The logarithms of the present values of an annuity of £1 for the remaining 43 rates tabulated were computed by the method given by Mr. Peter Gray, in his *Tables and Formulæ*, problem X, page 30, using for this purpose an excellent table of seven-place Gaussian logarithms by Wittstein, Hanover, 1866. The method of Mr. Gray—for the application of which to the computation of annuity-values he was the first to adapt and recompute the tables of Gauss—is very convenient, since it admits of verification at as many points as may be thought requisite; and because each value is derived from that preceding, we have the assurance that if the last computed value is correct, all the others are correct also, a computation in duplicate being unnecessary.*

When six places are sufficient, nothing can exceed the facility in use of Mr. Gray's own table of $\log(1+x)$, the arrangement of the proportional parts being especially convenient. For seven places, the table I have used is the best. I may here mention that in computing isolated values of annuities, tables of Gaussian logarithms of Mr. Gray's form are very useful, half the usual labour being saved by them, as may be seen in the following examples.

Required the amount† and present value of an annuity of £1 for 25 years at 5 per-cent.

$$
\begin{aligned}
\operatorname{Log}(1+i) &= 0\cdot02118{,}92991 \times 25 \\
\operatorname{Log}(1+i)^{25} &= 0\cdot52973{,}25 \\
\operatorname{Log}\{(1+i)^{25}-1\} &= 0\cdot37773\ 50 \\
\operatorname{Log} a_\infty = \operatorname{Colog} i &= 1\cdot30103\ 00 \\
\operatorname{Log} s_{\overline{25}|} &= 1\cdot67876{,}50 \\
s_{\overline{25}|} &= 47\cdot7271
\end{aligned}
$$

$$
\begin{aligned}
\operatorname{Log}(1+i)^{25} &= 0\cdot52973{,}25 \\
\operatorname{Log}(1+i)^{-25} &= 9\cdot47026{,}75 \\
\operatorname{Log}\{(1+i)^{25}-1\} &= 0\cdot37773{,}50 \\
\operatorname{Log} a_\infty &= 1\cdot30103{,}00 \\
\log a_{\overline{25}|} &= 1\cdot14903{,}25 \\
a_{\overline{25}|} &= 14\cdot09394
\end{aligned}
$$

* NOTE.—For further confirmation, every fifth value was computed by a direct process.

† In the Institute scheme of notation, a symbol is not provided for the amount of an annuity. The symbol $s_{\overline{n}|}$ has been here adopted.

Log $\{(1+i)^{25}-1\}$ is here formed from log $(1+i)^{25}$ by the inverse use of Wittstein's table.

The logarithms of $(1+i)^n$ were formed by addition with a card in the usual way.

From one to fifty-one years was thought to be a sufficient extent to which to compute the required values, this limit being chosen so as readily to obtain values for any number of weeks short of a year. I give a specimen.

TABLE B.—*Logarithms of the Amounts of £1 and of the Present Values of an Annuity of £1 at Yearly Rates.*

Years.	7 PER-CENT.		$7\frac{1}{8}$ PER-CENT.		$7\frac{1}{4}$ PER-CENT.	
n	Log $(1+i)^n$	Log $a_{\overline{n}\vert}$	Log $(1+i)^n$	Log $a_{\overline{n}\vert}$	Log $(1+i)^n$	Log $a_{\overline{n}\vert}$
11	0·32322,16	0·87498,44	0·32879,92	0·87228,49	0·33437,03	0·86959,42
12	0·35260,53	0·89996,74	0·35869,00	0·89707,92	0·36476,76	0·89420,13
13	0·38198,91	0·92208,42	0·38858,09	0·91901,29	0·39516,49	0·91595,32
14	0·41137,29	0·94178,30	0·41847,17	0·93853,40	0·42556,22	0·93529,79
15	0·44075,67	0·95941,90	0·44836,25	0·95599,73	0·45595,95	0·95259,02
16	0·47014,04	0·97527,77	0·47825,34	0·97168,89	0·48635,68	0·96811,59
17	0·49952,42	0·98959,32	0·50814,42	0·98584,21	0·51675,41	0·98210,85
18	0·52890,80	1·00255,86	0·53803,50	0·99865,03	0·54715,14	0·99476,12
19	0·55829,18	1·01433,55	0·56792,59	1·01027,50	0·57754,87	1·00623,55
20	0·58767·56	1·02506,05	0·59781,67	1·02085,27	0·60794,60	1·01666,77

From these two parallel columns the amount and present value of £1, the amount and present value of an annuity of £1, the annuity to repay £1 with interest, and the sinking fund to accumulate to £1, are at once obtained.

In order to render the tables easily applicable to loans repayable by instalments, it was deemed desirable to form a table of the logarithms of the annuity which £1 will purchase for the most usual rates and periods; so that it might readily be seen within a quarter per-cent of yearly interest what was the rate involved in any annuity payable at any of the intervals for which factors were computed. This was easy after the two tables already described were computed. By adding to the Cologs of $a_{\overline{n}|}$ the Cologs of the factors for each interval, the required logarithms of the annuity which £1 will purchase were immediately obtained.

This was done for the following yearly rates, namely, $2\frac{1}{2}$, 3, $3\frac{1}{2}$, 4, thence by quarters to $10\frac{1}{2}$, and for 11, $11\frac{1}{2}$, 12 and $12\frac{1}{2}$ per-cent, for every year from one to twenty-five, and a specimen is given in Table C.

In this will appear the peculiar facility of application of the

method which I associate with the name of De Moivre. By the usual mode the values for the several intervals would require to be separately computed. Here they are immediately obtained from the yearly annuities by the application of a constant factor. Moreover, since the interest is assumed as payable once a year only, the rate remains unchanged throughout the several intervals, that is to say, the rate for each interval is such as, if improved for the remainder of the intervals in the same ratio, the interest will amount to the yearly rate and no more, at the end of the year. The nominal rate is therefore here the real rate. The yearly rate with the corresponding rates for each interval being placed at the top of the respective columns, the equivalents are seen together.

As formerly intimated, a special advantage of this method consists in the facility of its application, by means of interpolation, to find the actual rate contained in any annuity payable in any of the given intervals, which fractional rate being found, the computer may, in estimating the yearly rate, at his option, multiply it by the number of intervals in the year, or raise it to the power whose exponent is that number, as ought truly to be done. A spade should always be called a spade, and not something a little different.

Table C.—*Specimen.—Logarithms of* $(1+i)^n$ *and of the Annuity which £1 will purchase.*

7 per-cent Yearly Interest.						
Years.	Log $(1+i)^n$	Half-Years. Log Annuity.	Quarters. Log Annuity.	Months. Log Annuity.	Fortnights. Log Annuity.	Weeks. Log Annuity.
n	...	·03440804	·01705852	·005654145	·002605645	·001301975
10	0·29383,78	8·84500,33	8·54028,48	8·06071,03	7·72425,85	7·42294,58
11	0·32322,16	8·81657,75	8·51185,90	8·03228,45	7·69583,27	7·39452,00
12	0·35260,53	8·79159,45	8·48687,60	8·00730,15	7·67084,97	7·36953,70
13	0·38198,91	8·76947,77	8·46475,92	7·98518,47	7·64873,29	7·34742,02
14	0·41137,29	8·74977,89	8·44506,04	7·96548,59	7·62903,41	7·32772,14
15	0·44075,67	8·73214,29	8·42742,44	7·94784,99	7·61139,81	7·31008,54

It may be asked,—What is truly the interest for any interval less than one year? The answer to this must be,—So much as can be obtained for the use of money in that interval. But what is the interest expressed in terms of the yearly rate? The answer must be,—As much as can be made of it. If therefore simple interest be paid for consecutive intervals less than one year, the

yearly interest must be greater than the rate of simple interest for the intervals, because the receiver has the option of investing such interest as paid. Accordingly, we find standard writers on annuities distinguish between nominal and real rates of interest. But as it would be inconvenient to quote a rate with a string of decimals attached, custom has established the practice of quoting the nominal rate instead of the real. Where this distinction is not kept in view, confusion must arise. The preferable mode, as far as regards annuities, would be to follow the older writers who, considering the yearly rate as the real one, necessarily estimated compound interest for any interval shorter than one year as less than simple interest. I am aware that this point has been much debated. As expressing the clearest view of it which I had the opportunity of perusing, I beg to mention the admirable letter of the late W. Orchard in vol. iv of the *Journal of the Institute.* That volume also contains a letter on the same subject by the celebrated Mr. Gompertz, who maintains the view demonstrated by the ever memorable De Moivre. Will it be considered out of place if I here quote Mr. Todhunter's reference to De Moivre in his *History of the Mathematical Theory of Probability*, pp. 135, 136? "It is recorded that Newton him-"self, in the later years of his life, used to reply to enquirers re-"specting mathematics in these words: 'Go to Mr. De Moivre, he "'knows these things better than I do.' In the long list of men "ennobled by genius, virtue, and misfortune, who have found an "asylum in England, it would be difficult to name one who has "conferred more honour on his adopted country than De Moivre."

Of living authorities, holding the same view, it may be sufficient to name Mr. Sprague and Mr. Peter Gray, as we find reported in the discussions at the Institute, Session 1866-7. Mr. Gray remarks: "I give my entire adherence to the view taken by Mr. "Sprague as to the true value of the interest for a fractional part "of a year. The function which expresses the amount of a sum of "money at the end of a given time, $a(1+i)^n$, is a continuous one; "and no sufficient reason can be assigned why it should hold for "certain values of n and suddenly cease to hold for others. It has "been well pointed out by our late Fellow, Mr. Orchard", (in the letter I have named) "that the other view leads to a most egregious "anomaly. And that is, that the difference between the value of "an annuity payable once a year, and one payable m times a year, "diminishes as m increases and vanishes when m becomes infinite."
May I be allowed to add a remark to those of Mr. Gray, namely, If it be an advantage to have an annuity payable oftener than once

a year for, say ten years, would not the advantage and therefore the value be at least as great, if not greater, were the annuity continued for ever?

To render complete the three tables described, I compiled a condensed table of ten-place logarithms. So many places for $\log(1+i)$ are generally requisite in order to have seven places true when it is multiplied by a number of three digits. It consists of 2,800 numbers with their logs, namely, 100,000 to 100,999 with differences—10,100 to 11,009 with first and second differences, and with a little table of corrections for the second difference to make the results true in the tenth place, and numbers 100 to 999, with reciprocals in addition to their logarithms. In this part of the table, the arguments may be considered as the present values of £1 in perpetuity, the reciprocals being the rates of interest. I have added the reciprocals in case it might be desired to use them at any time for interpolation, since the nearer the rates approach to each other the more exact will be the rate found by interpolation. The logarithms for $(1+i)$ are found by subtracting the log of the perpetuity from the log of the same perpetuity plus one. For example, the reciprocal of 200 is ·005, of 20, ·05, and of 2, ·5; therefore,

$$\begin{aligned} \text{Log } 201 &= 2{\cdot}30319{,}60574 \\ \text{Log } 200 &= 2{\cdot}30102{,}99957 \\ \hline \text{Log } 1{\cdot}005 &= 0{\cdot}00216{,}60617 \end{aligned} \qquad \begin{aligned} \text{Log } 21 &= 1{\cdot}32221{,}92947 \\ \text{Log } 20 &= 1{\cdot}30102{,}99957 \\ \hline \text{Log } 1{\cdot}05 &= 0{\cdot}02118{,}92990 \end{aligned}$$

$$\begin{aligned} \text{Log } 3 &= 0{\cdot}47712{,}12547 \\ \text{Log } 2 &= 0{\cdot}30102{,}99957 \\ \hline \text{Log } 1{\cdot}5 &= 0{\cdot}17609{,}12590 \end{aligned}$$

In the same manner is found $\log(1+i)$ for all the reciprocals.

Required the present value of an annuity of £1 for 260 intervals at ·002369668 per £1, this number being the reciprocal of 422.

$$\begin{aligned} \text{Log } 423 &= 2{\cdot}62634{,}03674 \\ \text{Log } 422 &= 2{\cdot}62531{,}24510 \\ \hline \text{Log } 1{\cdot}002369668 &= 0{\cdot}00102{,}79164 \times 260 \\ \hline \text{Log } (1+i)^{260} &= 0{\cdot}26725{,}83 \\ \hline \text{Log } (1+i)^{-260} &= 9{\cdot}73274{,}17 \\ \text{Log } \{(1+i)^{260}-1\} &= 9{\cdot}92960{,}74 \\ \text{Log } a_{\infty} = \text{Log } 422 &= 2{\cdot}62531{,}25 \\ \hline \text{Log } a_{\overline{260}|} &= 2{\cdot}28766{,}16, \end{aligned}$$

its number is £193·9374, the required value.

Another Solution.

$$\text{Log }(1+i)^{260}=0{\cdot}26725{,}83 \text{ subtract from}$$
$$\text{Log }422=2{\cdot}62531{,}25$$
$$2{\cdot}35805{,}42, \text{ its number is } 228{\cdot}0627 \text{ subtract from}$$
$$a_\infty = 422{\cdot}$$

The present value as before is $=£193{\cdot}9373$

Examples of the Use of the Tables.

(1) The present value of an annuity of £1 for 10 years being at 7 per-cent $=7{\cdot}02358$ (by Smart's Tables), what is the present value of an annuity of £1 per quarter at the equivalent rate, namely, $\cdot01705852$ per unit?

$a_{\overline{10}\vert}=$ 7·0235	8	
25301	4	Factor inverted.
280943	2	
7023	6	
210	7	
35	2	
1	4	
$4a^{(4)}_{\overline{10}\vert}=28{\cdot}8214$		Value required.

By Tables A and B.

$$\text{Log } a_{\overline{10}|}=0{\cdot}84655{,}86$$
$$\text{Constant}=0{\cdot}61315{,}66$$
$$\text{Log } 4a^{(4)}_{\overline{10}|}=1{\cdot}45971{,}52$$

its number is 28·8214 as before.

By direct Computation.

$$\text{Log } 1{\cdot}01705852=0{\cdot}00734{,}59423\times 40$$
$$\text{Log }(1+i)^{40}=0{\cdot}29383{,}77$$
$$\text{Log }(1+i)^{-40}=9{\cdot}70616{,}23$$
$$\text{Log }\{(1+i)^{40}-1\}=9{\cdot}98549{,}43$$
$$\text{Log } a_\infty=\text{Colog } i=1{\cdot}76805{,}87$$
$$1{,}45971{,}53 \text{ as before.}$$

(2) The amount of an annuity of £1 for 10 years at 7 per-cent being 13·816448, what is the amount of an annuity of £1 per quarter at the equivalent rate?

$s_{\overline{10}\vert}=$ 13·81644	8	
125301	4	Factor inverted.
5526579	2	
138164	5	
4144	9	
690	8	
27	6	
1	4	
$4s^{(4)}_{\overline{10}\vert}=56{\cdot}69608$		Amount required.

By Tables A and B.

$$\text{Log } a_{\overline{10}|}=0{\cdot}84655{,}86$$
$$\text{Log }(1+i)^{10}=0{\cdot}29383{,}78$$
$$\text{Constant}=0{\cdot}61315{,}66$$
$$1{\cdot}75355{,}30$$

its number is 56·696 08 as before.

By direct Computation.

$$\text{Log}\,(1+i)^{40\times\frac{1}{4}} = 0{\cdot}29383{,}78 = \text{Log}\,(1+i)^{10}$$

$$\text{Log}\,\{(1+i)^{40}-1\} = 9{\cdot}98549{,}43$$

$$\text{Log}\,a_{\infty} = 1{\cdot}76805{,}87$$

$$\text{Log}\,4s^{(4)}_{\overline{10|}} = 1{\cdot}75355{,}30 \text{ as before.}$$

(3) The annuity to repay £100 with interest at 7 per-cent for 10 years is 14·237750, what is the quarterly annuity to repay £100?

4·103521)14·237750(3·469643 The required Annuity.

12310753
1927187
1641408
285779
246211
39568
36932
2636
2462
174
164
10
12

By Tables A and B.

$$\text{Log}\,a_{\overline{10|}} = 0{\cdot}84655{,}86$$

$$\text{Constant} = 0{\cdot}61315{,}66$$

$$\text{Log}\,4a^{(4)}_{\overline{10|}} = 1{\cdot}45971{,}52$$

$$\text{Colog} = 8{\cdot}54028{,}48$$

$$\text{Log}\,100 = 2$$

$$0{\cdot}54028{,}48$$

its number is 3·4696,43 as before.

(4) What is the sinking fund, payable quarterly, to accumulate to £100 in 10 years at 7 per-cent yearly interest?

The annuity to repay £100 with interest is = 3·469643
The interest of £100 for one quarter is = 1·705852
The remainder is the required sinking fund = 1·763791

Proof.

Amount of £1 per quarter =	56·6960	8
Sinking fund inverted	19 7367	1
	566960	8
	396872	6
	34017	6
	1700	9
	396	8
	51	0
		6
	100·0000	

By Logarithms.

$$\text{Log}\,4s^{(4)}_{\overline{10|}} = 1{\cdot}75355{,}30$$

$$\text{Log}\,1{\cdot}763791 = 0{\cdot}24644\,71$$

$$\text{Log}\,100 = 2{\cdot}00000\,01$$

If the interest were convertible quarterly, and also one-fourth of the yearly rate, namely, ·0175, per £1 per quarter, the present value of an annuity of £1 would be $=28·5942$; but since the interest is assumed as convertible yearly, the value, namely, 28·8214, is slightly greater, the difference being ·2272, or about 4*s*. 6½*d*. Also the annuity to repay £100, with interest, would be greater, namely, 3·497209, from which, if the interest of £100 (1·75) be deducted, there remains 1·747209 for the sinking fund, which is smaller than when the interest is taken as convertible yearly. This may be of some importance when treating of assurance premiums payable at short intervals.

The half-yearly values in Smart's Tables which were pronounced "wrong" by Mr. Milne, at page 693 of his Work, and by Griffith Davies in note at page 41 of his, as "inapplicable to annual conversions", are, by the factors, rendered as useful as the yearly ones, as may be seen in the following example, namely:—

(5) The present value of an annuity of £1 for 12½ years at 7 per-cent is 8·153678, what is the present value of £1 per quarter at the equivalent quarterly rate?

$$a_{\overline{12\frac{1}{2}}|} = 8·153678$$

1253014	
3261471	2
81536	8
2446	1
407	7
16	3
	8

$$4a^{(4)}_{\overline{12\frac{1}{2}}|} = 33·45879$$

Value required.

By Logarithms.

$$\text{Log } a_{\overline{12\frac{1}{2}}|} = 0·91135,35$$
$$\text{Constant} = 0·61315,66$$
$$\text{Log } 4a^{(4)}_{\overline{12\frac{1}{2}}|} = 1·52451,01$$

its number is 33·45878 as before.

Another Solution.

$$\text{Log } (1+i) = 0·00734,59423 \times 50$$
$$\text{Log } (1+i)^{50} = 0·36729,71$$
$$\text{Log } (1+i)^{-50} = 9·63270,29$$
$$\text{Log}\{(1+i)^{50}-1\} = 0·12374,86$$
$$\text{Colog } i = 1·76805,87$$
$$\text{Log } 4a^{(4)}_{\overline{12\frac{1}{2}}|} = 1·52451,02$$

as before.

Similarly all the other usual values may be found for the half-years.

(6) A loan of £100 for 12 years is repaid by quarterly instalments of £3. 1*s*. 6*d*. each, What is the rate of interest? Turning 3 : 1 : 6 into a decimal and dividing by 100 we obtain ·03075; the logarithm of which is 8·4878451 for the annuity which £1 will purchase, the colog of this being $=1·51215,49$ is the true $\log 4a^{(4)}_{\overline{12}|}$, at the rate sought for. Inspection of Table C shows this to be slightly greater than 7 per-cent true yearly interest, and the rate is found by a simple interpolation, thus:—

7 per-cent	1·51312,40	1·5131240	·01735543	
$7\frac{1}{8}$ „	1·51042,86	1·5121549	·01705852	
	26954 :	9691 : :	29691 :	10675
				·01705852

The required rate is ·01716527 very nearly,

interpolation by means of the logarithmic values being as legitimate as by the natural numbers.

Proof of the Rate.

$$\text{Log } 1{\cdot}01716527 = 0{\cdot}00739{,}15232 \times 48$$

$$\text{Log } (1+i)^{48} = 0{\cdot}35479{,}31$$

$$\text{Log } (1+i)^{-48} = 9{\cdot}64520{,}69$$

$$\text{Log } \{(1+i)^{48} - 1\} = 0{\cdot}10159{,}79$$

$$\text{Log } a_\infty = \text{Colog } i = \bar{1}{\cdot}76534{,}94$$

$$\text{Log } 4a^{(4)}_{\overline{12}|} = 1{\cdot}51215{,}42$$

$$\text{True log } 4a^{(4)}_{\overline{12}|} = 1{\cdot}51215{,}49$$

$$\text{Error} = \cdot0000007$$

Log $a_{\overline{n}|}$ for $7\frac{1}{8}$ per-cent is found thus:—

$$\text{Log } a_{\overline{12}|} = 0{\cdot}89707{,}92$$

$$\text{Constant} = 0{\cdot}61334{,}94$$

$$\text{Log } 4a^{(4)}_{\overline{12}|} = 1{\cdot}51042{,}86$$

(7) How much of this loan is paid off at the end of $7\frac{1}{2}$ years? $12 - 7\frac{1}{2} = 4\frac{1}{2}$ years, or 18 quarters; then find the present value of the annuity for 18 quarters, and deduct it from the sum lent.

$$\text{Log } (1+i) = 0{\cdot}00739{,}15232 \times 18$$

$$\text{Log } (1+i)^{18} = 0{\cdot}13304{,}74$$

$$\text{Log } (1+i)^{-18} = 9{\cdot}86695{,}26$$

$$\text{Log } \{(1+i)^{18} - 1\} = 9{\cdot}55444{,}28$$

$$\text{Log } a_\infty = \text{Colog } i = \bar{1}{\cdot}76534{,}94$$

$$\text{Log } 3{\cdot}075 = 0{\cdot}48784{,}51$$

$$\text{Log } 4a^{(4)}_{\overline{4\frac{1}{2}}|} \times \text{an.} = 1{\cdot}67548{,}99$$

Lent	100·
Numb. =	47·27
Paid off	52·73

Another Solution.

Annuity =	3·075
Int. of £100 =	1·716527
Sinking fund =	1·358473

The question now becomes, to find the amount of an annuity equal to the sinking fund for 30 quarters.

Operation.

$$\text{Log } (1+i)^{30} = 0{\cdot}2217457$$

$$\text{Log } \{(1+i)^{30} - 1\} = 9{\cdot}8236510$$

$$\text{Log } a_\infty = \bar{1}{\cdot}7653494$$

$$\text{Log Sinking fund} = 0{\cdot}1330510$$

Log $4s^{(4)}_{\overline{7\frac{1}{2}}|} \times$ S.f. $= 1{\cdot}7220514$, the number to which is 52·73 as before.

(8) How much principal and interest are contained in the 31st instalment of this loan ? The respective amounts in the first intalment are seen above. Then for the principal raise the sinking fund to a power whose exponent is one less than the number of the instalment, the result deducted from the constant annuity leaves the interest, $31-1=30$.

$\text{Log } (1+i)^{30} = 0{\cdot}2217457$			
$\text{Log S.f.} = 0{\cdot}1330510$	Annuity $= 3{\cdot}075$		
$\text{Log Principal} = 0{\cdot}3547967$	Number $= 2{\cdot}258378$	Principal.	
	$0{\cdot}816622$	Interest.	

(9) What is the constant factor to transform yearly annuity-values at 7·185904 per-cent into quarterly ones ? By taking proportional parts *to eighths* of the differences, factors for any yearly rate are found.

The next less tabular rate is $7\frac{1}{8} = 7{\cdot}125$
Rate whose factor is wanted $= 7{\cdot}185904$

$\cdot 060904$ diff.

Tabular diff. 1927
9060 diff. inverted.

115·62
1·73

$117{\cdot}35 \times 8$ In place of dividing by ·125, we multiply by its reciprocal, 8.

Proportional part $= 939$
Next less factor $= 0{\cdot}6133494$

Required factor $= 0{\cdot}6134433$
$\text{Log } a_\infty = \text{Colog } i = 1{\cdot}1435186$ To find the quarterly interest, proceed as in the example.

$\text{Log } 4a^{(4)}_\infty$ for qrtr. $= 1{\cdot}7569619$

$\text{Log } i = \text{Colog } a_\infty = 8{\cdot}2430381$ Number $= \cdot 0175$ and $\cdot 0175 \times 100 = 1{\cdot}75$, the rate.

It is known that 1·75 per-cent paid quarterly = 7·185904 per-cent paid yearly.

It may be stated that annuities payable at stated intervals are readily changed into others of equivalent value for different intervals, by adding or subtracting the differences of their logarithmic factors.

I hope the following concluding remarks as to Annuities-certain may not be considered hypercritical. If we take the annuity to repay say £100 with interest in a certain number of years from the ordinary numerical tables, and for this annuity take the nearest penny which the tabulated decimal will give, we shall find that a difference of less than one half-penny in the repayment annuity creates an appreciable difference in the rate, which, when the annuity is large, it may be important to take account of.

(10) For example:—Taking, from Smart's tables, the annuity to repay £100 with interest in 20 half-years at $2\frac{1}{2}$ per-cent per half-year, we find it to be 6·414713, this being equal to £6. 8*s*. 3·53*d*., or taking the nearest penny, £6. 8*s*. 4*d*., which, decimally expressed, is 6·41666, &c. The half-yearly rate which this fractionally increased annuity gives is ·025032267, instead of ·025. Suppose £10,000 is lent, the repayment annuity being £641·666. The present value of this annuity for 20 half-years, at $2\frac{1}{2}$ per-cent, is £10,003·05, whereas at 2·5032267 per-cent, it is exactly £10,000.

To apply the factors to the transformation of life annuity-values and the obtaining assurance premiums payable in fractional intervals of a year:—

At the outset, I must say that it is my misfortune to know very little of the important subject of life contingencies, not having had time to give it attention, but which I hope some day to overtake.

For my present purpose, happily, it is not necessary to be able to apply the doctrine of probability to annuities. I begin where that has already been accomplished, with the values usually tabulated and payable yearly.

Since the factors apply to annuity-values, without reference to duration, namely, equally for all durations, it is evident this must include life annuity-values; and since every annuity short of a perpetuity must be for some definite term, or have a term corresponding to it, we may, if we know the rate of interest, discover the term from the annuity-value. We are thus furnished with a means of proof of what may be here adduced, namely, the same rules which govern annuities for terms certain.

I shall first give a few examples of deriving premiums for fractional intervals from yearly annuity-values, comparing them with those found in well-known works where interest is convertible at the same intervals. As may be expected, the premiums derived

by means of the factors, the interest being convertible yearly, will be somewhat greater.

Turning now to Dr. Farr's English Table No. 3, we find the value of an annuity on a male life, age 30, 3 per-cent, when reduced to the usual form (20·0143−1), is 19·0143. Here the half-yearly factor is 2·014889, and $(1-v) = \cdot0146707$, the formula being $1:(1+a_x)-(1-v)$. To obtain half-yearly premiums, we proceed as follows:—

$$\begin{array}{rr}
a_{30} = & 19\cdot0143 \\
\text{Factor inverted} = & 984102 \\
\hline
 & 380286 \\
 & 1901 \\
 & 761 \\
 & 152 \\
 & 17 \\
\hline
2a_{30}^{(2)} = & 38\cdot3117 \\
 & +\ 1\cdot \\
\hline
2\mathbf{a}_{30}^{(2)} = & 39\cdot3117
\end{array}
\qquad
\begin{array}{rl}
\dfrac{1}{39\cdot3117} & = \cdot0254377 \\
-(1-v) & = \cdot0146707 \\
\hline
\text{Half-yearly prem.} & = \cdot010767 \\
\text{Dr. Farr's} \quad ,, & = \cdot010606 \\
\hline
\text{Difference} & = \cdot000161
\end{array}$$

, being equal to 4*d*. on an assurance of £100.

At age 40, the yearly value (reduced) is 16·4744:—

$$\begin{array}{rr}
a_{40} = & 16\cdot4744 \\
\text{Factor inverted} = & 984102 \\
\hline
 & 329488 \\
 & 1647 \\
 & 659 \\
 & 132 \\
 & 15 \\
\hline
2a_{40}^{(2)} = & 33\cdot1941 \\
 & +\ 1\cdot \\
\hline
2\mathbf{a}_{40}^{(2)} = & 34\cdot1941
\end{array}
\qquad
\begin{array}{rl}
\dfrac{1}{34\cdot1941} & = \cdot0292448 \\
-(1-v) & = \cdot0146707 \\
\hline
\text{Half-yearly prem.} & = \cdot014574 \\
\text{Dr. Farr's} \quad ,, & = \cdot014336 \\
\hline
\text{Difference} & = \cdot000238
\end{array}$$

, being equal to 6*d*. on an assurance of £100.

For quarterly premiums, the factor is 4·044723 and $(1-v) = \cdot0073625$:—

$$\begin{array}{rr}
a_{30} = & 19\cdot0143 \\
\text{Factor inverted} = & 274404 \\
\hline
 & 760572 \\
 & 7606 \\
 & 761 \\
 & 133 \\
 & 4 \\
\hline
4a_{30}^{(4)} = & 76\cdot9076 \\
 & +\ 1\cdot \\
\hline
4\mathbf{a}_{30}^{(4)} = & 77\cdot9076
\end{array}
\qquad
\begin{array}{rl}
\dfrac{1}{77\cdot9076} & = \cdot0128357 \\
-(1-v) & = \cdot0073625 \\
\hline
\text{Quarterly Prem.} & = \cdot0054732 \\
\text{Dr. Farr's} \quad ,, & = \cdot005351 \\
\hline
\text{Difference} & = \cdot000122
\end{array}$$

, being equal to 3*d*. on an assurance of £100.

$$
\begin{array}{rr}
a_{40} = & 16\cdot4744 \\
\text{Factor inverted} = & 274404 \\
\hline
 & 65\cdot8976 \\
 & 6590 \\
 & 659 \\
 & 115 \\
 & 3 \\
\hline
4a_{40}^{(4)} = & 66\cdot6343 \\
 & +\ 1\cdot \\
\hline
4\mathbf{a}_{40}^{(4)} = & 67\cdot6343
\end{array}
$$

$$
\begin{array}{rl}
\dfrac{1}{67\cdot6343} = & \cdot0147854 \\
-(1-v) = & \cdot0073625 \\
\hline
\text{Quarterly Prem.} = & \cdot007423 \\
\text{Dr. Farr's}\ \ ,, \quad = & \cdot007241 \\
\hline
\text{Difference} = & \cdot000182
\end{array}
$$

, being equal to $4\frac{1}{2}d.$ on an assurance of £100.

Turning again to Mr. Ansell's work on Friendly Societies, Table VII, the value of an annuity of £1, at age 30, at 3 per-cent, is 18·1823, and at 40 is 15·6673. Here the factor is 12·16412 for months, and $(1-v)$ is ·0024602. Required the monthly premiums to assure £1 at these ages?

$$
\begin{array}{rr}
a_{30} = & 18\cdot1823 \\
\text{Factor inverted} = & 146121 \\
\hline
 & 181823 \\
 & 36365 \\
 & 1818 \\
 & 1091 \\
 & 73 \\
 & 2 \\
\hline
12a_{30}^{(12)} = & 221\cdot172 \\
 & +\ \ 1 \\
\hline
12\mathbf{a}_{30}^{(12)} = & 222\cdot172
\end{array}
$$

$$
\begin{array}{rl}
\dfrac{1}{222\cdot172} = & \cdot0045010 \\
-(1-v) = & \cdot0024602 \\
\hline
\text{Monthly prem.} = & \cdot0020408 \\
\text{Mr. Ansell's}\ \ ,, \quad = & \cdot001964 = \tfrac{1}{12} \text{ of his value in Table XXII.} \\
\hline
\text{Difference} = & \cdot000077
\end{array}
$$

, being equal to $2d.$ on an assurance of £100.

$$
\begin{array}{rr}
a_{40} = & 15\cdot6673 \\
\text{Factor inverted} = & 146121 \\
\hline
 & 156673 \\
 & 31335 \\
 & 1567 \\
 & 940 \\
 & 62 \\
 & 2 \\
\hline
12a_{40}^{(12)} = & 190\cdot579 \\
 & +\ \ 1 \\
\hline
12\mathbf{a}_{40}^{(12)} = & 191\cdot579
\end{array}
$$

$$
\begin{array}{rl}
\dfrac{1}{191\cdot579} = & \cdot0052198 \\
-(1-v) = & \cdot0024602 \\
\hline
\text{Monthly prem.} = & \cdot0027596 \\
\text{Mr. Ansell's}\ \ ,, \quad = & \cdot0026450 = \tfrac{1}{12} \text{ of his prem. in Table XXII.} \\
\hline
\text{Difference} = & \cdot000115
\end{array}
$$

, being equal to $2\frac{3}{4}d.$ on an assurance of £100.

I now give an example of obtaining single and annual premiums

in order to deduce from them a means of proving the correctness of premiums found for fractional intervals. Turning to Mr. Morgan's work on Assurances (1821), page 37, we find the value of an annuity at age 20, Northampton Table and 3 per-cent interest, to be 18·6385. The reciprocal of this, being the annuity which £1 will purchase, is ·05365239. If from this the interest of £1 (=·03) be deducted, there remains ·02365239, the portion of the annuity to replace the purchase-money, or the sinking fund. Then $18·6385 \times ·02365239 = ·440845$, and this is the sum which, if invested at the end of the year, will amount to £1 on the extinction of the life. If this were an annuity-certain, ·440845 would be the present value of £1 due at the end of its term.

It may be remarked that this, the present value of £1, namely, ·440845, will be the same, whether the annuity-value whence it may be derived be yearly or fractional, because the interest for the several intervals is such as, improved for the remainder of the year in the same ratio, will amount to the yearly interest, and no more, at the end of the year.

Since an assurance premium is payable at the beginning of a year, the value already found, namely, ·440845, must be discounted for a year, which brings it to ·428005, being the present value of £1 due at the end of the year in which, according to the table, death happens, or the single premium to insure £1 on the life.

If the single premium be divided by the annuity-value, 18·6385, the quotient would be the annual premium, if paid at the end of each year, for an assurance of £1; but since the premiums are payable at the beginning of each year, the annual premium supposed payable at the end of the year must be discounted for a year, or, which amounts to the same, the single premium must be divided by the annuity plus unity, in this instance 19·6385, the quotient being ·0217942, which is the annual premium required. The reciprocal of ·0217942 is 45·8838, which is the sum an annual premium of £1 will insure on the life, called by Griffith Davies "the improved amount of an annuity of £1 on an assigned life, payable at the beginning of the year." These values, namely, ·428005, ·0217942 and 45·8838 agree with those given by Mr. Davies at page 242 and in Table XXXVIII of his *Treatise on Annuities*, which may be accepted as abundant proof of their correctness.

My object in arriving at the single and annual premiums by the circuitous method just described, is to obtain a method of proof for annual premiums as well as for those for fractional intervals, by finding what would be the improved amounts of such sums invested

at the beginning of each year or interval for the period of duration corresponding to the annuity-value, which duration is very easily found from the single premium, thus :—the reciprocal of it, namely, of ·428005, is the amount of £1 in the time sought for, the logarithm of which divided by the log of $(1+i)$ gives the answer required, which in the present instance is 28·70957 years. The amount of £1 invested at the beginning of each year for 28·70957 years will amount to 45·8838, and so in proportion for any other sum or premium invested, which amount is due at the end of the year in which death happens.

The same reasoning applies to premiums for fractional intervals, thus :—for quarters, discounting ·440845 for one quarter, the single premium is ·437599, and dividing by the quarterly annuity (derived from the yearly one, in the manner previously shown) plus unity the quarterly premium is ·005728672 for an assurance of £1 payable at the end of the quarter in which death occurs, the duration in this case being 111·8383 quarters, being three-quarters of a year less than that of the assurance paid for by yearly premiums. Similarly, the duration for a monthly annuity is 333·515 months, being eleven months less than the yearly, and for a weekly annuity 1441·898 weeks, being fifty-one weeks less than the yearly duration, the assurance being payable at the end of the interval in which death happens and for which the premiums were paid.

Here it may be asked, Is it possible to invest monies so that compound interest may be obtained, convertible at short intervals, as weekly, monthly, or even quarterly ? I should think not. Hence it may be found that contingent liabilities provided for by periodic payments computed on the assumption of interest being converted into principal at short intervals cannot be met. This has been well pointed out by Mr. Orchard, in the letter to which I have already referred, and now recommend for careful perusal.

I have thus endeavored to show some of the uses of the constant factors. It is possible they may have other uses with which I am not at present acquainted. I have aimed throughout at being practical ;—whether I have succeeded must be left to the judgement of more experienced heads than mine.

APPENDIX.—TABLE I.—*Logarithms of Constant Factors.*

Yearly Rates.	Half-Yearly.	Quarterly.	Monthly.	Fortnightly.	Weekly.	Momently.	Yearly Rates.
2½	0·30371,92	0·60609,18	1·08410,73	1·42014,00	1·72127,32	0·00537,30	2½
2⅝	0·30385,24	0·60629,13	1·08435,10	1·42039,56	1·72153,38	0·00563,87	2⅝
2¾	0·30398,54	0·60649,06	1·08459,45	1·42065,10	1·72179,42	0·00590,42	2¾
2⅞	0·30411,84	0·60668,98	1·08483,77	1·42090,61	1·72205,44	0·00616,95	2⅞
3	0·30425,12	0·60688,88	1·08508,07	1·42116,09	1·72231,43	0·00643,44	3
3⅛	0·30438,38	0·60708,75	1·08532,34	1·42141,54	1·72257,40	0·00669,91	3⅛
3¼	0·30451,64	0·60728,61	1·08556,59	1·42166,97	1·72283,34	0·00696,35	3¼
3⅜	0·30464,88	0·60748,45	1·08580,82	1·42192,38	1·72309,24	0·00722,77	3⅜
3½	0·30478,11	0·60768,27	1·08605,02	1·42217,76	1·72335,13	0·00749,16	3½
3⅝	0·30491,33	0·60788,07	1·08629,20	1·42243,12	1·72360,98	0·00775,52	3⅝
3¾	0·30504,54	0·60807,85	1·08653,35	1·42268,44	1·72386,81	0·00801,86	3¾
3⅞	0·30517,74	0·60827,61	1·08677,48	1·42293,75	1·72412,62	0·00828,17	3⅞
4	0·30530,92	0·60847,36	1·08701,59	1·42319,03	1·72438,41	0·00854,45	4
4⅛	0·30544,09	0·60867,08	1·08725,67	1·42344,28	1·72464,16	0·00880,71	4⅛
4¼	0·30557,25	0·60886,79	1·08749,73	1·42369,51	1·72489,89	0·00906,94	4¼
4⅜	0·30570,40	0·60906,48	1·08773,76	1·42394,72	1·72515,59	0·00933,14	4⅜
4½	0·30583,54	0·60926,15	1·08797,77	1·42419,89	1·72541,27	0·00959,32	4½
4⅝	0·30596,66	0·60945,80	1·08821,76	1·42445,04	1·72566,93	0·00985,47	4⅝
4¾	0·30609,77	0·60965,43	1·08845,72	1·42470,17	1·72592,56	0·01011,60	4¾
4⅞	0·30622,87	0·60985,04	1·08869,66	1·42495,27	1·72618,16	0·01037,70	4⅞
5	0·30635,96	0·61004,64	1·08893,58	1·42520,35	1·72643,74	0·01063,77	5
5⅛	0·30649,04	0·61024,21	1·08917,47	1·42545,41	1·72669,28	0·01089,82	5⅛
5¼	0·30662,11	0·61043,77	1·08941,34	1·42570,44	1·72694,80	0·01115,84	5¼
5⅜	0·30675,16	0·61063,31	1·08965,19	1·42595,44	1·72720,30	0·01141,84	5⅜
5½	0·30688,20	0·61082,83	1·08989,01	1·42620,42	1·72745,78	0·01167,81	5½
5⅝	0·30701,23	0·61102,33	1·09012,81	1·42645,38	1·72771,23	0·01193,76	5⅝
5¾	0·30714,25	0·61121,82	1·09036,59	1·42670,31	1·72796,66	0·01219,68	5¾
5⅞	0·30727,26	0·61141,28	1·09060,35	1·42695,21	1·72822,06	0·01245,57	5⅞
6	0·30740,25	0·61160,73	1·09084,08	1·42720,10	1·72847,43	0·01271,44	6
6⅛	0·30753,24	0·61180,16	1·09107,79	1·42744,95	1·72872,79	0·01297,28	6⅛
6¼	0·30766,21	0·61199,57	1·09131,47	1·42769,79	1·72898,12	0·01323,10	6¼
6⅜	0·30779,17	0·61218,96	1·09155,13	1·42794,60	1·72923,41	0·01348,89	6⅜
6½	0·30792,12	0·61238,34	1·09178,77	1·42819,38	1·72948,69	0·01374,66	6½
6⅝	0·30805,06	0·61257,69	1·09202,39	1·42844,15	1·72973,94	0·01400,40	6⅝
6¾	0·30817,99	0·61277,03	1·09225,99	1·42868,89	1·72999,17	0·01426,11	6¾
6⅞	0·30830,90	0·61296,35	1·09249,56	1·42893,60	1·73024,38	0·01451,81	6⅞
7	0·30843,81	0·61315,66	1·09273,11	1·42918,29	1·73049,56	0·01477,47	7
7⅛	0·30856,70	0·61334,94	1·09296,64	1·42942,95	1·73074,71	0·01503,11	7⅛
7¼	0·30869,58	0·61354,21	1·09320,14	1·42967,59	1·73099,83	0·01528,73	7¼
7⅜	0·30882,45	0·61373,46	1·09343,62	1·42992,21	1·73124,94	0·01554,32	7⅜
7½	0·30895,31	0·61392,69	1·09367,08	1·43016,80	1·73150,02	0·01579,89	7½
7⅝	0·30908,16	0·61411,90	1·09390,52	1·43041,38	1·73175,07	0·01605,43	7⅝
7¾	0·30920,99	0·61431,10	1·09413,93	1·43065,93	1·73200,10	0·01630,95	7¾
7⅞	0·30933,82	0·61450,28	1·09437,32	1·43090,45	1·73225,11	0·01656,44	7⅞
8	0·30946,63	0·61469,44	1·09460,69	1·43114,95	1·73250,10	0·01681,91	8
8⅛	0·30959,43	0·61488,58	1·09484,04	1·43139,42	1·73275,06	0·01707,35	8⅛
8¼	0·30972,23	0·61507,71	1·09507,36	1·43163,88	1·73299,99	0·01732,77	8¼
8⅜	0·30985,01	0·61526,81	1·09530,66	1·43188,31	1·73324,90	0·01758,16	8⅜
8½	0·30997,77	0·61545,90	1·09553,94	1·43212,71	1·73349,79	0·01783,53	8½
8⅝	0·31010,53	0·61564,97	1·09577,20	1·43237,10	1·73374,66	0·01808,88	8⅝
8¾	0·31023,28	0·61584,03	1·09600,44	1·43261,46	1·73399,50	0·01834,19	8¾
8⅞	0·31036,02	0·61603,07	1·09623,66	1·43285,79	1·73424,32	0·01859,49	8⅞
9	0·31048,74	0·61622,09	1·09646,85	1·43310,10	1·73449,11	0·01884,76	9
9⅛	0·31061,45	0·61641,09	1·09670,02	1·43334,39	1·73473,87	0·01910,01	9⅛
9¼	0·31074,16	0·61660,08	1·09693,17	1·43358,66	1·73498,62	0·01935,23	9¼
9⅜	0·31086,85	0·61679,05	1·09716,30	1·43382,90	1·73523,34	0·01960,43	9⅜
9½	0·31099,53	0·61698,00	1·09739,41	1·43407,13	1·73548,04	0·01985,61	9½
9⅝	0·31112,20	0·61716,93	1·09762,49	1·43431,32	1·73572,71	0·02010,76	9⅝
9¾	0·31124,86	0·61735,85	1·09785,55	1·43455,50	1·73597,36	0·02035,89	9¾
9⅞	0·31137,51	0·61754,75	1·09808,59	1·43479,65	1·73621,99	0·02060,99	9⅞
10	0·31150,14	0·61773,63	1·09831,61	1·43503,78	1·73646,60	0·02086,07	10

TABLE II.—*Fractional Rates of Interest corresponding to Yearly Rates.*

Yearly i	Half-Yearly. $\{(1+i)^{\frac{1}{2}}-1\}$	Quarterly. $\{(1+i)^{\frac{1}{4}}-1\}$	Monthly. $\{(1+i)^{\frac{1}{12}}-1\}$	Fortnightly. $\{(1+i)^{\frac{1}{26}}-1\}$	Weekly. $\{(1+i)^{\frac{1}{52}}-1\}$	Momently. $\log_e(1+i)$	Yearly i
$2\frac{1}{2}$	·012422,84	·0061922,46	·0020958,36	·00095016,71	·00047497,07	·024692,61	$2\frac{1}{2}$
$2\frac{5}{8}$	·013039,98	·0064988,72	·0021616,15	·00099708,84	·00049842,00	·025911,38	$2\frac{5}{8}$
$2\frac{3}{4}$	·013656,75	·0068052,18	·0022632,80	·0010439,55	·00052184,13	·027128,67	$2\frac{3}{4}$
$2\frac{7}{8}$	·014273,14	·0071112,84	·0023648,31	·0010907,66	·00054523,46	·028344,47	$2\frac{7}{8}$
3	·014889,16	·0074170,72	·0024662,70	·0011375,24	·00056860,01	·029558,80	3
$3\frac{1}{8}$	·015504,80	·0077225,82	·0025675,95	·0011842,26	·00059193,77	·030771,66	$3\frac{1}{8}$
$3\frac{1}{4}$	·016120,07	·0080278,13	·0026688,09	·0012308,74	·00061524,77	·031983,05	$3\frac{1}{4}$
$3\frac{3}{8}$	·016734,97	·0083327,68	·0027699,10	·0012774,68	·00063853,00	·033192,97	$3\frac{3}{8}$
$3\frac{1}{2}$	·017349,50	·0086374,46	·0028708,99	·0013240,07	·00066178,48	·034401,43	$3\frac{1}{2}$
$3\frac{5}{8}$	·017963,65	·0089418,48	·0029717,75	·0013704,93	·00068501,20	·035608,43	$3\frac{5}{8}$
$3\frac{3}{4}$	·018577,44	·0092459,76	·0030725,42	·0014169,25	·00070821,18	·036813,97	$3\frac{3}{4}$
$3\frac{7}{8}$	·019190,85	·0095498,29	·0031731,96	·0014633,03	·00073138,41	·038018,07	$3\frac{7}{8}$
4	·019803,90	·0098534,07	·0032737,40	·0015096,27	·00075452,90	·039220,71	4
$4\frac{1}{8}$	·020416,58	·010156,71	·0033741,73	·0015558,98	·00077764,66	·040421,92	$4\frac{1}{8}$
$4\frac{1}{4}$	·021028,89	·010459,74	·0034744,95	·0016021,16	·00080073,72	·041621,68	$4\frac{1}{4}$
$4\frac{3}{8}$	·021640,84	·010762,50	·0035747,07	·0016482,80	·00082380,06	·042820,00	$4\frac{3}{8}$
$4\frac{1}{2}$	·022252,42	·011064,99	·0036748,09	·0016943,91	·00084683,70	·044016,89	$4\frac{1}{2}$
$4\frac{5}{8}$	·022863,63	·011367,21	·0037748,03	·0017404,49	·00086984,62	·045212,34	$4\frac{5}{8}$
$4\frac{3}{4}$	·023474,47	·011669,15	·0038746,85	·0017864,54	·00089282,85	·046406,37	$4\frac{3}{4}$
$4\frac{7}{8}$	·024084,96	·011970,83	·0039744,59	·0018324,07	·00091578,38	·047598,98	$4\frac{7}{8}$
5	·024695,08	·012272,23	·0040741,24	·0018783,07	·00093871,26	·048790,16	5
$5\frac{1}{8}$	·025304,84	·012573,37	·0041736,80	·0019241,54	·00096161,47	·049979,93	$5\frac{1}{8}$
$5\frac{1}{4}$	·025914,23	·012874,24	·0042731,28	·0019699,49	·00098448,99	·051168,29	$5\frac{1}{4}$
$5\frac{3}{8}$	·026523,26	·013174,83	·0043724,68	0020156,92	·0010073,38	·052355,23	$5\frac{3}{8}$
$5\frac{1}{2}$	·027131,93	·013475,17	·0044716,99	·0020613,82	·0010301,61	·053540,77	$5\frac{1}{2}$
$5\frac{5}{8}$	·027740,24	·013775,24	·0045708,23	·0021070,21	·0010529,56	·054724,90	$5\frac{5}{8}$
$5\frac{3}{4}$	·028348,19	·014075,04	·0046698,39	·0021526,07	·0010757,25	·055907,63	$5\frac{3}{4}$
$5\frac{7}{8}$	·028955,78	·014374,58	·0047687,48	·0021981,42	·0010984,68	·057088,97	$5\frac{7}{8}$
6	·029563,01	·014673,85	·0048675,51	·0022436,25	·0011211,84	·058268,91	6
$6\frac{1}{8}$	·030169,89	·014972,85	·0049662,46	·0022890,57	·0011438,74	·059447,46	$6\frac{1}{8}$
$6\frac{1}{4}$	·030776,41	·015271,59	·0050648,35	·0023344,37	·0011665,38	·060624,62	$6\frac{1}{4}$
$6\frac{3}{8}$	·031382,57	·015570,07	·0051633,17	·0023797,66	·0011891,76	·061800,40	$6\frac{3}{8}$
$6\frac{1}{2}$	·031988,37	·015868,28	·0052616,94	·0024250,43	·0012117,88	·062974,80	$6\frac{1}{2}$
$6\frac{5}{8}$	·032593,82	·016166,24	·0053599,66	·0024702,70	·0012343,73	·064147,82	$6\frac{5}{8}$
$6\frac{3}{4}$	·033198,92	·016463,93	·0054581,30	·0025154,46	·0012569,33	·065319,47	$6\frac{3}{4}$
$6\frac{7}{8}$	·033803,66	·016761,36	·0055561,90	·0025605,69	·0012794,66	·066489,74	$6\frac{7}{8}$
7	·034408,04	·017058,52	·0056541,45	·0026056,45	·0013019,75	·067658,65	7
$7\frac{1}{8}$	·035012,08	·017355,43	·0057519,95	·0026506,68	·0013244,57	·068826,19	$7\frac{1}{8}$
$7\frac{1}{4}$	·035615,76	·017652,08	·0058497,41	·0026956,41	·0013469,13	·069992,37	$7\frac{1}{4}$
$7\frac{3}{8}$	·036219,08	·017948,47	·0059473,82	·0027405,64	·0013693,44	·071157,20	$7\frac{3}{8}$
$7\frac{1}{2}$	·036822,07	·018244,60	·0060449,19	·0027854,36	·0013917,49	·072320,66	$7\frac{1}{2}$
$7\frac{5}{8}$	·037424,70	·018540,47	·0061423,52	·0028302,58	·0014141,29	·073482,78	$7\frac{5}{8}$
$7\frac{3}{4}$	·038026,97	·018836,09	·0062396,82	·0028750,30	·0014364,84	·074643,54	$7\frac{3}{4}$
$7\frac{7}{8}$	·038628,90	·019131,45	·0063369,07	·0029197,53	·0014588,12	·075802,97	$7\frac{7}{8}$
8	·039230,48	·019426,55	·0064340,30	·0029644,25	·0014811,16	·076961,04	8
$8\frac{1}{8}$	·039831,72	·019721,40	·0065310,50	·0030090,47	·0015033,94	·078117,78	$8\frac{1}{8}$
$8\frac{1}{4}$	·040432,60	·020015,98	·0066279,67	·0030536,21	·0015256,47	·079273,18	$8\frac{1}{4}$
$8\frac{3}{8}$	·041033,14	·020310,32	·0067247,80	·0030981,45	·0015478,75	·080427,25	$8\frac{3}{8}$
$8\frac{1}{2}$	·041633,33	·020604,40	·0068214,93	·0031426,19	·0015700,77	·081579,99	$8\frac{1}{2}$
$8\frac{5}{8}$	·042233,18	·020898,22	·0069181,03	·0031870,44	·0015922,54	·082731,40	$8\frac{5}{8}$
$8\frac{3}{4}$	·042832,68	·021191,79	·0070146,11	·0032314,21	·0016144,07	·083881,49	$8\frac{3}{4}$
$8\frac{7}{8}$	·043431,84	·021485,11	·0071110,18	·0032757,47	·0016365,35	·085030,25	$8\frac{7}{8}$
9	·044030,65	·021778,18	·0072073,23	·0033200,26	·0016586,37	·086177,70	9
$9\frac{1}{8}$	·044629,11	·022071,00	·0073035,25	·0033642,55	·0016807,15	·087323,83	$9\frac{1}{8}$
$9\frac{1}{4}$	·045227,25	·022363,56	·0073996,30	·0034084,36	·0017027,68	·088468,65	$9\frac{1}{4}$
$9\frac{3}{8}$	·045825,03	·022655,87	·0074956,33	·0034525,67	·0017247,97	·089612,16	$9\frac{3}{8}$
$9\frac{1}{2}$	·046422,48	·022947,93	·0075915,34	·0034966,52	·0017468,00	·090754,36	$9\frac{1}{2}$
$9\frac{5}{8}$	·047019,58	·023239,75	·0076873,35	·0035406,87	·0017687,79	·091895,26	$9\frac{5}{8}$
$9\frac{3}{4}$	·047616,34	·023531,31	·0077830,37	·0035846,74	·0017907,34	·093034,87	$9\frac{3}{4}$
$9\frac{7}{8}$	·048212,76	·023822,62	·0078786,38	·0036286,13	·0018126,63	·094173,17	$9\frac{7}{8}$
10	·048808,85	·024113,69	·0079741,40	·0036725,03	·0018345,69	·095310,18	10

Discussion.

The President (Mr. A. H. Bailey) said he was sure they would all appreciate this contribution by a new member, who had shown a great amount of industry in the preparation of the tables which he had exhibited that evening. The paper would have a practical value for actuaries. Assurance companies often make advances in consideration of annuities for terms of years, payable most frequently half-yearly, but in some cases quarterly. Such a rate of interest as $4\frac{1}{8}$ per-cent payable half-yearly has of late been by no means uncommon in these transactions, and in such cases tables of compound interest at $2\frac{1}{16}$ are wanted, but none such are available. Colonel Oakes's tables ascend by steps of $\frac{1}{8}$th, but not of $\frac{1}{16}$th. He thought some of them therefore might derive assistance in their work from Mr. McKenzie's labors.

Mr. G. King said they were of course all very well aware that the values and amounts of annuities payable yearly can be transformed into similar functions payable quarterly, or at any other intervals, by means of constant factors, if the rate of interest is kept the same; but he did not think it was a common thing in practice so to do. When they grant loans to corporations and other bodies in exchange for annuities payable half-yearly, they take the interest convertible half-yearly, thus securing a somewhat higher interest. However, these factors are worthy of attention. Mr. Sprague, in his article on "Annuities" in the *Encyclopædia Britannica*, gave another form of factor. The form given in most of the books is derived from looking at the rate of interest in the denominator of the expression for the value of the annuity. In the numerator the expression is identical, whether the annuity be payable yearly or at any other interval. But in the denominator we get a different expression, and by taking the ratios of the denominators the factors are obtained. Mr. Sprague looked at it differently. He accumulated to the end of the year the payments of the annuity payable at short intervals, and formed them in that way into a new annuity payable yearly, and by means of the accumulation got an expression for the factor. For instance, for an annuity payable half-yearly, Mr. Sprague's constant factor is $\frac{1}{2}\{(1+i)^{\frac{1}{2}}+1\}$. It is of course identical in value with the other, but in a different form.

Towards the close of the paper an application of the constant factors to life annuities is given, but they do not seem exactly suitable for the purpose. In the case of annuities-certain, there is only one difference between the annuity payable yearly and at other intervals, and it arises simply from the interest. In the case of life annuities, there is the same difference of interest, and in addition the difference of mortality. These differences diverge in opposite directions, and tend to neutralize each other, and hence the closeness of the results obtained by Mr. McKenzie, as compared with the figures of Dr. Farr and others. In the cases of annuities on old lives the differences would be greater.

Mr. McKenzie had pointed out in certain tables of annuities a considerable number of errors. Now it is very desirable to have some simple and ready means of checking a table with which we are not familiar. To recompute all the values would be absurd, but if by

some simple formula a check can be got, it is a great advantage. In investigating Mr. Makeham's annuities of different orders, he (Mr. King) had found a method of checking printed tables by mere addition. Supposing they had tables of present values of annuities for any number of years up, say, to 100, like the tables in Jones's book, if they simply summed them, they would find that the sum of the column was equal to $(n-a_{\overline{n}|}):i$. Also as regarded the columns of amounts, the sum is equal to $\{(1+i)s_{\overline{n}|}-n\}:i$, the small letter s being used to denote the amount of an annuity, there being no symbol for that function in the Institute scheme of notation. If the equation holds, the table is correct. If not, there is an error, and we must divide the table into sections, and apply the formula again and so localize the error.

Mr. C. WALFORD remarked that, apart from the scientific value of the paper, anything which recalled the life of De Moivre had for him a great charm, embracing as it did the circumstances which drove him from his own country and induced him to come here—the period of the South Sea Bubble, when the Government of this country was pressed very much in its mode of State financing, and had recourse to lotteries. Out of that system of finance by means of State lotteries arose a very long series of problems, which must have been exceedingly interesting to De Moivre, and in all probability have provided him with a considerable source of revenue. With regard to these lotteries, there were forty combinations of chance. You could insure with a broker as to whether you would draw a prize or a blank, with another broker as to whether your number would be drawn before a certain hour or a certain day; and with another against the chance of your number consisting of two, three, four, or five figures. Every combination of chance that ingenuity or want of money could devise was applied to the system of lotteries.

Mr. PETER GRAY was much pleased with the formulas of verification brought before them by Mr. King. He believed they were quite new, and they ought to be very useful. Mr. Walford's references to De Moivre reminded him of an incident in his own experience. In the preface to a work published thirty years ago he spoke of De Moivre as "the ingenious De Moivre". He submitted the preface to Prof. De Morgan, by whom it was returned with the notandum:— "Ingenious! why it ought to be *illustrious* at the very least." Of course he adopted the correction.

Mr. T. G. ACKLAND thought these factors peculiarly applicable to the valuation of the affairs of building societies, where monthly payments usually arise. If we have, for example, to compute the value of monthly instalments, with interest at 6 per-cent per annum, it was far from satisfactory to take the tabular value at ½ per-cent per month for the number of months, as that plan assumes that interest is convertible monthly, and gives in the year a rate exceeding 6 per-cent. The better plan in the case of monthly payments would be to employ the factors advocated in the paper before them.

Mr. G. W. BERRIDGE remarked that in the case of an annuity-certain, whether it was payable yearly or half-yearly, the same gross amount was payable; but in case of a life annuity this is not so.

Supplementary Note by Mr. McKenzie on the Application of the Factors to Life-Annuity-Values and Assurance Premiums.

It has been objected that the factors are not suited for these applications, since in the preceding paper no account is taken of any portion the annuitant may live to receive during the year in which he dies. The factors, as there applied, transform the yearly annuity-values into their exact equivalents for fractional intervals of a year, and the premiums thereby obtained are also the exact equivalents of the yearly ones; so that the annuitant, in the case under consideration, being the life office, would have received to the end of the year preceding death the fractional payments necessary to provide for the assurance,—as it would, in fact, have received were the premiums paid yearly; and any further payments received during the year in which death happens would be surplus, which, under the more usual conditions of assurance, would be returnable in the shape of increased benefit to the assured. To obviate the objection, however, I proceed to apply corrections to the factors, which, without asserting these are the best which could be chosen, give very close results, with equal facility of application to any table of mortality for which yearly annuity-values have been computed. It is to be noted that the fractional premiums, as found by means of the factors, the interest being convertible yearly, will be somewhat greater than those with which they are to be compared, where interest is convertible at the same intervals. Reducing the factors to De Moivre's form, by dividing each by its own interval, or, better, subtracting from the logarithmic factors the logarithms of their respective intervals, and, in the case of half-yearly factors adding one-half, quarterly factors three-quarters, and monthly factors eleven-twelfths of the remainders so obtained, to the logarithmic factors, and taking out the natural numbers, we render the factors better adapted to the finding of assurance premiums for these intervals. The factors at 3 per-cent so corrected are, half-yearly 2·022375, quarterly 4·078593, and monthly 12·31654. Taking the same examples as in the paper, and substituting in the operations the factors corrected, as above given, we obtain, for

Half-yearly premiums at age 30	·010675	At age 40	·014469
Dr. Farr's „ „ „	·010606	Dr. Farr's	·014336
Difference	·000069.	Difference	·000133.
Quarterly premiums at age 30	·005368	At age 40	·007302
Dr. Farr's „ „ „	·005351	Dr. Farr's	·007241
Difference	·000017.	Difference	·000061.
Monthly premium at age 30	·001985	At age 40	·002695
Mr. Ansell's „ „ „	·001964	Mr. Ansell's	·002645
Difference	·000021.	Difference	·000050.

A further proof of the correspondence of these premiums with those given in Dr. Farr's and Mr. Ansell's works, may be obtained by computing the times in which they will respectively amount to the sums assured. I find the durations to be—

At age 30,	Half-years,	Dr. Farr's	58·616	equal to	29·308	years.
,,	,,	By factors	58·507	,,	29·254	,,
At age 40,	,,	Dr. Farr's	47·586	,,	23·793	,,
,,	,,	By factors	47·369	,,	23·684	,,
At age 30,	Quarters,	Dr. Farr's	116·675	,,	29·169	,,
,,	,,	By factors	116·857	,,	29·214	,,
At age 40,	,,	Dr. Farr's	94·634	,,	23·658	,,
,,	,,	By factors	94·359	,,	23·590	,,
At age 30,	Months,	Mr. Ansell's	327·629	,,	27·302	,,
,,	,,	By factors	327·255	,,	27·271	,,
At age 40,	,,	Mr. Ansell's	266·120	,,	22·175	,,
,,	,,	By factors	263·294	,,	21·941	,,

The operation of correcting the factors is as follows:—

3 *per-cent Yearly Interest.*

HALF-YEARLY.		QUARTERLY.		MONTHLY.	
Log factor=	0·30425,12	Log factor=	0·60688,88	Log factor=	1·08508,07
—log 2=	0·30103,00	—log 4=	0·60206,00	—log 12=	1·07918,12
$\frac{1}{2}$ of	0·00322,12	$\frac{3}{4}$ of	0·00482,88	$\frac{11}{12}$ of	0·00589,95
	=0·00161,06		=0·00362,16		=0·00540,79
	+0·30425,12		+0·60688,88		+1·08508,07
Corrected log factor	0·30586,18	Corrected log factor	0·61051,04	Corrected log factor	1·09048,86
Number	2·022375	Number	4·078593	Number	12·31654

being the factors corrected for life-annuity-values.

On the Rates of Fatal Accidents in various Occupations. By WILLIAM J. H. WHITTALL, F.I.A., *of the Clerical, Medical, and General Life Assurance Society.*

[Read before the Institute, 2 Jan. 1882.]

THE rapid growth of Accident Assurance, the passing of the Employers' Liability Act, and the periodically recurring discussions respecting schemes for a National Accident Fund, have created a demand for statistics on the subject of accidents, which cannot be said to have been satisfied.

The Tables given in this paper are believed to be unique as regards this country. The object of their compilation was to show the Rates of Fatal Accidents in various occupations. They are based upon a comparison of the male population in each occupation in England and Wales, as enumerated at the Census of 1871, with the average of the number of the violent deaths occurring to male persons following that occupation in England and Wales during the three years, 1870, 1871 and 1872. For the extraction of the particulars relating to these deaths from the records of the General Register Office, special permission was obtained by Dr. Farr; and it is with his cordial approval that the facts are now laid before the members of the Institute.

The Census of 1871 was taken on the 3rd April of that year. The population enumerated was therefore slightly in excess of the mean population to which the average of the deaths for the years 1870, 1871 and 1872 would have reference. The difference would, however, be slight; and for the present purpose it was thought that the population as enumerated might be considered suitable.

As the subject of accidents to children formed no part of the present investigation, all accidents occurring under 10 years of age were disregarded, the population being, of course, similarly treated. To make the investigation complete, the whole of the deaths were classified with respect to age; but as the figures for several of the occupations are small, it was not considered desirable to present the whole of the facts in this respect. The influence of age on accidents is, however, treated for the more numerous occupations further on.

Besides all accidents occurring under 10 years of age, the deaths from exposure to cold, homicide, and suicide and executions, were excluded. Mr. Walford, in his paper on Fatal Accidents read before the Statistical Society in February last, states that in order to give the returns of the Registrar-General practical value it is necessary to exclude, besides the deaths from the causes just mentioned, the deaths from sunstroke and lightning, and also all the deaths from "violence *not otherwise classed*". With regard to the deaths from sunstroke and lightning, it depends mainly upon the special object of an investigation whether they should be excluded. Perhaps it would have been better in the present instance if such had been the case. An examination, however, of the details of the deaths from "violence *not otherwise classed*",

does not suggest to me the desirability of excluding these also *. On the other hand, I should think that a large number of the deaths resulting from poisons, which seem to be retained by Mr. Walford, would not be generally regarded as "accidents". To form an idea of how these various causes may influence the results, I give the following statement of the deaths affected by them :—

Cause of Death.	1870.	1871.	1872.	Total.
Sunstroke . .	69	47	57	173
Lightning . .	12	23	35	70
Poisons (all) . .	182	165	169	516
TOTAL . .	263	235	261	759

Many of these deaths are no doubt legitimately included, and the remainder are not likely to cause the risk of accident to be overstated; for there are counteracting influences at work. For example, large numbers of deaths are known to occur which might be distinctly traced to the effects of accidents which happened some time previously, but which are not registered as caused by them.

The total number of deaths classified is 25,455. An examination of the Registrar-General's reports for the years 1870, 1871 and 1872 gives the following results:—

Year.	Total Accidents (exclusive of Exposure to Cold, Homicide, Suicide, and Executions).	Do. (under 10 years of age).	Balance.
1870	10,823	2,772	8,051
1871	11,225	2,857	8,368
1872	11,664	2,807	8,857
TOTAL .	33,712	8,436	25,276

* The greater number of the adult deaths classified under "Violence *not otherwise classed*" are ascribed to "Accident (not otherwise described)", "Manslaughter (manner not stated)" and "Injury (manner not stated)"—most of which it is probably right to retain. Some which are ascribed to "Murder and Suicide (manner not stated)", should be excluded, but they are very few in number.

It would from this appear that the number of deaths included in the present abstract is too great by 179; but the explanation is that in making the classification all cases of manslaughter were not rigorously excluded. Where, notwithstanding a verdict of manslaughter, the crimes did not appear to have been wilful, and were thus in the nature of accidents to the victims, the deaths caused by them were included. Many instances in which this might be the case will suggest themselves. The majority of them were in connection with mines and railways.

The fact that almost every one of the deaths was the subject of a coroner's inquest, is suggestive of care in their registration; and their classification has been made on exactly the same lines as the classification of occupations in the Census. As a preliminary test as to whether correctness has resulted, we may criticize the number of deaths constituting what may be called the "residuum", *i.e.*, those which were insufficiently described to be placed to any definite occupation, and compare them with the population which was similarly disposed of in the Census. First, then, it was found that 4,426 deaths were placed to "general labourer (branch undefined)"; and the population so described at the Census being 509,010, this would give an average annual death rate from accidents of 29 per 10,000. This is no doubt excessive; but, as will be seen further on, it is possible to surmise with probable correctness the occupations over which these surplus deaths should have been distributed. It was further found that there were 1,604 deaths which it was necessary to place to "no stated occupation"; the population so described in the Census being 157,490. This would give an average annual rate per 10,000 of 33·9, which is possibly too high. But assuming that the real rate applicable here is as low as 15 per 10,000, there would only remain a surplus of 895 deaths to be accounted for. With the exception, then, of some such small residuum, all the deaths may be said to have been classified. Considering the nature and extent of the investigation, this cannot be considered unsatisfactory.

A small proportion of the deaths classified are not included in the tables, as they belong to obscure occupations, where the smallness of the numbers involved renders them scarcely worthy of notice.

In Table No. I have been included all the results where neither the nature of the occupations nor the results themselves suggested any disturbing influence.

Table I.—*Showing, for various Occupations, the number of Fatal Accidents in England and Wales in the years* 1870, 1871 *and* 1872*; the Population in England and Wales, as enumerated at the Census of* 1871*; and the Rate of Fatal Accidents.*

Occupation.	Fatal Accidents in 1870.	1871.	1872.	Total of the 3 Years.	Population, 1871.	Average Annual Rate of Fatal Accidents per 10,000.	Remarks.
Author, &c.; Student	8	8	7	23	61,085	1·3	
Accountant	1	3	1	5	9,832	1·7	
Protestant Minister	1	...	4	5	9,264	1·8	
Locksmith	4	...	...	4	7,154	1·9	
Nurseryman, &c.	..	...	3	3	5,113	2·0	
Bookseller; Stationer	2	3	4	9	15,186	2·0	
Pawnbroker	1	1	2	4	6,449	2·1	
Tool Maker—Dealer	1	3	1	5	7,453	2·2	
Whitesmith	2	...	4	6	8,588	2·3	
Hosier; Haberdasher	...	1	2	3	4,381	2·3	
Wine and Spirit Merchant	2	5	1	8	10,576	2·5	
Corn, &c., Merchant	5	...	4	9	11,963	2·5	
Schoolmaster	5	6	4	15	19,378	2·6	
Clergyman	3	12	2	17	20,694	2·7	
Glass Manufacture	4	6	6	16	18,338	2·9	
Grocer; Tea Dealer	26	26	24	76	88,591	2·9	
Medical Student—Assistant	...	1	3	4	4,514	3·0	
Draper; Mercer	13	17	21	51	55,224	3·1	
Auctioneer; Appraiser; Valuer	1	3	2	6	6,247	3·2	
Wood Turner; Worker	...	4	3	7	7,206	3·2	
Box and Packing-case Maker	2	3	...	5	5,213	3·2	
Watch and Clock Maker	5	7	9	21	20,693	3·4	
Provision Curer—Dealer	2	4	3	9	8,770	3·4	
Goldsmith; Silversmith	4	11	5	20	19,007	3·5	
Tobacco, Cigar, &c., Manufacturer	4	3	4	11	10,109	3·6	
Earthenware Manufacture	10	7	14	31	29,085	3·6	
Telegraph Service (not Government)	1	1	1	3	2,707	3·7	Includes Co.'s Staffs at Offices, &c.
Printer	13	16	20	49	44,066	3·7	
Coach Maker	12	6	7	25	22,706	3·7	
Teacher; Professor; Lecturer	10	...	5	15	13,349	3·7	
Glove Maker	1	2	...	3	2,726	3·7	
Baker	23	10	25	58	52,724	3·7	
Currier	4	6	6	16	14,203	3·8	
Calico—Cotton Printer	5	3	2	10	8,710	3·8	
Domestic Servant (General)	32	28	20	80	68,335	3·9	
Manufacturing Chemist	8	...	5	13	10,735	4·0	
Ironmonger, &c.	5	6	8	19	15,863	4·0	
Earthenware, &c., Dealer	1	...	3	4	3,336	4·0	
Chemist; Druggist	8	9	7	24	19,190	4·2	
Surveyor; Estate Agent	4	3	3	10	7,904	4·2	
Warehouseman (not Manchester)	11	15	21	47	36,804	4·3	
Tailor	51	41	51	143	111,843	4·3	
Shoe and Boot Maker	80	90	87	257	197,465	4·3	
Cabinet Maker; Upholsterer	21	27	15	63	48,328	4·4	
Spinning and Weaving Machine Maker	6	4	3	13	9,631	4·5	
Tallow Chandler	2	2	1	5	3,710	4·5	
Hair Dresser	2	9	5	16	11,885	4·5	
Agricultural Implement, &c., Maker	1	2	2	5	3,617	4·6	
File Maker—Dealer	4	2	5	11	7,978	4·6	
Tin-plate Worker, &c.	5	12	6	23	16,439	4·7	
Barrister	1	2	2	5	3,580	4·7	

TABLE I—*(continued)*.

Occupation.	Fatal Accidents in 1870.	1871.	1872.	Total of the 3 Years.	Population, 1871.	Average Annual Rate of Fatal Accidents per 10,000.	Remarks.
Cheesemonger	2	3	1	6	4,249	4·7	
Anchor-smith; Chain-smith	1	2	3	6	4,153	4·8	
Merchant	7	7	9	23	15,903	4·8	
Commercial Traveller	11	9	6	26	17,895	4·8	
Sugar Refiner	2	1	1	4	2,781	4·8	
Basket Maker	1	4	7	12	8,407	4·8	
Wood Carver; Carver and Gilder	6	4	4	14	9,784	4·8	
Civil Service	14	11	12	37	25,717	4·8	
Maltster	4	5	6	15	10,274	4·9	
Nail Manufacture	6	8	4	18	12,339	4·9	
Tanner	7	3	3	13	8,624	5·0	
Saddler; Harness—Whip Maker	8	9	15	32	21,179	5·0	
Fellmonger	...	1	2	3	2,017	5·0	
Coffee House Keeper	...	3	2	5	3,305	5·0	
Confectioner	2	6	6	14	9,386	5·0	
Stone Merchant—Cutter—Dresser	5	3	1	9	6,046	5·0	
Brush—Broom Maker	3	3	7	13	8,539	5·1	
Civil Engineer	3	2	3	8	5,234	5·1	
Woodman	4	5	3	12	7,855	5·1	
Gas Works Service	7	9	5	21	13,561	5·2	
Butcher	39	36	41	116	72,675	5·3	
Commercial Clerk	55	71	73	199	124,787	5·3	Includes Law, Banking, and Insurance Clerks.
Solicitor	8	8	4	20	12,314	5·4	
Police	19	13	15	47	28,330	5·5	
Musician; Music Master	3	10	6	19	11,575	5·5	
Musical Instrument Maker—Dealer	3	5	4	12	7,156	5·6	
Paper Manufacture	8	8	1	17	10,131	5·6	
Hatter; Hat Manufacture	10	4	9	23	13,533	5·7	
Horsekeeper; Groom; Jockey	34	36	36	109	63,878	5·7	Includes Domestic Groom.
Paperhanger	...	4	2	6	3,442	5·8	
Municipal—Parish—Union Officer	7	8	4	19	10,955	5·8	Includes Fire Engine Service.
Wire Worker—Drawer	6	3	4	13	7,433	5·8	
Bookbinder	6	4	4	14	7,917	5·9	
Coach — Omnibus — Cab Owner; Livery Stable Keeper	4	3	3	10	5,558	6·0	
Painter (Artist)	2	5	2	9	5,005	6·0	
Wheelwright	19	22	15	56	30,310	6·2	
Navy Pensioner	5	7	2	14	7,296	6·4	
Builder	18	14	13	45	23,128	6·5	
Broker; Agent; Factor	9	15	22	46	22,776	6·7	
Rope—Cord Maker	9	5	7	21	10,192	6·9	
Ginger Beer, &c., Manufacture	1	3	1	5	2,409	6·9	
Church—Chapel Officer	5	2	...	7	3,378	6·9	
Undertaker	...	2	1	3	1,459	6·9	
Cooper	14	18	8	40	19,240	6·9	
Blacksmith	73	77	85	235	112,027	7·0	
Actor	4	...	...	4	1,899	7·0	
Farrier; Veterinary Surgeon	6	4	4	14	6,650	7·0	
Greengrocer	14	15	11	40	18,980	7·0	
Shopkeeper (branch undefined)	19	12	16	47	22,208	7·1	
Inn—Hotel Keeper; Beerseller	58	55	47	160	74,367	7·2	
Messenger; Porter; Errand Boy	78	87	85	250	115,773	7·2	Includes Govmt. Messengers, Letter Carriers, and Dockyard Workmen.
Miller	21	24	20	65	29,715	7·3	
Cutler	13	18	7	38	17,059	7·4	

TABLE I—*(continued)*.

Occupation.	Fatal Accidents in 1870.	1871.	1872.	Total of the 3 Years.	Population, 1871.	Average Annual Rate of Fatal Accidents per 10,000.	Remarks.
Trimming Maker—Dealer	2	...	1	3	1,356	7·4	
Carpenter; Joiner	150	157	177	484	205,615	7·8	
Coal Merchant—Dealer	5	13	18	36	15,300	7·8	
Farmer; Grazier (and Relatives)	243	213	261	717	302,035	7·9	
Gun and Ammunition Manufactures	11	14	6	31	13,008	7·9	Includes Gunpowder Manufacture.
Farm Bailiff	8	18	14	40	16,476	8·1	
Chimney Sweeper	5	5	5	15	6,206	8·1	
Gas Fitter	5	12	4	21	8,615	8·1	
Cork Cutter	2	2	1	5	2,021	8·2	
Cowkeeper; Milkseller	9	12	20	41	16,435	8·3	
Brewer	18	21	25	64	25,562	8·4	
Cook (not Domestic Servant)	4	...	2	6	2,375	8·4	
Millwright	6	7	6	19	7,538	8·4	
Fishmonger	11	15	6	32	12,642	8·4	
Toll Collector; Turnpike Keeper	2	3	1	6	2,365	8·5	
Gamekeeper	11	10	11	32	12,426	8·6	
Lodging House Keeper	4	3	3	10	3,840	8·7	
Poulterer	...	6	1	7	2,696	8·7	
Mason; Pavior	88	77	96	261	98,119	8·9	Includes Marble Mason.
Plasterer	20	26	20	66	24,575	9·0	
Ship Builder; Shipwright	38	28	45	111	40,605	9·1	
Soap Boiler	1	2	2	5	1,819	9·2	
Scavenger	3	2	1	6	2,174	9·2	
Physician; Surgeon	13	14	14	41	14,684	9·3	
Agricultural Machine Proprietor—Attendant	3	1	2	6	2,152	9·3	
Cellarman	4	1	4	9	3,205	9·4	
Fancy Goods Manufacturer—Dealer	7	...	1	8	2,849	9·4	
Cotton, &c., Warehouseman—Dealer	4	5	...	9	3,214	9·4	
Slater; Tiler	9	2	7	18	6,078	9·9	
Button Maker	...	2	5	7	2,370	9·9	
Meter; Weigher	...	2	2	4	1,324	10·1	
Hay—Straw Dealer	5	1	2	8	2,566	10·4	
Coast Guard; Royal Navy Reserve	3	5	4	12	3,775	10·6	
Rat Catcher	4	1	...	5	1,552	10·7	
Coke Burner—Dealer	5	4	4	13	3,998	10·8	
Sawyer	25	28	38	91	27,944	10·9	
Coachman; Cabman; Flyman	52	74	57	183	56,171	10·9	Includes Domestic Coachman.
Waterworks Service; Well Sinker	3	1	7	11	3,326	11·0	
Plumber; Painter; Glazier	115	113	128	356	103,380	11·5	
Steam Navigation Service	11	6	6	23	6,608	11·6	Includes Co.'s servants on land.
Huckster; Costermonger	3	3	6	12	3,350	11·9	
Cattle, &c., Salesman	3	11	10	24	5,835	13·7	
Inn—Hotel Servant	36	40	42	118	28,534	13·8	Includes Club and Dining-room Waiters.
Horse Proprietor—Dealer	1	2	3	6	1,364	14·7	
Army Pensioner	12	8	13	33	7,442	14·8	
Hawker; Pedlar	43	47	35	125	27,277	15·3	
Thatcher	12	1	6	19	4,143	15·3	
Salt Manufacture	2	7	3	12	2,553	15·7	
Dyer; Scourer	17	23	20	60	11,394	17·6	
Rag Gatherer—Dealer	4	7	5	16	2,312	23·0	
Pilot	8	12	5	25	3,039	27·4	
Barge—Lighter—Waterman	114	119	121	354	29,445	40·1	
Horse Breaker	6	8	4	18	1,253	47·9	

I think that the majority of the results in this table are correct, so far as they go. It is not contended that the "average rate" which has been placed against each occupation shows the exact rate of fatal accidents applicable to persons of that occupation. There are several reasons against this:—

1. There is a possibility of errors in the observations.
2. The years under observation may not have been average years.
3. In many occupations the rate of fatal accidents is constantly changing, owing to alterations in methods of manufacturing, &c.
4. In a great number of occupations, the numbers exposed to risk are so small that they cannot be said to possess any ascertainable rate of fatal accidents at all.

Nevertheless, I thought it well to present the table as it is. A rough guide is sometimes better than no guide at all. If there are in it several steps in the dark, they are at least steps in the direction of light.

In Table II are contained similar particulars to those in Table I for some occupations as to which it would be well to offer a few words of explanation.

TABLE II.—*Showing similar Particulars to Table I, for certain Occupations requiring explanation.*

Occupation.	Fatal Accidents in				Population, 1871.	Average Annual Rate of Fatal Accidents per 10,000.
	1870.	1871.	1872.	Total of the 3 Years.		
Brass Manufacture	4	6	3	13	20,974	2·1
Army Officer	3	3	4	10	9,842	3·4
Needle Manufacture	1	...	2	3	2,614	3·8
Textile Manufactures	154	182	187	523	379,122	4·6
Iron Manufacture	103	96	97	296	178,047	5·5
Soldier	40	54	56	150	70,675	7·1
Navy Officer	3	2	5	10	4,362	7·6
Engine—Machine Maker	96	90	87	273	106,434	8·6
Quarrier	88	80	78	246	40,636	20·2
Fisherman	44	50	43	137	20,672	22·1
Pin Manufacture	1	1	...	2	284	23·5
Railway Employees:—						
Engine Driver; Stoker	64	79	65	208	13,715	50·6
Clerk; Station Master	13	20	14	47	22,082	7·1
Attendant; Servant	194	234	303	731	48,827	49·9
Platelayer; Labourer; Navvy	225	334	338	897	45,064	66·4
Total Railway Employees	496	667	720	1883	129,688	...

Brass Manufacture.—Here the population includes, besides brass moulders and founders, all engaged in the manufacture of brass fittings; and other divisions in which the risk of accident would vary.

Army Officer.—Several of these may have been otherwise described. It is also doubtful whether the number enumerated nearly represents the average number resident in the country.

Needle and Pin Manufactures.—The smallness of the numbers observed here, of course renders the results unreliable; but they seemed so curious that I have included them.

Textile Manufactures.—The various textile manufactures were found to be so difficult to distinguish, that it was ultimately found necessary to combine the figures relating to the whole of them. There did not appear to be any *primâ facie* reason for supposing that the rate of accidents would vary very much in these manufactures.

Iron Manufacture.—Although a great many of the deaths under this head were in connection with smelting and puddling, the figures furnish no clue to the rate of accidents among those employed in the actual manufacture of iron; as the population under this description also includes iron founders and others working in manufactured iron, and even iron and iron ore merchants and agents.

Soldier.—As the soldiers kept in the country at one time would probably not vary very considerably, the resultant rate is fairly reliable.

Navy Officer.—It is very improbable that the Census enumeration would give the average number of naval officers resident in the country. Some, also, may be returned under other descriptions.

Engine-machine Maker.—It is possible that some of the occupations included under this head may have been confused with some branches included under "Iron Manufacture".

Quarrier.—In very few cases was the description of quarrier stated in the Registers. It was therefore found necessary to combine with the stone quarriers the less numerous cases of slate and limestone quarriers.

Fisherman.—In this occupation the mortality is probably much understated. Only the cases where the bodies of drowned fishermen are recovered would be registered; and these would be a small proportion of the whole.

Railways.—The annual rates brought out by the figures relating to railway employees look suspiciously large. Yet it is extremely unlikely that the number of deaths has been overstated,

although at first sight this would be suggested by the Board of Trade Reports on Railway Accidents, as the following comparison will show:—

Year.	No. of Railway Employees killed in England and Wales.	
	As reported to the Board of Trade.*	As extracted from the Registers of Death.
1870	63	496
1871	164	667
1872	512	720
1873	620	...
1874	641	...
1875	629	...

In noticing these figures, however, it must be remembered that the Act requiring railway companies to report the deaths of their servants from accidents did not come into operation until the 1st November 1871, and that the Board of Trade Reports prior to 1872 are therefore admittedly defective as regards the numbers of employees killed. It is very improbable that absolute correctness was suddenly arrived at in this respect in 1873, or, indeed, that the whole of the deaths traceable to accidents ever will be reported to the Board of Trade. It might well be, therefore, if the Board of Trade figures had been stated at the true amount suggested by the numbers for the years 1873, 1874 and 1875, that they would not point to any large overstatement in the present results.

With regard to the other factor—the exposed to risk—it will be seen from Table II that the total number of railway employees in England and Wales enumerated at the Census in 1871 was 129,688. According to Lord Aberdeen's return, presented to Parliament in the autumn of 1874, the number of the officers and servants of the railway companies of England and Wales at the end of 1873 was 228,958. Of these, 33,651 are described as "Inspectors, Foremen and Artificers". If, as I assume, these are men employed in the manufacture and repair of engines and carriages, they would not be included in the Census populations quoted in Table II. The same remark doubtless applies to many of the 16,531 described in the return as "Miscellaneous". Making a deduction for these items, we have, say, 190,000 servants at the end of 1873, as returned by the companies, as against the 129,688

* These do not agree with the numbers of "Servants, &c." killed, as stated in the "Miscellaneous Statistics" of the Board of Trade. The latter include Trespassers, Suicides, &c.

returned in the Census in April 1871. Even if we allow for the increase in the number of railway employees which would take place between the dates of the two returns, we are left no alternative but to conclude that the railway populations as returned in the Census Reports are understated. The possible deficiency in this respect, however, would hardly be sufficient to account for the difference between the rates arrived at in Table II and those obtained by Mr. Neison in his Report on the Rates of Accidents in Mines and Railways, issued last year, in which he found the fatal accidents among railway employees to vary from 25 per 10,000 on a passenger line, to 35 per 10,000 on a goods line.

We have now to deal with the large number of deaths attributed to "General Labourer", as previously referred to; and the other occupations which are probably affected by it. In Table No. III are given particulars, in the same form as before, for those occupations to which it is likely that some of the deaths classified under "General Labourer" should have been placed.

TABLE III.—*Showing similar Particulars to Table I, for certain further Occupations requiring explanation.*

Occupation.	Fatal Accidents in 1870.	1871.	1872.	Total of the 3 Years.	Population, 1871.	Average Annual Rate of Fatal Accidents per 10,000.	Remarks.
Road Labourers	...	...	...	...	8,288	...	
Farm Servant (indoor)	1	...	1	2	134,157	...	
Coal Heaver—Labourer	10	1	...	11	24,717	1·5	
Shepherd (outdoor)	9	8	8	25	23,323	3·6	
Gardener	61	48	41	150	114,509	4·4	Includes Domestic Gardener.
Timber, &c., Merchant—Dealer	5	7	6	18	12,004	5·0	
Bricklayer; Bricklayer's Labourer	54	70	59	183	99,939	6·1	
Brickmaker—Dealer	22	30	17	69	36,163	6·4	
Agricultural Labourer (outdoor)	591	611	598	1,800	761,362	7·9	
Harbour—Dock Service; Dock Labourer	46	18	17	81	28,737	9·4	
Canal and Inland Navigation Service	...	4	7	11	3,128	11·7	
Carman; Carrier; Carter; Drayman	195	165	214	574	73,572	26·0	
General Labourer (branch undefined)	1,293	1,540	1,593	4,426	509,010	29·0	
TOTAL	2,287	2,502	2,561	7,350	1,828,909	13·4	

Almost all these occupations are of such a character that a person following it might have been described as a "labourer".

There is, moreover, a decided appearance about the resultant rates of their being understated. This, it is true, does not apply to "carman, &c."; but as, in the course of the investigation, it was found to be impossible to distinguish in rural districts between "carters" and agricultural and other labourers, they have all been included in this table. As a test whether the occupations given are sufficient to account for the large number of deaths registered as those of general labourers, the whole have been totalled; and the resulting average rate of 13·4 per 10,000 would seem to show that they are.

The rate of accidents among miners having been made the subject of special enquiry, the deaths were separately abstracted for each county. The results are tabulated in Table No. IV.

TABLE IV.—*Showing similar Particulars to Table I, for the Coal Miners in each County in England and Wales, and for the Miners (other than Coal) in England and Wales.*

County.	Fatal Accidents in			Total in the 3 Years 1870-2.		Population, 1871.	Annual Average Rate of Fatal Accidents per 10,000.
	1870.	1871.	1872.	Original.	Adjusted.		
CHESHIRE—							
Coal Miners . . .	18	11	16	45	45	3,704	40·5
Miners (other than Coal)	...	...	2	2	2	..	...
CORNWALL—							
Miners (other than Coal)	41	51	47	139	139	...	...
CUMBERLAND—							
Coal Miners . . .	19	19	13	51	54	4,792	37·6
Miners (other than Coal)	24	19	15	58	55	...	...
DERBY—							
Coal Miners . . .	26	69	41	136	136	13,347	34·0
Miners (other than Coal)	...	1	4	5	5	...	...
DEVONSHIRE—							
Miners (other than Coal)	4	6	10	20	20	...	...
DURHAM—							
Coal Miners . . .	151	127	159	437	447	45,783	32·5
Miners (other than Coal)	14	1	3	18	8	...	...
GLOUCESTER—							
Coal Miners . . .	8	8	11	27	31	3,636	28·4
Miners (other than Coal)	2	1	1	4	...	...	...
LANCASHIRE—							
Coal Miners . . .	238	182	215	635	668	49,484	45·0
Miners (other than Coal)	15	10	12	37	4	...	...
LEICESTER—							
Coal Miners . . .	13	8	6	27	27	2,788	32·3

TABLE IV—*(continued)*.

County.	Fatal Accidents in			Total in the 3 Years 1870-2.		Population, 1871.	Annual Average Rate of Fatal Accidents per 10,000.
	1870.	1871.	1872.	Original.	Adjusted.		
LINCOLN—							
Miners (other than Coal)	...	1	1	2	2	...	...
MONMOUTH—							
Coal Miners . . .	40	69	42	151	154	14,103	36·4
Miners (other than Coal)	5	...	...	5	2	...	...
NORTH RIDING—							
Coal Miners . . .	6	4	1	11	11	165	222·2
Miners (other than Coal)	24	21	27	72	72	...	...
NORTHUMBERLAND—							
Coal Miners . . .	47	48	45	140	145	15,187	31·8
Miners (other than Coal)	2	1	6	9	4	...	...
NORTH WALES—							
Coal Miners . . .	40	33	32	105	111	8,863	41·7
Miners (other than Coal)	8	1	12	21	15	...	...
NOTTINGHAM—							
Coal Miners . . .	18	17	22	57	57	10,900	17·4
SALOP—							
Coal Miners . . .	20	9	13	42	54	4,884	36·9
Miners (other than Coal)	1	3	12	16	4	...	...
SOMERSET—							
Coal Miners . . .	4	14	9	27	29	5,503	17·6
Miners (other than Coal)	2	1	1	4	2	...	...
SOUTH WALES—							
Coal Miners . . .	185	177	187	549	566	39,383	47·9
Miners (other than Coal)	9	9	20	38	21	...	...
STAFFS—							
Coal Miners . . .	162	147	183	492	506	37,801	44·6
Miners (other than Coal)	9	7	6	22	8	...	...
WARWICK—							
Coal Miners . . .	6	5	7	18	19	2,234	28·3
Miners (other than Coal)	...	1	...	1	...	...	...
WEST RIDING—							
Coal Miners . . .	100	113	144	357	363	39,055	31·0
Miners (other than Coal)	3	...	7	10	4	...	...
WORCESTER—							
Coal Miners . . .	12	18	8	38	41	2,698	50·7
Miners (other than Coal)	...	1	2	3	...	...	...
ENGLAND AND WALES—							
Coal Miners . . .	1,113	1,078	1,154	3,345	3,464	304,310	37·9
Miners (other than Coal)	163	135	188	486	367	66,285	18·5

The column in this table in which some of the facts have been arbitrarily adjusted requires a word of explanation. In the original investigation it was endeavoured to distinguish, mainly by a consideration of the manner of death, between accidents caused in connection with the occupation, and those not so caused. In the case of the miners' deaths *not* occurring in connection with mines, a difficulty was found in satisfactorily distinguishing between coal and other miners; but as there appeared no reason why in the cases of these deaths there should be any great difference in the rates for the two classes, the relative populations in each county were compared, and the arbitrary alterations appearing in the Table were made.

The large number of miners returned at the Census as "miner (branch undefined)", makes it difficult to arrive satisfactorily at the number of coal-miners. But an examination of the numbers of the mining population in the several counties served to determine pretty closely whether those "undefined" were coal-miners or otherwise. The total of the county coal-mining populations thus arrived at (304,310), is taken as the coal-mining population of the whole country. The total miners of all descriptions enumerated at the Census being 370,595, the difference (66,285), after deducting the 304,310 coal-miners, is assumed to be the population among whom the 367 deaths attributed to "miners (other than coal)" occurred. The deaths under the latter head being few in number, the rates have not been inserted for the separate counties. The coal-mining population in many counties being small, too much weight is not to be attached to every result. The rate figuring against the North Riding of Yorkshire, for instance, is evidently abnormal.

It will be noticed that the total fatal accidents to coal-miners for the three years under observation were 3,464. Of these, 2,968 were traced as being due to the nature of the occupation. This would give an annual average rate of 32·5 per 10,000 as the measure of the occupation risk. The total of the violent deaths returned in the Registrar-General's Annual Reports for 1870, 1871, and 1872, as connected with coal mines, is 2,970; and this would seem to confirm the results of the present investigation, as far as regards the deaths.

I have endeavoured to compare these results with those arrived at by Mr. Neison in his report previously referred to, based upon the returns of the Inspectors of Mines. According to the latter, the number of lives lost in the coal and ironstone mines of Great

Britain in the years 1870, 1871, and 1872, was 3,286: deducting 304 for the Scotch mines included in the return, there remain 2,982 as the number reported for England and Wales. The number of persons estimated by the Inspectors to be employed in or about the coal and ironstone mines of Great Britain in 1871, was 370,881. Deducting 46,861 for Scotland, there remain 324,020 for England and Wales. 2,982 fatal accidents in three years among this number would give an average annual rate of 30·7 per 10,000, as compared with the 32·5 per 10,000 given by the present experience.

As the "ironstone-miners" returned by the Inspectors are not included in the "coal-miners" returned in the Registrar-General's and the Census Reports, we have an explanation of the larger number of deaths and exposed to risk which the Inspectors' reports exhibit; and a lighter mortality among the ironstone-miners might perhaps account for the slight difference in the two resulting rates.

I have already referred to an endeavour which was made in the present investigation to distinguish between deaths which arose in connection with the occupation of the deceased and deaths not so occasioned. In Table No. V (p. 203) are given the results of this endeavour as regards the most important occupations.

As a comparison of the manner of death with the nature of the occupation was in a great number of the cases the only mode of deciding in which category a death should be placed, it will be imagined that considerable difficulty was frequently experienced in the course of classifying the data upon which this table is based. Still, the ample particulars which, in the case of most of the deaths, appear in the Registers, combined with the care and judgment which was exercised in the classification, have, it is hoped, secured to this table an amount of reliability which may enable it to prove of service. Perhaps most of the percentages exhibited in the table only confirm our preconceived ideas of the relative danger of the various occupations; but I think it is worthy of special note that miners are, according to the facts here shown, subject to so considerable a fatality independently of their occupation.

It was mentioned at the commencement of the paper that the whole of the data were classified according to ages. This was originally done in groups of ten years each. Some of the results obtained through having the deaths thus classified, have been embodied in Table VI (pp. 204, 205).

TABLE V.—*Showing, for certain Occupations, the number of Fatal Accidents considered to have occurred in England and Wales during the years* 1870, 1871 *and* 1872, IN CONNECTION WITH THE OCCUPATION, *and the percentages thereof on the* TOTAL *Fatal Accidents.*

Occupation.	Fatal Accidents, 1870-2, in connection with Occupation.	Total Fatal Accidents, 1870-72.	Percentage of (1) on (2).
	(1)	(2)	
Agricultural Labourer	1,231	1,800	68·4
Barge—Lighter—Waterman	291	354	82·2
Blacksmith	84	235	35·7
Brewer	31	64	48·4
Bricklayer	123	183	67·2
Brickmaker—Dealer	29	69	42·0
Builder	22	45	48·9
Butcher	25	116	21·6
Carman; Carrier; Carter; Drayman	434	574	75·6
Carpenter; Joiner	256	484	52·9
Coachman; Cabman; Flyman	103	183	56·3
Cooper	9	40	22·5
Cowkeeper; Milkseller	15	41	36·6
Cutler	12	38	31·6
Dock Service	66	81	81·5
Dyer; Scourer	30	60	50·0
Earthenware Manufacture	13	31	41·9
Engine and Machine Maker	165	273	60·4
Fisherman	110	137	80·3
Gas Works Service	13	21	61·9
General Labourer	2,395	4,426	54·1
Glass Manufacture	3	16	18·8
Groom; Horsekeeper; Jockey	53	109	48·6
Horse Breaker	14	18	77·8
Mason; Pavior	146	261	55·9
Miller	43	65	66·2
Millwright	13	19	68·4
Miner (Coal)	2,968	3,464	85·7
Miner (other than Coal)	315	367	85·8
Paper Manufacture	9	17	52·9
Pilot	17	25	68·0
Plasterer	42	66	63·6
Plumber; Painter; Glazier	238	356	66·9
Printer	7	49	14·3
Quarrier	208	246	84·6
Railway Attendant—Servant	697	731	95·3
Railway Engine Driver—Stoker	206	208	99·0
Railway Labourer—Platelayer—Navvy	828	897	92·3
Sawyer	47	91	51·6
Shipbuilder; Shipwright	91	111	82·0
Slater; Tiler	12	18	66·7
Thatcher	16	19	84·2
Wheelwright	18	56	32·1

Table VI.—*Showing, for certain Occupations and Groups of Ages, the Population in* 1871, *and the Fatal Accidents in* 1870, 1871 *and* 1872, *in England and Wales; and the Rates of Fatal Accidents:—distinguishing those considered to have occurred in connection with the Occupation.*

Occupation.	Age.	Population, 1871.	Fatal Accidents, 1870-2.		Average Annual Rate per 10,000.	
			In connection with Occupation.	Total.	In connection with Occupation.	Total.
Railway Engine Driver; Stoker . . .	10–20	1,355	29	29	71·3	71·3
	20–45	10,931	153	155	46·7	47·3
	45–65	1,380	23	23	55·6	55·6
	65–upwds.	49	1	1	68·0	68·0
Railway Attendant; Servant	10–20	5,270	93	96	58·8	60·7
	20–45	36,178	498	516	45·9	47·5
	45–65	6,734	88	98	43·6	48·5
	65–upwds.	645	18	21	93·0	108·5
Carman; Carrier; Carter; Drayman . . .	10–20	9,640	34	46	11·8	15·9
	20–45	44,905	222	289	16·5	21·5
	45–65	16,133	150	201	31·0	41·5
	65–upwds.	2,894	28	38	32·3	43·8
Agricultural Labourer .	10–20	161,035	267	384	5·5	7·9
	20–45	324,528	419	575	4·3	5·9
	45–65	195,840	378	526	6·4	9·0
	65–upwds.	79,959	167	315	7·0	13·1
Engine and Machine Maker . . .	10–20	19,882	25	40	4·2	6·7
	20–45	69,131	97	164	4·7	7·9
	45–65	15,738	38	57	8·0	12·1
	65–upwds.	1,683	5	12	9·9	23·8
Carpenter . . .	10–20	31,076	16	54	1·7	5·8
	20–45	117,298	123	207	3·5	5·9
	45–65	44,539	84	150	6·3	11·2
	65–upw .	12,702	33	73	8·7	19·2
Bricklayer . . .	10–20	13,751	9	19	2·2	4·6
	20–45	59,929	53	82	2·9	4·6
	45–65	21,047	48	62	7·6	9·8
	65–upwds.	5,212	13	20	8·3	12·8
Mason; Pavior . .	10–20	15,109	13	32	2·9	7·1
	20–45	53,927	64	101	4·0	6·2
	45–65	21,027	43	83	6·8	13·2
	65–upwds.	5,125	16	31	10·4	20·2

TABLE VI—*(continued)*.

Occupation.	Age.	Population, 1871.	Fatal Accidents, 1870-2. In connection with Occupation.	Fatal Accidents, 1870-2. Total.	Average Annual Rate per 10,000. In connection with Occupation.	Average Annual Rate per 10,000. Total.
Plumber; Painter; Glazier	10–20	17,508	11	24	2·1	4·6
	20–45	63,338	114	181	6·1	9·6
	45–65	19,550	94	121	16·0	20·6
	65–upwds.	2,986	19	30	21·2	33·5
Quarrier . . .	10–20	6,454	13	18	6·7	9·3
	20–45	23,058	103	124	14·9	17·9
	45–65	9,391	78	86	27·7	30·5
	65–upwds.	1,733	14	18	26·9	34·6
Iron Manufacture . .	10–20	38,677	40	72	3·4	6·2
	20–45	109,395	88	167	2·7	5·1
	45–65	26,629	16	45	2·0	5·6
	65–upwds.	3,346	2	12	2·0	12·0
Miners (of all descriptions)	10–20	98,527	880	1,025	29·8	34·7
	20–45	206,262	1,728	1,980	27·9	32·0
	45–65	57,043	618	731	36·1	42·8
	65–upwds.	8,763	57	95	21·7	36·1
Total Males in England and Wales . . .	10–20	2,305,483	...	4,978	...	7·2
	20–45	3,772,431	...	11,212	...	9·9
	45–65	1,603,317	...	6,526	...	13·6
	65–upwds.	490,420	...	2,739	...	18·6

The following groupings of ages were adopted for this table:—

(i) . . . . . Ages 10–20
(ii) . . . . . „ 20–45
(iii) . . . . . „ 45–65
(iv) . . . . . „ 65 and upwards.

The first would represent the periods of youth and apprenticeship, naturally characterized by want of thought and inattention to precaution. The rates shown by this grouping are generally, as might be expected, higher than those brought out by the next. The second grouping would represent the period of life in which the worker should be in his prime. The third covers the period when muscular and vital force is declining and activity is abating. The fourth includes those whose fast-fading strength would render them peculiarly liable to accident.

There is, generally, a marked steady increase in the rates for the last three groups in almost every occupation—the only exception in the table being the miners, who show, curiously enough, an improvement in the last group.* Of several of the occupations included in this table (notably, "Agricultural Labourer", "Bricklayer", and "Iron Manufacture"), it has been previously explained that the figures do not accurately express the rates obtaining in them; but they probably express with correctness the relative rates in the various groups of ages.

At the commencement of the paper the belief was expressed that the facts now presented are of an original character. It may be readily imagined that the labour attending the classification of over 25,000 deaths according to age and occupation, was very considerable. When, therefore, the special purpose for which the investigation was undertaken had been served, and the opportunity for publishing the results arose, it seemed important that such an experience should be presented as much as possible in its original shape. There appeared to me to be two further reasons why this should be done. First, because an attempt to combine the results for the purpose of arriving at more practical conclusions would probably have ended in destroying the significance of many original facts worthy of notice. Second, because it would be better for any person desiring either to make use of these results, or to go over similar ground in a future investigation, to be able to refer to the facts in their crude form.

It is possible that further investigation and manipulation of the experience will lead to the establishment of more practical truths than the present paper can be said to point to; but for the present I must be content to conclude with a repetition of the warning before given against attaching too much weight to every individual result.

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* It is suggested to me by Mr. Newbatt, that this is very likely due to the fact that many miners of more advanced years, though still described as "miners", are not actually engaged in the pit, but are employed in less risky occupations about the surface.

APPENDIX.

TABLE *showing the Rates of Fatal Accidents for all Occupations included in Tables* I, II, *and* III, *arranged in alphabetical order.*

N.B.—Those printed in heavier type are taken from Tables II and III, and should be read in connection with the observations made on those Tables.

Occupation.	Annual Average Rate of Fatal Accidents per 10,000.	Occupation.	Annual Average Rate of Fatal Accidents per 10,000.
Accountant	1·7	**Coal Heaver—Labourer**	**1·5**
Actor	7·0	Coal Merchant—Dealer	7·8
Agricultural Implement, &c., Maker	4·6	Coast Guard; Royal Navy Reserve	10·6
Agricultural Labourer (out-door)	**7·9**	Coffee House Keeper	5·0
Agricultural Machine Proprietor—Attendant	9·3	Coke Burner—Dealer	10·8
Anchorsmith; Chainsmith	4·8	Commercial Clerk	5·3
Army Officer	**3·4**	Commercial Traveller	4·8
Army Pensioner	14·8	Confectioner	5·0
Auctioneer; Appraiser; Valuer	3·2	Cook (not domestic servant)	8·4
Author, &c.; Student	1·3	Cooper	6·9
Baker	3·7	Cork Cutter	8·2
Barge—Lighter—Waterman	40·1	Corn, &c., Merchant	2·5
Barrister	4·7	Cotton, &c., Warehouseman—Dealer	9·4
Basket Maker	4·8	Cowkeeper; Milkseller	8·3
Blacksmith	7·0	Currier	3·8
Bookbinder	5·9	Cutler	7·4
Bookseller; Stationer	2·0	Domestic Servant (general)	3·9
Box—Packing-case Maker	3·2	Draper; Mercer	3·1
Brass Manufacture	**2·1**	Dyer; Scourer	17·6
Brewer	8·4	Earthenware, &c., Dealer	4·0
Bricklayer; Bricklayer's Labourer	**6·1**	Earthenware Manufacture	3·6
Brick Maker—Dealer	**6·4**	**Engine—Machine Maker**	**8·6**
Broker; Agent; Factor	6·7	Fancy Goods Manufacturer—Dealer	9·4
Brush and Broom Maker	5·1	Farm Bailiff	8·1
Builder	6·5	**Farm Servant (indoor)**	...
Butcher	5·3	**Farmer; Grazier; (and relatives)**	**7·9**
Button Maker	9·9	**Farrier; Veterinary Surgeon**	**7·0**
Cabinet Maker; Upholsterer	4·4	Fellmonger	5·0
Calico and Cotton Printer	3·8	**File Maker; Dealer**	4·6
Canal and Inland Navigation Service	**11·7**	**Fisherman**	**22·1**
Carman; Carrier; Carter; Drayman	**26·0**	Fishmonger	8·4
Carpenter; Joiner	7·8	Game Keeper	8·6
Cattle, &c., Salesman	13·7	**Gardener**	**4·4**
Cellarman	9·4	Gas Fitter	8·1
Cheesemonger	4·7	Gas Works Service	5·2
Chemist; Druggist	4·2	**General Labourer (branch undefined)**	**29·0**
Chimney Sweeper	8·1	Ginger Beer, &c. Manufacture	6·9
Church—Chapel Officer	6·9	Glass Manufacture	2·9
Civil Engineer	5·1	Glove Maker	3·7
Civil Service	4·8	Goldsmith; Silversmith	3·5
Clergyman	2·7	Greengrocer	7·0
Coach Maker	3·7	Grocer; Tea Dealer	2·9
Coachman; Cabman; Flyman	10·9	Gun—Ammunition Manufacture	7·9
Coach—Omnibus—Cabowner; Livery-stable Keeper	6·0	Hair Dresser	4·5
		Harbour—Dock Service; Dock Labourer	**9·4**
		Hatter; Hat Manufacture	5·7

TABLE *showing the Rates of Fatal Accidents, &c.*—(continued).

Occupation.	Annual Average Rate of Fatal Accidents per 10,000.	Occupation.	Annual Average Rate of Fatal Accidents per 10,000.
Hawker; Pedlar	15·3	**Railway Engine Driver—Stoker**	**50·6**
Hay—Straw Dealer	10·4	**Railway Platelayer — Labourer—Navvy**	**66·4**
Horse Breaker	47·9	Rat Catcher	10·7
Horse Keeper; Groom; Jockey	5·7	**Road Labourers**	...
Horse Proprietor—Dealer	14·7	Rope—Cord Maker	6·9
Hosier; Haberdasher	2·3	Saddler; Harness—Whip Maker	5·0
Huckster; Costermonger	11·9	Salt Manufacture	15·7
Inn—Hotel Keeper; Beerseller	7·2	Sawyer	10·9
Inn—Hotel Servant	13·8	Scavenger	9·2
Iron Manufacture	**5·5**	Schoolmaster	2·6
Ironmonger, &c.	4·0	**Shepherd (out-door)**	**3·6**
Locksmith	1·9	Shipbuilder; Shipwright	9·1
Lodging House Keeper	8·7	Shoe—Boot Maker	4·3
Maltster	4·9	Shop Keeper (branch undefined)	7·1
Manufacturing Chemist	4·0	Slater; Tiler	9·9
Mason; Pavior	8·9	Soap Boiler	9·2
Medical Student—Assistant	3·0	**Soldier**	**7·1**
Merchant	4·8	Solicitor	5·4
Messenger; Porter; Errand Boy	7·2	Spinning—Weaving Machine Maker	4·5
Meter; Weigher	10·1	Steam Navigation Service	11·6
Miller	7·3	Stone Merchant—Cutter—Dresser	5·0
Millwright	8·4	Sugar Refiner	4·8
Municipal—Parish—Union Officer	5·8	Surveyor; Estate Agent	4·2
Musical Instrument Maker—Dealer	5·6	Tailor	4·3
Musician; Music Master	5·5	Tallow Chandler	4·5
Nail Manufacture	4·9	Tanner	5·0
Navy Officer	**7·6**	Teacher; Professor; Lecturer	3·7
Navy Pensioner	6·4	Telegraph Service (not Government)	3·7
Needle Manufacture	**3·8**	**Textile Manufactures**	**4·6**
Nurseryman, &c.	2·0	Thatcher	15·3
Painter (Artist)	6·0	**Timber, &c. Merchant—Dealer**	**5·0**
Paper Manufacture	5·6	Tinplate Worker, &c.	4·7
Paperhanger	5·8	Tobacco—Cigar, &c. Manufactures	3·6
Pawnbroker	2·1	Toll Collector; Turnpike Keeper	8·5
Physician; Surgeon	9·3	Tool Maker—Dealer	2·2
Pilot	27·4	Trimming Maker—Dealer	7·4
Pin Manufacture	**23·5**	Undertaker	6·9
Plasterer	9·0	Warehouseman (not Manchester)	4·3
Plumber; Painter; Glazier	11·5	Watch—Clock Maker	3·4
Police	5·5	Waterworks Service; Well Sinker	11·0
Poulterer	8·7	Wheelwright	6·2
Printer	3·7	Whitesmith	2·3
Protestant Minister	1·8	Wine—Spirit Merchant	2·5
Provision Curer—Dealer	3·4	Wire Worker—Drawer	5·8
Quarrier	**20·2**	Wood Carver; Carver and Gilder	4·8
Rag Gatherer—Dealer	23·0	Woodman	5·1
Railway Attendant—Servant	**49·9**	Wood Turner—Worker	3·2
Railway Clerk—Station Master	**7·1**		

Discussion.

The President (Mr. A. H. Bailey) said, that no doubt from his connections, Mr. Whittall has had access to the records of the Registrar-General's Office not possessed by us, and he has availed himself of that opportunity to deduce some very interesting results. Of course for the purpose to which we should first be disposed to apply these statistics—the computation of, or the formation of ideas as to premiums to be charged for accident insurances—these results must be used with much caution. The proportion of fatal cases of accident to the number of partial cases is very small—just as in fire insurance, for instance, the number of claims for total loss is a very small proportion of the claims generally. At the same time the aggregate of small sums paid for partial losses amounts at the end of the year to a very substantial sum. He spoke with great diffidence as to how far Mr. Whittall's results would help us in determining rates of premiums for accidents in each of the occupations. He did not know, for example, whether it can be said that the ratio of fatal accidents in particular occupations forms any clue to the number of partial accidents. Mr. Whittall has shown us that the ratio of fatal accidents in coal mines is frightful, so frightful that he (the President) thought that any company, or any one advising a company, would not like to undertake the risk at any possible premium. The ratio is 40 in 10,000 in Cheshire, 50 in 10,000 in Worcestershire, and curiously enough 31 in 10,000 in the West Riding. When an explosion does occur in a coal mine the result is so disastrous that if the company had to pay a sum equal to three years' wages of the men who are killed, it would be swamped. And yet, on the other hand, it is possible that the slight accidents may be very small.

Mr. Walford remarked that thirty years of close attention to the problems connected with accident insurance had given him no doubt an exceptional experience, but the first thing he would say was how glad he was to see one of the younger members of the Institute taking up a practical question of this sort and handling it efficiently, and further, that any young man who is in earnest in seeking an actuarial question, or one in regard to general statistics, need not look very far to find questions of the most deep and practical interest. Mr. Whittall had offered a criticism quite fairly upon some statistics on this question, or on a branch of this question, which he (Mr. Walford) submitted to another society some months ago. He said, with regard to his (Mr. Walford's) having excluded deaths caused by lightning and by sunstroke from his first results, that he thinks that, on the same principle, he might have excluded deaths from poison. He would take the opportunity of saying why he did not do that. The statistics which were prepared had this peculiarity about them, that, while they were based upon general results, the general statistics as drawn from the Registrar's report—they were put into a shape with regard very much to experience in connection with the question of accident assurance companies, and also with some regard to the possible experience which would arise under the Employers' Liability Act. Now, with regard to poisons, it is clear that there can be no reason for excluding deaths from poison, because the greater proportion of these

may be and are supposed to be of accidental origin, for of course in the Registrar's return all the known deaths from suicide, whether resulting from poison or from any other cause, had been included under the heading "Suicide". But there is another reason why deaths from poisoning should not be excluded. There are many occupations in this country wherein the liability to being poisoned in their pursuit is very great; for instance, the occupation of making bronze paper for printing and fine art work, and also a much more hazardous occupation—that of preparing chemicals—a very large industry in some parts of the West Riding of Yorkshire. With regard to both of these occupations—and one could name some others, but these will be sufficient—there are many instances wherein deaths from poison resulting to a workman or workwoman in these occupations might be held to be the result of carelessness on the part of the employer. All young persons who go into bronze paper works are liable to be seized with sickness which may be fatal to them but for a very simple and well-known remedy, and that remedy is drinking copiously of new milk. Most of us perhaps remember a case that occurred not many months ago in London where a poor lad was taken ill, and no one suggested to him in a forcible manner the drinking of new milk. Another case had come within the speaker's knowledge as regards a manufacturer in a large way of business. He was conducting some experiments, and he inhaled fumes, and he forgot, and his brothers forgot, the application of new milk, and this poor man succumbed. In either of these cases—certainly in the case of the lad—it is possible that the employer might have been liable, and for this reason, that there were no rules suspended in the workshops stating this simple remedy, and unless notice of proper precautions be given, the employer may be held to be liable for the consequences. I think that that is a type of the numerous questions which may arise under the Employers' Liability Act. With regard to deaths from lightning, he did not himself quite see how any question could arise wherein an employer might be held to be liable in consequence of such a death, unless indeed the not providing of efficient lightning conductors should entail responsibility. In experimental legislation one never can tell what may happen. With respect to sunstroke, death or injury from that cause has been held to be, in the case of Sinclair *v.* The Maritime Passengers' Insurance Company, not a death falling within the policy of accident insurance. One can very well understand that in coffee plantations in tropical climates if an employer failed to provide sheds or shelters in the daytime, he might be held to be liable for death to his workmen from exposure to heat. But in this country, it did not seem necessary to include deaths from sunstroke with regard to accident insurance or the Employers' Liability Act. The next, and a most important question from a practical point of view, is the liability to non-fatal accidents arising from occupations; it is indeed far more important than the liability to death. Is there any necessary connection between non-fatal accidents and fatal accidents? Is there any ratio of probability between the one and the other? He thought not. He did not think that the extreme test of the coal miners is a good one. The fact is, with regard to these miners, and also with respect to certain classes of railway servants, that so

many men are killed during the early and middle stages of life that there are not very many old ones left to kill, and that has to be taken very seriously into account. But with respect to deaths ascertained in certain occupations, his experience, briefly stated, was that the deaths do not, to a very large extent, certainly in the middle class of life, happen by reason of the occupation. He noticed that the worst risks in table No. 1 are those connected with horses. Well, those are the worst risks all round. There are nearly twice as many people killed by horses in the course of the year as are killed by all the railways running. Those persons connected with horses will also have a succession of non-fatal injuries occurring to them, and causing them to claim very heavily upon the insurance funds. There are other occupations that involve much liability to non-fatal injuries, but wherein the fatal accidents very often happen from causes entirely outside their pursuits, as in a railway train or by drowning, for instance. He might give an extreme case the other way. Dentists do not seem to be exposed to great risk in their occupation, yet they come out badly in some tables he had prepared as respects fatal risks; mainly from railway fatalities, and from driving, riding, and boating. And so, going through the list of fatal accidents in the occupations included in the tables, it will transpire that, speaking generally, there is no relation between the number of non-fatal accidents which a man may have in his occupation, and the chance of his being killed in his occupation. With respect to the employers' liability companies which have caused all of us who have had experience in relation to accidents to think very seriously upon such questions as are now before us, he felt bound to say, after watching very carefully the proceedings, and after having been consulted by several of them on various points, that he could not account for the principles, if any principle is involved at all, on which those companies are conducting their business. He should say that if they come out of it well, it will be more by luck than by good judgment. Some of the wildest things he had ever known in modern times have been done by the managers of the employers' liability companies. He said this with all respect to those gentlemen, because they have done their best, and, perhaps, they have forgotten that in all new businesses there is a great apparent immunity from accident while the business is growing. All the young accident insurance companies seem to have what our American friends call a good time in their infancy. But the period does arrive, generally in about the third or fourth year, when they feel the pinch considerably. Of course it must be so. While the new business is rapidly growing, there can be but a very small current risk attaching to the first year of insurance; but even that does not seem to account for the fact altogether. There are always new points coming up with regard to accident insurance, which tend to puzzle one extremely. The cycles of accidents are very remarkable. Not only have we cycles of disaster with as great a regularity as there seem to be panics in the commercial world, but we have something much more remarkable than that. We have cycles of accidents in certain districts, and nothing can be more surprising than the fact that a very large number of accidents, frequently of quite a similar character, will result in one part of the kingdom and will continue for some months, and then,

after a year or more, suddenly these accidents will cease altogether in that part of the kingdom, and will break out in one or two distant places. He could not tell why, nor have we yet devised any means of testing these causes with precision. There is one solution only, and it is a solution which has, perhaps, but a small bearing. It is this—that agents talk about the accidents which have occurred, and, of course, in their want of wisdom, frequently invite in their canvassing persons to insure who may be, more than others, liable to the same sort of accident. And the next phase is, the persons who are more than ordinarily risky suggest themselves to the agents. What has come to pass during the last few years then is this, there is a sort of double process of selection going on against us; first the agent suggests to the persons exposed to risk the propriety of insuring, and next the persons exposed to the risk have themselves learned the propriety of insuring. He found, by keeping a careful note of the claim registers of one of the large accident companies which are under his observation constantly, and also of the registers of some other large insurance companies of the same class in other countries, that there is a selection going on against the company which is much more serious in relation to non-fatal than to fatal injuries. In fact, the real question in accident insurance is the non-fatal and not the fatal claims; and so they present no analogy to life assurance in any form. And what is to be the ultimate effect of this continuing and increasing selection against the company, he did not pretend to know. The effect now is that rates which were very abundant some twenty years ago are becoming very much narrowed down; and to make things apparently worse, the companies have thought it desirable to bring their policyholders into participation in profits. That, perhaps, has something to be said in its favor in the way of sustaining the income, but he spoke of it only in its connection with its helping to reduce the margin of profit. There is the selection against the companies, and there is the competition in the way of superior advantages offered by the younger and less experienced companies to policyholders. There is also a tendency to cut down the rates, or to introduce new features of a hazardous class; and hence business of the class which is particularly in view in this paper is not in a very hopeful position. He did not, on the other hand, take an extremely despondent view. These cycles of competition arise and subside alternately. We shall have to watch this business and analyze it. Altogether the business of insuring against the risk of accident, either fatal or non-fatal, is one which should be undertaken with extreme care, and the business connected with the employers' liability companies is one which requires even greater care, because the legal nature of the contracts is not yet clearly defined.

Mr. Neison said that after much thoughtful enquiry upon this subject, and turning the matter over in his mind in every possible way, he was forced to the conclusion that the statistics presented in the reports of the Registrar-General of births, deaths, and marriages, did not afford any satisfactory basis for the determination of the rates of fatal accidents. If we turn to Table No. 1 of Mr. Whittall's paper, we can hardly help observing the remarkably favorable accidental rate which, according to this table, is incidental

to some occupations which are notoriously known to accident insurance companies to be hazardous risks. He need only mention one or two, such as butchers and horsekeepers. He could not help thinking that the explanation is this,—that possibly some of these occupations should have been grouped together. If we look at "horsekeeper, groom, and jockey", we shall find the rate of fatal accident is given at 5·7 per 10,000, but the most unfavorable entry in this table is that at the end, namely, "horsebreaker", in which the rate of accident is put down at nearly 48 per 10,000. He was not at all sure the proper way would not have been to put these two classes together. There is so much akin in the two that we have no certainty whatever that the living and the deaths have been classed under the proper groups. Perhaps many of the deaths of horsebreakers ought to have been placed in the category of grooms, and *vice versa*. Further on in the paper it was said, in reference to textile manufactures, "There did not appear to be any *primâ facie* reason for supposing that the rate of accidents would vary very much in these manufactures." He would mention that, for reasons connected with the Employers' Liability Act, he had had occasion to go very carefully through the statistics of the Registrar-General to see whether he could get any information upon this special class of manufactures; but it was simply impossible to satisfactorily separate the different occupations, and he was accordingly driven to another source for information; and that source, though it is not a very good one, is very much better than the statistics in the Registrar-General's table. He referred to the reports of the inspectors of factories, and there it will be found that the differences in the rates of accident amongst the various textile occupations is most marked. For instance, take scutch mills, where the rate of accident, especially in the north of Ireland, is something frightful. Then, if we take flax mills, the rate of accident among them is very considerable; and, though the rate of accident in the cotton mills is not nearly so heavy as it is in the flax mills, the fatal accidents in the cotton mills are far in excess. Speaking generally, we may take it that, whereas the flax mills and the cotton mills taken as a whole are the most disastrous in their accidents, the silk mills are the least. Putting it in a rough ratio we may take it that the rate of accident is $2\frac{1}{2}$ times greater in the one than in the other. Turning to the very interesting abstract given of the rate of accident among railway employees, he would like to ask Mr. Whittall one or two questions on the statistics therein presented. Mr. Whittall, first of all, very properly draws attention to the fact that the accidents returned to the Board of Trade are not reliable. There is no doubt whatever about that being actually the case, and the explanation of it is that the companies reported only accidents occurring on the line. A perusal of the statute will show that the Legislature meant all accidents to be reported to the Board of Trade; but practically what grew up was a custom of reporting only accidents happening on the line, and consequently any accidents occurring in the sheds or factories or buildings in connection with the company, were not reported to the Board of Trade, though this matter is now altered. He would like to ask Mr. Whittall how he arrived at the figures

given in the table of the numbers killed in 1870 to 1872. Returning to the reports of the Registrar-General, there occurs among the special tables devoted to the scheduling of fatal accidents, an entry of this description—" violent deaths on railways "—not " violent deaths of those connected with railways ", and consequently the deaths there returned are far in excess of those occurring among the employees only. It was therefore first of all essential for Mr. Whittall to go through all these records and find out how many of these deaths were in respect of railway employees, and how many were in respect of outsiders. That was an extremely difficult task; in fact, impossible. When the late Government appointed a Royal Commission on railway accidents some years ago, his (Mr. Neison's) attention was directed to this subject, and in the report issued by that Royal Commission there is a statement for four years, of the number of fatal accidents reported to the Board of Trade in those years. Roughly speaking, this was the conclusion that was come to —that one out of every five of these accidents, or slightly more, was in respect to trespassers. In other words, they were persons who trespassed on the line, and this will show at once that they could not have been in the employ of the company; and consequently, that of itself is sufficient to knock off one-fifth of the deaths at once. But then there must have been an inquest in every case, and, of course, the Registrar would have been required to put in some occupation. And if, for instance, the Registrar puts in the occupation of a laborer, he did not see how Mr. Whittall could determine whether that was a laborer whose employment was outside the company's service, or whether it was a laborer who was employed in connection with the company's service. Then, a considerable number of deaths—some 300 out of the 1,800 given by Mr. Whittall for those three years—are stated in Dr. Farr's reports as being " in manner not stated "; and notwithstanding the very elaborate schedule which they give of the cause of death, they could not even find out for the purposes of the report how the death had arisen; and if there is so much of doubt even on the point of whether the person was run over, or whether there was a collision, or whether he got jammed between the buffers, he was afraid that there could not be very satisfactory evidence as to the point of whether he was a laborer in connection with the company, or whether he was concerned outside, or whether it was a case of suicide. But turning from the deaths to the number of employees at risk, Mr. Whittall states the figures as 129,688; and he has taken them quite accurately. But it is known very well that that does not represent the number of persons in connection with the railways of this country, and, as a result, one of the first things that had to be done when the Royal Commission on accidents on railways was issued, was to find out really what was the number of employees; and, for that purpose, the return alluded to here by Mr. Whittall—Lord Aberdeen's return in 1874—was issued. In that return every company was called upon to produce a statement of the number of persons they employed, very much as is done under the Mines Regulation Act. Well, as Mr. Whittall states, the return given was 274,000 for the whole of the United Kingdom; and the figures for 1873 would not be so very different from what

the facts were in 1871. It will be seen that, practically, it is adding a very considerable number on to the population at risk. Instead of being 129,000, as Mr. Whittall takes it, he ought, at least, to make it somewhere about 220,000. Now, he believed that this inapplicability of the census results for getting the railway population is not an exceptional case. In the report of the Royal Commission on factories in 1876 the same difficulty was met with. It was desired in connection with the Employers' Liability Bill to find out what was the rate of accident; but that was merely one point that we were desirous of ascertaining. It was wished to ascertain also the population in connection with the different factories. The census referred to was taken, but it was found impossible to deduce any satisfactory conclusions from the census, and special returns were called for by the House of Commons; and any gentleman curious in the matter has only to turn to appendix B to the report of the Royal Commission on Factories, and he will there see a most instructive table, drawn up by the secretary, showing the occupation in the census and the occupation in the manufacturers' returns. He (Mr. Neison) spent two or three weeks in trying to account for the differences, but at last he was obliged to give it up as simply impossible. Turning to the rate of accidents among railway employees which Mr. Whittall gives, as shown by him (the speaker) and shown by Mr. Whittall's own results, it appears that the advantages of his (Mr. Neison's) ratios over Mr. Whittall's are just these—that Mr. Whittall is in doubt, for the reason already given, whether his population ought to be increased by ten thousand or one hundred thousand, and for the same reason he must have experienced considerable difficulties in making up his mind whether the accidents returned by the Registrar were accidents really to laborers, or to persons connected with the company or not. Happily he (Mr. Neison) was not under such difficulties. The panic created at the time that the Employers' Liability Bill was coming before the House of Commons was so great, that railway and other companies were anxious to see whether they could arrive at the truth, and, consequently, he had returns from some of the large railway companies in this country, not returns submitted by the companies themselves, but really the experience of the benevolent funds which existed in connection with the companies, and therefore there was no possibility that the accounts had been cooked in any way,—if such a term might be used,—simply because the facts had been drawn up for other purposes years before. Well, the results were very astonishing. He would take the first, so far as magnitude is concerned. It was a very large company indeed. The experience ran over ten years, and embraced, in round numbers, 200,000 years of risk. The rate of mortality was practically 33 per 10,000.

The PRESIDENT—What description of trade?

Mr. NEISON—A railway company, a large goods line. Mr. Whittall's figures are an accident rate of 48 or 49 per 10,000. Then we went to another company of almost equal magnitude, but there there was the advantage of having the facts for sixteen years. Again the years at risk were somewhat over 100,000. The rate of fatal accidents was found to be 34 per 10,000, instead of 33, as in the other case.

The President—Were these weekly wage men?

Mr. Neison—All the men that were on the company's funds. In some companies they insist upon everybody belonging to the fund. In other companies they insist only on those connected with the traffic belonging to it. There is no hard and fast line.

The President—There is more than one fund in some railway companies, but these were the weekly wages men—not the clerks and station-masters?

Mr. Neison—The station-masters are taken in. That is a risky occupation, as we saw lately at Waterloo Junction.

The President—But there is within my own knowledge more than one fund. One class is called the black coat men, and the other the fustian.

Mr. Neison—The distinction between those funds is, that whereas one is a sickness and death fund, the other is a superannuation fund. But these are the men who were paid weekly. Then there was a third company—and here is a remarkable difference—a company in which there was very little goods traffic, and we had the experience of some ten years, and we found that the accident rate was 25 per 10,000. Shortly after these facts were put before him—he did not know whether it was in consequence of those facts being submitted to the Government—he was waited upon by the men to know whether there were any statistics on the other side of the question, so that he had the benefit of having the experience of the large union connected with the railway service. That embraced some 25,000 men. He could only get the experience for a few years, but there, again, the rate of mortality was found to be 34 per 10,000. The unanimity with which these statistics come out is wonderful. Then he had the statistics of the railway guards' friendly society. It includes the guards all over the kingdom. There, again, their accidents came to 35 per 10,000, so that for all practical purposes we may assume that the rate of mortality from fatal accidents in goods lines is about 34 per 10,000, and in passenger lines it comes down to about 25. If Mr. Whittall would kindly just make allowance for the differences between the population at the time of Lord Aberdeen's return and at the time of the census, he will find the facts will work out at a mean mortality of somewhat over 30 per 10,000. He (Mr. Neison) turned up the experience of a large number of friendly societies which had come before him, to find what was the rate of fatal accidents, and he found that whereas in rural districts it was as high as 12 per 10,000, it is not quite 10 in towns, where, from ignorance of the subject, he expected to find the rate of accident larger; but it is the reverse, and he believed that that coincides with the experience of accident insurance companies, who find rural risks are heavier than town risks. The President raised the question whether the rate of non-fatal accidents bore any relation to the rate of fatal accidents? There is no relation whatever. He further raised the question, what was the proportion in mines of persons injured by non-fatal accidents? While, in all cases of permanent relief funds, we find that fatal accidents range among miners from 23 to 38 or 40 per 10,000, the non-fatal accidents are at the rate of 14 to 20 per-cent; or, in other words, speaking generally, nearly one

out of every five miners meets with an accident of some sort during the year. In the railway service of course the ratio of non-fatal accidents is not so high; it is found to be one in twelve. He was sorry to say that the records given by the registrars under the head of occupation are very delusive indeed. Before the recent census was taken, so strongly did he feel on that point that, in conjunction with one or two other gentlemen, he had interviews with the past and present presidents of the Local Government Board and Sir Brydges Henniker, and they did all they could to enforce upon the registrars a little more care in filling up the occupation column. But the fact is, it is difficult enough to get the registrars to fill in the age column properly, and, rightly or wrongly, they have an idea that the occupation column is quite beside the question, and that no use is made of it. It is not only so with regard to the registrars of births, deaths, and marriages, but, within the last fortnight, he had statistics from some hospitals, and he found in one case the occupations were only of two sorts for everybody that came to the hospital. It was a woman's hospital. The course of procedure was: "Are you a dressmaker?" "No." "Oh, machinist then;" and consequently in column after column of these records there are but two sorts of entries—either dressmaker or machinist.

Mr. M. N. Adler said that in the paper was a mass of information of quite a novel character so far as this country is concerned. In Prussia they have such statistics; and there they are laboring under similar difficulties to those which appear to have been brought out in the discussion—namely, that the returns derived from the Registrar-General's reports differ very much from the returns given by inspectors of factories, mines, and railways. The discrepancy is very great, and only a few days ago he had seen quite a treatise on the subject by Dr. Engel, who actually analyzes the statistics for each province, and shows the percentage of discrepancies in the two returns, between the records of deaths given by the doctors and the local registrars, and those given by the inspectors. Mr. Neison has very closely looked into the statistics of accidents, and a report which was not actually published, but was a private document, which investigates rates of accidents in mines and on railways, affords a great deal of information which is very valuable indeed.* In this work Mr. Neison brings out that the rate of fatal accidents for coal and ironstone mines, looking at all the English counties together, is 23 for every 10,000 employed. It is a curious fact that an investigation by Dr. Heym, based upon the Prussian mine inspectors' returns for 10 years, brings out a figure wonderfully near. It is 23·26. This is a remarkable coincidence, because, in looking at the statistics of accidents, one is really baffled by discrepancies on so many points. He did not know to what extent Mr. Whittall's results have been made available to companies granting insurances under the Employers' Liability Act, but these figures can hardly be applied, and are not of much practical use. Under the Employers' Liability Act, an employer

* This report is entitled "The Rate of Fatal and Non-Fatal Accidents in and about Mines and on Railways, with the Cost of Insurance against such Accidents". A copy is in the Library of the Institute.

is not only liable in respect of a fatal accident, but also if the workman sustains any personal injury. The question was asked by the President, whether there were any statistics available to show the relative proportion of fatal to non-fatal accidents. There is very great difficulty in getting at any reliable figures. The Germans are, however, very painstaking, and from a mass of statistical matter of interest which has been published recently, he had extracted some figures which refer to one of the leading mining institutions in Germany, at Chemnitz, and the result is this, that to 91 deaths there are 213 total disablements, by which the men are permanently prevented from continuing to do any hard work. There were 369 partial disablements and 8,177 accidents of a temporary or lighter character, so that there would be something like 100 accidents to one death.

Mr. WALFORD—That is the experience here.

Mr. ADLER—Mr. Whittall states that almost every one of the deaths recorded by the Registrar-General under the heading of violent deaths, was the subject of a coroner's inquest. If every one of the cases represented by the figures given here was the subject of a coroner's inquest, these tables, which are intended to give the percentage of fatal accidents, must be very incomplete, because you can well understand that a man meets with an accident but does not die at once. He is taken to his home or to an hospital. There he lingers, and months afterwards perhaps, mortification sets in and he dies. Such a death would not be a case for a coroner's inquest, and yet the death has clearly arisen from the accident.

Mr. HUMPHREYS—Mr. Whittall, near the beginning of the paper, makes this remark, "For example, large numbers of deaths are known to occur which might be distinctly traced to the effects of accidents which happened some time previously, but which are not registered as caused by them." That almost answers Mr. Adler's question.

Mr. WALFORD—The accident offices allow three months after the accident for a case to terminate fatally.

Mr. HUMPHREYS—There are a great many deaths from accident not included in this return.

Mr. NEISON—Under the Mines Regulation Act, if a man dies any time within twelve months, you are bound to have an inquest. That has been always in force under the Mines Regulation Act.

Mr. WHITTALL, in reply, said—In his opening remarks, the President referred to the want of disablement experience. Of course this paper did not pretend to give a complete experience for the use of an accident company or any other company to trade upon. It is palpable that one must have disablement experience as well as death experience to calculate rates for accident insurance, inasmuch as the rates of disablement are undoubtedly quite independent of the rates of fatal accidents. The present is intended to be a scientific paper dealing with a special subject of enquiry. Speaking of the use of this paper, there is one use to which it might be applied, although it is not mentioned in the paper, and it has not been referred to by any of the speakers. It may be used in our ordinary life assurance practice. We very often have a risky occupation submitted, and it is then a

great question, what is the amount of risk to which a man is subjected on account of his occupation, and what extra premium should he be charged. Some of the figures here given may be useful in showing that we have often charged excessive rates. If you take railway employees even at the rates brought out in the paper, the rate is only one-half per-cent, which would be covered by an extra annual premium of 10*s*. per-cent, or, perhaps, less than that—perhaps 6*s*., according to Mr. Neison. But yet if it were proposed to add only something like three to six years on account of his occupation to the life of a railway guard, actuaries would rather laugh at the idea. Mr. Walford made some very suggestive observations about poisons which are very instructive. His statement that a great many of them properly come under the head of accidents adds all the more to the correctness of the tables in the paper, as those deaths are included in them. They were all taken account of in the first small table given in the paper in order to see what was the maximum possible error on account of their being included. Mr. Neison, in his observations, began by referring in table 1 to some anomalous rates which were brought out there, and he referred more particularly to horsekeepers and horse-breakers. Now, if Mr. Neison were a registrar, and registered a horsekeeper who was killed under the name of a horsebreaker, he could hardly be judged eligible for his post.

Mr. Neison—It is groom and jockey as well as horsekeeper.

Mr. Whittall—Well, those are very well-defined occupations, and men would hardly make such errors. The registers are very carefully kept; moreover, these are all cases in which coroner's inquests were held, which fact reduces the chance of error very considerably. There is no doubt that they would be very much more carefully registered and described than ordinary deaths. The further great point relied on in giving these statistics is, that these deaths have been classified on exactly the same lines, and practically in the same office, and by the same men, as the census returns. If there was a liability to error in the one, probably the liability to similar error in the other would counteract its effect. Then Mr. Neison referred to textile manufactures, and he mentions the statement that there was no *primâ facie* reason for supposing that the rates would vary much in them. His new information to the contrary is very interesting. He (Mr. Whittall) experienced the same difficulty which Mr. Neison refers to in trying to deal with the textile manufactures entirely from the census returns. No doubt Mr. Neison touched a weak point in the figures when he spoke about trespassers killed on railways possibly being described as railway laborers, when they ought, properly, to have been put down as ordinary laborers. But, bearing in mind that there was a coroner's inquest in every case, there is great probability of their having been correctly named. Then he has criticized the populations given in the paper, and very naturally draws an inference that they are understated from what is stated there about Lord Aberdeen's return, which brings out a very much larger railway population. He desired to explain that it was only since that paragraph was penned that he had seen the details of the return of 1873, and he found that they were given in a much fuller manner than he thought; and while he had only quoted the total of 274,535 in the

paper, yet in reality they are pretty minutely classified in the different classes. On referring to the instructions which are issued for the guidance of the classifying clerks of the census, he found that a great number of those men who are included in Lord Aberdeen's return would not be included at all in the census populations adopted; so that he had done himself an injustice in quoting that larger number which are included in Lord Aberdeen's return, and comparing this with the 156,083 which his figures amount to. Also with regard to the deaths quoted here as being reported to the Board of Trade, he would shortly say that since these were inserted, he had discovered a seeming discrepancy between the return of the Royal Commissioners on Railway Accidents from which the facts were extracted, and the miscellaneous statistics issued by the Board of Trade, and also purporting to give the number of fatal accidents on railways as returned by the companies. The number given in the miscellaneous statistics of the Board of Trade are very much in excess of those quoted in the paper. This shows there is a less probability of the figures being overstated. Mr. Neison referred to his own railway experience, and he explained, as he has also explained in that valuable report which Mr. Adler has referred to, that it was the result of the experience of some railway funds. But it may be rather doubtful from the description he has given us of them, whether he has not got a superior class of men in these funds. As the figures in the paper include all the railway men in the kingdom, they will include not only the well-kept and well-regulated men on the larger lines where there is stricter discipline and greater immunity from accidents: but they will also include all the men on small railways—small local railways in the coal-mining and other industrial districts. It is quite possible that the mortality amongst the class of men employed on these smaller, and probably more dangerous, railways, might be higher than amongst the class from which Mr. Neison's funds are recruited. Notwithstanding what Mr. Neison says, therefore, he could not think that the discrepancy in the respective railway rates was due to overstatement in his (Mr. Whittall's) entirely.

Insurances against Issue.

In January 1877 we gave a table of the Issue Insurances in force according to the latest returns of the offices to the Board of Trade. It then appeared that there were 27 companies transacting this class of business, and that the total sum assured, after deduction of reassurances, was £875,558, in respect of which premiums had been received amounting to £62,238. We now give a similar table brought down to the present date; and it will be seen from this that the business has increased during the five years to the extent of about 50 per-cent, the number of companies transacting

the business being now 41, and the totals, after deduction of reassurances, being :—

Sums Assured, £1,264,166 | Premiums, £97,495

Name.	Date of Valuation.	Number of Cases.	Net Amounts, deducting Reinsurances.		Gross Amounts.			
			Sums insured.	Premiums received.	Sums insured.	Premiums received.	Reserve.	Average Premium per-cent.
London & Prov. Law	31 Dec. 1875	79	221,628	13,485	238,628	14,755	10,068	6·18
Eagle . . .	30 June 1877	29	179,355	19,195	279,210	27,124	10,656	9·71
Equity and Law .	31 Dec. 1879	46	164,322	12,492	199,657	14,825	14,825	7·43
Law Union . .	30 Nov. 1879	39	84,520	5,877	103,520	7,129	3,564	6·89
Standard . . .	15 Nov. 1875	15	63,220	6,400	63,220	6,400	3,316	10·12
Norwich Union .	30 June 1876	16	63,050	4,412	63,050	4,412	2,206	7·00
Legal and General .	31 Dec. 1876	7	41,000	2,121	45,200	2,305	1,153	5·10
Guardian . . .	31 Dec. 1879	14	39,800	2,896	48,800	3,164	3,164	6·48
North British . .	31 Dec. 1875	9	39,500	2,690	143,000	5,480	9,315	3·83
London Assurance .	31 Dec. 1875	8	36,462	3,111	36,462	3,111	3,031	8·53
Pelican . . .	30 June 1875	7	36,015	3,016	36,015	3,016	3,016	8·37
Law . . .	31 Dec. 1879	10	32,500	3,354	69,500	9,182	9,182	13·21
Rock . . .	20 Aug. 1875	4	26,000	1,820	26,000	1,820	580	7·00
Hand-in-Hand . .	31 Dec. 1877	4	25,000	2,552	25,000	2,552	2,250	10·21
Universal . . .	31 Dec. 1879	12	24,400	1,964*	46,900	3,776	3,897	8·05
Reliance . . .	31 Dec. 1877	8	22,807	1,603	24,807	1,771	1,954	7·14
Alliance . . .	31 Dec. 1878	6	20,500	1,254	20,500	1,254	627	6·12
Union . . .	30 June 1877	5	20,000	1,365	20,000	1,365	793	6·83
Liv. & London & Globe	31 Dec. 1878	6	16,400	1,166	16,400	1,166	595	7·11
Commercial Union .	31 Dec. 1877	6	14,500	879	14,500	879	847	6·06
City of Glasgow .	20 Jan. 1879	8	14,500	1,019	15,500	1,071	360	6·91
West of England .	31 Dec. 1877	6	11,600	75	11,600	75	*nil*	(?)
Imperial . . .	31 Jan. 1876	4	10,325	306	10,325	306	306	2·96
Westminster . .	31 Dec. 1876	6	8,362	612	8,362	612	570	7·32
Sovereign . . .	31 Dec. 1879	3	8,000	726	8,000	726	590	9·08
Edinburgh . .	31 Mch. 1878	2	5,000	420	5,000	420	378	8·40
University . .	30 Apr. 1880	2	4,800	353	4,800	353	335	7·35
London & Lancashire	31 Dec. 1877	3	4,000	241	6,500	372	56	5·72
General . . .	31 Dec. 1877	2	4,000	273	4,000	273	137	6·83
Scottish Equitable .	1 Mch. 1878	2	4,000	462	4,000	462	439	11·55
Patriotic . . .	31 Mch. 1879	2	4,000	480	4,000	480	287	12·00
Provident . . .	31 Dec. 1877	1	2,500	210	2,500	210	210	8·40
Queen . . .	31 Dec. 1878	2	2,000	40	2,000	40	20	2·00
U. K. Temperance .	31 Dec. 1875	1	2,000	210	2,000	210	140	10·50
Whittington . .	30 Apr. 1878	1	1,600	80	1,600	80	80	5·00
Scottish Imperial .	31 Dec. 1875	1	1,500	79	1,500	79	59	5·27
Law Property . .	31 Dec. 1875	1	1,000	*not stated*	2,000	*not stated*	*nil*	(?)
Caledonian . .	14 May 1878	1	1,000	84	1,000	84	84	8·40
Northern . . .	31 Dec. 1875	1	1,000	53	1,000	53	53	5·30
Atlas . . .	25 Dec. 1879	1	1,000	20	1,000	20	5	(?)
Briton Medical & Gen.	31 Dec. 1874	3	1,000	100	1,000	100	100	10·00
41 Companies .	...	383	1,264,166	97,495	1,618,056	121,512	89,248	7·51

* Estimated as $= \frac{244}{469} \times 3776$.

The only other circumstances which appear to call for remark from us are, *Firstly*—That the average premium has slightly increased. As we know of no reason for believing that the rates charged by companies for such insurances have increased, but rather the contrary, we are inclined to attribute this increase in the average premium to the circumstance that the average age of the lives insured against issue, is less than formerly. *Secondly,* That several policies included in the present returns are subject to the payment of annual premiums, whereas on the former occasion all the policies seem to have been effected at single premiums. It may be useful to point out that the former can be safely accepted only in exceptional circumstances, since the risk under issue insurances in general, rapidly diminishes as the duration of the policy increases.

The Logic of an Unfortunate Experience.

[From the *Insurance Monitor* of New York.]

As was stated by our Gossip last month, the Loomis saw and planing mill in Brooklyn was burned out for the eleventh time, yet the owner's record is a reputable one; nevertheless, his unfortunate experience has naturally rendered it extremely difficult for him to obtain protection. A somewhat similar experience is narrated by the Cincinnati *Price Current* as follows:—

"A few days ago application was made to one of our city companies by a broker, for insurance upon a new steamboat owned in another western city, which was promptly declined. He presented the fact that the boat was new and well built, which was admitted; that the trade in which it was to engage was not dangerous, which was also admitted; and that the owners were among the best men in the country, which was heartily endorsed. 'Then why decline?' 'Solely for the reason that the ownership of that boat has been unfortunate, having lost several boats. We admit that they have good boats employed in good trades; that they seem to be careful in the employment of their crews; and that, personally, the ownership is of the highest character. And yet the boats of that concern do burn and sink in greater ratio than do steamboats generally. We have paid money to learn these facts; and without going into an investigation of the whys and wherefores, we propose to derive some benefit in the future by our experience in the past.'"

Every office can furnish scores of just such cases; and the question is, what is the true interpretation to be placed on an unfortunate experience of this kind? Suppose that no reason is known, or can be fairly assigned, for what appears to be a special moral hazard. Is the underwriter to say that there is at any rate some fatality connected with that man's property? Or is he to

argue that there is a special danger which he cannot divine? Or, on the other hand, to simply say that the combination was purely accidental and betokens no additional danger to a new risk? The first view is very apt to be secretly cherished by many, while at the same time it is obviously wholly untenable. Apart from some particular hazard which has caused the fires, no fatality attaches to any risk. If the hazard be not present in some shape, an insurance after the eleventh disaster is just as safe as after a first. But the repetition of a fire is an indication of the presence of such a hazard; it is an evidence which grows wonderfully stronger with each added instance. It may have been a hundred to one that the first fire would not have occurred. It is nearly ten thousand to one that two fires would not quickly follow, and the probability increases enormously each time if the risk were of the ordinary kind. Thus, without going into the mathematics of the subject, it is clear that the hypothesis of a pure coincidence decreases enormously with every fresh loss. Among the great number of risks on a company's books, such coincidences are to be looked for, and due allowance made for their occurrence. A single repetition may not justify a belief in the presence of a special danger. But it does justify and demand a careful enquiry, whether such a danger does not exist; while a third disaster, following quickly after, may well, to use the language of the courts, throw the burden of proof on the claimant, that his was a case of pure accident or that his risk was not far greater than it seemed.

Take the case of the Brooklyn mill. It is almost impossible to believe that the hazard on that establishment was at all measured by the price charged, that any company again assuming that risk would not be guilty of the grossest folly if it consented to write for any figures which did not fully recognize the exceptional experience; and such figures might well render the mill practically uninsurable. So in the case of the Cincinnati steamboats, the position of the underwriters was strictly logical. No matter whether they were able to assign a reason for the successive losses or not, the fact that they had occurred, was good ground for assuming that the chances of another repetition were greater than in ordinary lines.

But here we must pause. The measure of that chance, the measure of the additional charge which would justify a policy, must be purely a matter of judgement. For this, no rule can be given. The law is a very broad one, that every additional fire increases in a high degree the probability that the risk is far greater than was assumed, and few things are more important to the underwriter than to know the past experience of the applicant.

[*See letter from* MR. D. CARMENT, *p.* 232.]

H^{M} $3\frac{1}{2}$ *per-cent—Death or* 50.

Duration.	20	21	22	23	24	25	26	Duration.
1	1·825	1·898	2·004	2·139	2·290	2·443	2·603	1
2	3·689	3·864	4·100	4·380	4·677	4·982	5·295	2
3	5·619	5·920	6·296	6·715	7·158	7·608	8·075	3
4	7·637	8·074	8·585	9·144	9·724	10·321	10·950	4
5	9·752	10·320	10·964	11·654	12·374	13·125	13·923	5
6	11·957	12·654	13·425	14·248	15·115	16·026	17·008	6
7	14·249	15·068	15·967	16·930	17·949	19·035	20·211	7
8	16·618	17·562	18·595	19·704	20·889	22·160	23·539	8
9	19·067	20·140	21·313	22·581	23·943	25·407	26·997	9
10	21·598	22·806	24·133	25·569	27·115	28·780	30·587	10
11	24·215	25·573	27·061	28·673	30·411	32·282	34·314	11
12	26·931	28·445	30·103	31·899	33·833	35·918	38·184	12
13	29·751	31·429	33·264	35·248	37·386	39·694	42·210	13
14	32·681	34·531	36·546	38·725	41·075	43·621	46·401	14
15	35·726	37·750	39·953	42·335	44·912	47·711	50·772	15
16	38·886	41·092	43·490	46·090	48·908	51·974	55·335	16
17	42·168	44·563	47·171	50·001	53·074	56·426	60·098	17
18	45·575	48·173	51·003	54·078	57·424	61·073	65·067	18
19	49·119	51·932	54·998	58·335	61·964	65·920	70·255	19
20	52·810	55·852	59·170	62·778	66·701	70·982	75·672	20
21	56·658	59·945	63·523	67·413	71·646	76·266	81·333	21
22	60·676	64·216	68·066	72·253	76·809	81·789	87·260	22
23	64·869	68·672	72·809	77·305	82·206	87·571	93·474	23
24	69·244	73·325	77·760	82·587	87·856	93·633	26	
25	73·812	78·182	82·936	88·115	93·779	25		
					24			
26	78·581	83·259	88·354	93·912			41	
27	83·565	88·575	94·034	23		42		
28	88·783	94·147	22		43		86·743	8
29	94·254	21				85·389	74·120	7
	20			44	83·645	71·476	62·081	6
			45					
				81·319	68·072	58·206	50·581	5
4	…	…	78·060	63·530	53·218	45·531	39·578	4
3	…	…	57·169	46·564	39·030	33·404	29·039	3
2	…	…	37·243	30·357	25·455	21·788	18·945	2
1	…	…	18·210	14·852	12·453	10·663	9·270	1
			45	44	43	42	41	

H^M $3\frac{1}{2}$ *per-cent—Death or* 50—(continued).

Duration.	27	28	29	30	31	32	33	Duration.
1	2·763	2·936	3·127	3·339	3·584	3·859	4·171	1
2	5·618	5·972	6·362	6·803	7·305	7·869	8·505	2
3	8·570	9·111	9·718	10·400	11·171	12·036	13·004	3
4	11·623	12·368	13·202	14·137	15·188	16·361	17·675	4
5	14·790	15·750	16·822	18·020	19·359	20·852	22·525	5
6	18·078	19·264	20·584	22·051	23·689	25·515	27·571	6
7	21·495	22·916	24·489	26·237	28·185	30·366	32·824	7
8	25·046	26·706	28·543	30·582	32·862	35·417	38·302	8
9	28·731	30·642	32·753	35·103	37·731	40·683	44·021	9
10	32·558	34·728	37·133	39·810	42·809	46·182	49·991	10
11	36·531	38·979	41·693	44·718	48·111	51·921	56·218	11
12	40·665	43·405	46·447	49·843	53·644	57·908	62·721	12
13	44·969	48·020	51·412	55·192	59·416	64·159	69·509	13
14	49·456	52·838	56·593	60·771	65·444	70·686	76·605	14
15	54·142	57·867	61·998	66·598	71·736	77·507	84·033	15
16	59·032	63·114	67·642	72·680	78·314	84·649	91·821	16
17	64·133	68·592	73·535	79·038	85·199	92·137		
18	69·460	74·312	79·693	85·693	92·418			
19	75·021	80·289	86·141	92·672				
20	80·834	86·548	92·901					
21	86·919	93·109						
22	93·300							

Duration.	40	39	38	37	36	35	34	Duration.
							91·465	15
						91·061	83·338	14
					90·598	82·549	75·586	13
				90·065	81·646	74·430	68·182	12
			89·443	80·605	73·107	66·675	61·098	11
		88·707	79·390	71·582	64·951	59·255	54·313	10
9	87·824	77·955	69·802	62·963	57·148	52·149	47·814	9
8	76·231	67·699	60·644	54·717	49·674	45·342	41·585	8
7	65·173	57·902	51·882	46·818	42·515	38·818	35·617	7
6	54·610	48·530	43·489	39·254	35·654	32·567	29·901	6
5	44·504	39·552	35·451	32·003	29·079	26·580	24·419	5
4	34·825	30·954	27·746	25·055	22·783	20·839	19·153	4
3	25·551	22·713	20·364	18·402	16·744	15·324	14·092	3
2	16·668	14·816	13·294	12·021	10·944	10·023	9·218	2
1	8·154	7·253	6·513	5·892	5·369	4·918	4·523	1

H^M 3½ *per-cent—Death or* 55.

Duration.	20	21	22	23	24	25	26	27	Duration.
1	1·434	1·476	1·549	1·648	1·758	1·865	1·975	2·078	1
2	2·889	3·002	3·172	3·377	3·591	3·804	4·012	4·218	2
3	4·394	4·601	4·874	5·180	5·495	5·803	6·111	6·425	3
4	5·969	6·278	6·649	7·053	7·459	7·862	8·273	8·699	4
5	7·622	8·026	8·493	8·984	9·482	9·984	10·502	11·053	5
6	9·346	9·843	10·394	10·974	11·567	12·172	12·810	13·493	6
7	11·136	11·717	12·353	13·024	13·716	14·437	15·202	16·024	7
8	12·983	13·647	14·372	15·138	15·941	16·784	17·682	18·647	8
9	14·885	15·636	16·453	17·327	18·247	19·218	20·254	21·364	9
10	16·846	17·686	18·607	19·594	20·638	21·742	22·917	24·176	10
11	18·866	19·809	20·840	21·946	23·118	24·355	25·674	27·086	11
12	20·959	22·009	23·155	24·385	25·685	27·060	28·526	30·104	12
13	23·127	24·290	25·556	26·910	28·343	29·860	31·485	33·239	13
14	25·375	26·655	28·042	29·523	31·093	32·763	34·558	36·502	14
15	27·707	29·104	30·615	32·228	33·945	35·779	37·757	39·904	15
16	30·121	31·639	33·278	35·034	36·908	38·918	41·092	43·448	16
17	32·620	34·263	36·040	37·948	39·992	42·190	44·565	47·135	17
18	35·206	36·984	38·909	40·981	43·207	45·599	48·179	50·972	18
19	37·888	39·811	41·895	44·143	46·556	49·146	51·940	54·961	19
20	40·674	42·753	45·008	47·436	50·040	52·837	55·851	59·109	20
21	43·574	45·820	48·251	50·863	53·666	56·674	59·917	63·432	21
22	46·597	49·014	51·624	54·429	57·436	60·665	64·154	67·943	22
23	49·746	52·338	55·136	58·138	61·357	64·823	68·577	72·659	23
24	53·022	55·798	58·786	61·993	65·441	69·163	73·199	77·599	24
25	56·432	59·394	62·582	66·011	69·705	73·699	78·042	82·782	25
26	59·977	63·134	66·537	70·204	74·162	78·451	83·123	88·228	26
27	63·663	67·031	70·666	74·587	78·830	83·437	88·460	93·957	27
28	67·504	71·099	74·981	79·179	83·729	88·675	94·077	27	
29	71·513	75·350	79·501	83·997	88·874	94·187	26		
30	75·704	79·804	84·245	89·058	94·289	25			
					24				
31	80·094	84·477	89·227	94·383				44	
32	84·700	89·386	94·470	23			45		
33	89·539	94·552	22			46		88·570	10
34	94·630	21			47		87·675	77·731	9
	20			48		86·583	75·988	67·431	8
					85·222	73·862	64·882	57·626	7
			49	83·475	71·210	61·772	54·310	48·282	6
		50							
			81·150	67·807	57·894	50·263	44·235	39·362	5
4	...	77·899	63·276	52·917	45·217	39·294	34·616	30·828	4
3	...	56·942	46·290	38·742	33·136	28·824	25·414	22·652	3
2	...	37·026	30·121	25·233	21·604	18·808	16·598	14·805	2
1	...	18·068	14·710	12·337	10·571	9·211	8·137	7·258	1
		50	49	48	47	46	45	44	

H^M $3\frac{1}{2}$ *per-cent—Death or* 55—(continued).

Duration.	28	29	30	31	32	33	34	35	Duration.
1	2·186	2·303	2·430	2·579	2·743	2·925	3·124	3·340	1
2	4·439	4·677	4·947	5·252	5·588	5·958	6·360	6·796	2
3	6·761	7·136	7·554	8·023	8·538	9·098	9·708	10·373	3
4	9·166	9·683	10·258	10·897	11·592	12·349	13·173	14·083	4
5	11·658	12·325	13·062	13·872	14·753	15·713	16·768	17·937	5
6	14·241	15·064	15·965	16·952	18·025	19·202	20·501	21·948	6
7	16·921	17·901	18·970	20·139	21·418	22·826	24·386	26·130	7
8	19·695	20·836	22·080	23·445	24·943	26·598	28·438	30·485	8
9	22·567	23·875	25·305	26·879	28·612	30·531	32·657	35·017	9
10	25·539	27·026	28·656	30·453	32·436	34·627	37·047	39·734	10
11	28·621	30·299	32·143	34·179	36·420	38·889	41·617	44·637	11
12	31·822	33·706	35·778	38·060	40·565	43·324	46·367	49·737	12
13	35·155	37·258	39·565	42·098	44·879	47·936	51·307	55·050	13
14	38·629	40·957	43·505	46·301	49·364	52·731	56·454	60·595	14
15	42·248	44·806	47·606	50·670	54·028	57·728	61·827	66·392	15
16	46·013	48·812	51·869	55·214	58·887	62·943	67·442	72·464	16
17	49·931	52·977	56·302	59·948	63·960	68·394	73·325	78·836	17
18	54·005	57·308	60·921	64·889	69·262	74·105	79·497	85·529	18
19	58·242	61·821	65·742	70·055	74·815	80·097	85·981	92·572	19
20	62·656	66·531	70·782	75·465	80·643	86·391	92·804	35	
21	67·263	71·454	76·061	81·142	86·765	93·015	34		
22	72·078	76·612	81·600	87·106	93·206	33			
23	77·124	82·024	87·419	93·382	32			36	
24	82·417	87·709	93·542	31			37		
25	87·978	93·691	30			38		92·316	18
		29					92·031	85·029	17
26	93·829				39	91·713	84·474	78·105	16
	28			40	91·355	83·854	77·293	71·513	15
			41	90·949	83·157	76·387	70·457	65·231	14
		42	90·484	82·366	75·367	69·278	63·942	59·234	13
	43	89·945	81·460	74·210	67·951	62·503	57·722	53·497	12
11	89·315	80·410	72·885	66·446	60·883	56·035	51·772	48·000	11
10	79·183	71·350	64·721	59·046	54·136	49·848	46·072	42·725	10
9	69·555	62·724	56·942	51·983	47·682	43·919	40·601	37·651	9
8	60·389	54·504	49·515	45·225	41·498	38·230	35·339	32·772	8
7	51·653	46·657	42·410	38·751	35·563	32·759	30·279	28·084	7
6	43·315	39·150	35·603	32·537	29·855	27·496	25·417	23·577	6
5	35·337	31·957	29·070	26·561	24·365	22·440	20·744	19·251	5
4	27·694	25·055	22·787	20·814	19·091	17·580	16·257	15·101	4
3	20·359	18·416	16·744	15·291	14·021	12·914	11·953	11·115	3
2	13·304	12·031	10·938	9·984	9·154	8·439	7·819	7·276	2
1	6·519	5·896	5·358	4·888	4·485	4·140	3·838	3·576	1
	43	42	41	40	39	38	37	36	

H^{M} $3\frac{1}{2}$ *per-cent—Death or* 60.

Duration.	20	21	22	23	24	25	26	27	28	Duration.
1	1·168	1·192	1·245	1·322	1·408	1·489	1·570	1·640	1·711	1
2	2·347	2·423	2·551	2·712	2·877	3·036	3·184	3·323	3·470	2
3	3·563	3·713	3·923	4·161	4·402	4·626	4·841	5·053	5·277	3
4	4·838	5·069	5·354	5·666	5·969	6·259	6·544	6·830	7·142	4
5	6·178	6·483	6·840	7·213	7·579	7·936	8·293	8·665	9·072	5
6	7·575	7·951	8·368	8·801	9·233	9·659	10·099	10·563	11·068	6
7	9·026	9·461	9·936	10·433	10·931	11·438	11·967	12·527	13·134	7
8	10·518	11·010	11·548	12·109	12·686	13·278	13·900	14·558	15·266	8
9	12·050	12·603	13·203	13·840	14·500	15·183	15·900	16·656	17·465	9
10	13·624	14·238	14·913	15·630	16·377	17·152	17·965	18·818	19·732	10
11	15·240	15·927	16·681	17·483	18·319	19·186	20·093	21·049	22·077	11
12	16·910	17·674	18·510	19·399	20·324	21·283	22·288	23·355	24·505	12
13	18·636	19·482	20·403	21·378	22·392	23·446	24·559	25·744	27·027	13
14	20·422	21·352	22·357	23·418	24·524	25·682	26·909	28·224	29·652	14
15	22·271	23·283	24·371	25·522	26·729	27·998	29·351	30·805	32·380	15
16	24·179	25·273	26·449	27·698	29·012	30·403	31·892	33·489	35·208	16
17	26·146	27·326	28·598	29·951	31·383	32·906	34·533	36·271	38·143	17
18	28·175	29·449	30·823	32·290	33·851	35·508	37·272	39·157	41·177	18
19	30·273	31·648	33·133	34·726	36·417	38·206	40·112	42·141	44·313	19
20	32·446	33·931	35·538	37·257	39·076	41·001	43·050	45·227	47·562	20
21	34·702	36·307	38·039	39·882	41·835	43·898	46·086	48·422	50·934	21
22	37·051	38·777	40·630	42·604	44·688	46·889	49·232	51·739	54·440	22
23	39·493	41·338	43·319	45·420	47·637	49·988	52·497	55·187	58·093	23
24	42·023	43·995	46·099	48·330	50·692	53·201	55·891	58·781	61·908	24
25	44·649	46·742	48·973	51·344	53·863	56·548	59·428	62·532	65·893	25
26	47·364	49·582	51·950	54·473	57·160	60·032	63·121	66·452	70·058	26
27	50·171	52·523	55·040	57·726	60·595	63·670	66·979	70·549	74·422	27
28	53·077	55·576	58·253	61·116	64·182	67·470	71·011	74·841	79·001	28
29	56·095	58·750	61·600	64·655	67·929	71·443	75·236	79·345	83·819	29
30	59·232	62·058	65·095	68·353	71·845	75·605	79·670	84·084	88·903	30
31	62·501	65·512	68·747	72·218	75·949	79·972	84·334	89·085	94·284	31
32	65·914	69·119	72·563	76·267	80·254	84·567	89·256	94·378	28	
33	69·480	72·891	76·562	80·516	84·784	89·416	94·466	27		
34	73·207	76·842	80·758	84·986	89·565	94·548	26			
35	77·112	80·988	85·173	89·703	94·625	25				
36	81·210	85·349	89·831	94·696	24					
37	85·521	89·952	94·762	23						
38	90·070	94·825	22							
39	94·885	21								
	20									

Duration.	55	54	53	52	51	50	49	48	47	Duration.
									89·735	12
								89·099	80·072	11
							88·350	78·837	70·942	10
						87·453	77·382	69·142	62·291	9
					86·360	75·642	67·021	59·955	54·067	8
				84·994	73·519	64·483	57·202	51·222	46·231	7
			83·241	70·867	61·387	53·909	47·869	42·900	38·751	6
		80·909	67·463	57·520	49·891	43·858	38·975	34·957	31·595	5
4	77·652	62·937	52·557	44·873	38·964	34·280	30·486	27·358	24·746	4
3	56·614	45·957	38·433	32·852	28·551	25·137	22·365	20·084	18·185	3
2	36·738	29·868	25·007	21·396	18·611	16·391	14·591	13·117	11·890	2
1	17·903	14·574	12·213	10·461	9·102	8·019	7·145	6·432	5·834	1
	55	54	53	52	51	50	49	48	47	

H^M $3\frac{1}{2}$ *per-cent—Death or* 60—(continued).

Duration.	29	30	31	32	33	34	35	36	37	Duration.
1	1·790	1·871	1·970	2·078	2·196	2·322	2·455	2·595	2·747	1
2	3·627	3·804	4·006	4·228	4·467	4·720	4·986	5·271	5·588	2
3	5·526	5·803	6·114	6·452	6·813	7·193	7·596	8·038	8·530	3
4	7·488	7·871	8·295	8·749	9·230	9·742	10·296	10·904	11·585	4
5	9·520	10·011	10·546	11·116	11·724	12·379	13·091	13·879	14·766	5
6	11·621	12·220	12·867	13·558	14·303	15·109	15·994	16·977	18·071	6
7	13·791	14·498	15·261	16·083	16·974	17·945	19·016	20·197	21·498	7
8	16·028	16·847	17·736	18·699	19·747	20·896	22·156	23·535	25·053	8
9	18·335	19·276	20·300	21·414	22·633	23·964	25·413	26·998	28·730	9
10	20·720	21·791	22·962	24·241	25·634	27·145	28·790	30·579	32·530	10
11	23·191	24·403	25·733	27·179	28·744	30·414	32·283	34·280	36·466	11
12	25·756	27·123	28·613	30·225	31·971	33·856	35·894	38·114	40·552	12
13	28·427	29·949	31·599	33·385	35·308	37·383	39·634	42·094	44·799	13
14	31·203	32·879	34·697	36·652	38·758	41·035	43·516	46·232	49·226	14
15	34·080	35·919	37·900	40·030	42·330	44·828	47·552	50·543	53·847	15
16	37·065	39·062	41·211	43·528	46·039	48·770	51·757	55·045	58·675	16
17	40·152	42·311	44·641	47·160	49·895	52·878	56·149	59·747	63·722	17
18	43·344	45·677	48·201	50·936	53·912	57·167	60·736	64·663	69·009	18
19	46·649	49·170	51·902	54·870	58·107	61·648	65·531	69·813	74·557	19
20	50·080	52·802	55·759	58·978	62·490	66·331	70·554	75·218	80·394	20
21	53·647	56·587	59·786	63·269	67·071	71·238	75·826	80·903	86·554	21
22	57·364	60·538	63·993	67·755	71·870	76·387	81·372	86·903	93·074	22
23	61·244	64·666	68·390	72·454	76·906	81·805	87·225	93·254	37	
24	65·299	68·981	72·997	77·386	82·204	87·521	93·420	36		
25	69·537	73·502	77·831	82·574	87·795	93·572	35			
26	73·976	78·246	82·917	88·049	93·713	34				
27	78·635	83·237	88·284	93·844	33					
28	83·537	88·504	93·965	32						
29	88·709	94·078	31							
30	94·184	30								
	29									

Duration.	46	45	44	43	42	41	40	39	38	Duration.
									92·879	21
								92·664	86·174	20
							92·428	85·758	79·841	19
						92·167	85·300	79·234	73·839	18
					91·874	84·792	78·566	73·051	68·134	17
				91·547	84·225	77·825	72·185	67·175	62·697	16
			91·178	83·588	76·998	71·224	66·119	61·575	57·508	15
		90·759	82·872	76·070	70·150	64·948	60·339	56·229	52·543	14
13	90·282	82·060	75·025	68·945	63·640	58·968	54·821	51·115	47·791	13
12	81·134	73·841	67·590	62·173	57·437	53·260	49·543	46·220	43·240	12
11	72·491	66·052	60·522	55·720	51·516	47·800	44·490	41·532	38·872	11
10	64·301	58·650	53·787	49·560	45·852	42·572	39·651	37·033	34·671	10
9	56·517	51·595	47·358	43·667	40·430	37·566	35·008	32·705	30·624	9
8	49·098	44·861	41·208	38·026	35·237	32·762	30·540	28·536	26·716	8
7	42·017	38·419	35·321	32·624	30·253	28·140	26·237	24·511	22·936	7
6	35·243	32·253	29·682	27·439	25·459	23·689	22·083	20·617	19·281	6
5	28·758	26·347	24·271	22·451	20·841	19·391	18·064	16·852	15·757	5
4	22·548	20·679	19·066	17·647	16·383	15·233	14·177	13·222	12·358	4
3	16·588	15·227	14·052	13·009	12·070	11·212	10·431	9·721	9·088	3
2	10·855	9·976	9·212	8·522	7·899	7·336	6·817	6·352	5·946	2
1	5·332	4·905	4·529	4·183	3·878	3·597	3·340	3·116	2·921	1
	46	45	44	43	42	41	40	39	38	

H^M 3½ *per-cent—Death or* 65.

Duration.	20	21	22	23	24	25	26	27	28	29	30	Duration.
1	·990	1·002	1·042	1·106	1·178	1·243	1·305	1·356	1·407	1·461	1·518	1
2	1·981	2·034	2·137	2·270	2·406	2·532	2·644	2·744	2·848	2·957	3·082	2
3	3·003	3·117	3·289	3·485	3·680	3·854	4·014	4·165	4·322	4·498	4·695	3
4	4·076	4·258	4·491	4·745	4·986	5·206	5·416	5·620	5·841	6·088	6·362	4
5	5·205	5·448	5·738	6·037	6·323	6·592	6·852	7·119	7·408	7·730	8·081	5
6	6·383	6·682	7·016	7·359	7·691	8·009	8·331	8·665	9·029	9·424	9·851	6
7	7·605	7·947	8·324	8·713	9·093	9·470	9·857	10·262	10·698	11·169	11·670	7
8	8·858	9·243	9·664	10·099	10·536	10·977	11·434	11·910	12·418	12·960	13·542	8
9	10·141	10·569	11·035	11·526	12·025	12·534	13·060	13·606	14·184	14·805	15·463	9
10	11·454	11·926	12·448	12·999	13·564	14·140	14·734	15·349	16·004	16·698	17·451	10
11	12·798	13·325	13·905	14·520	15·151	15·794	16·454	17·143	17·870	18·657	19·510	11
12	14·182	14·768	15·411	16·089	16·785	17·492	18·225	18·984	19·801	20·686	21·649	12
13	15·611	16·258	16·964	17·705	18·464	19·241	20·042	20·889	21·802	22·794	23·868	13
14	17·087	17·796	18·563	19·365	20·192	21·035	21·922	22·862	23·881	24·980	26·160	14
15	18·609	19·379	20·206	21·075	21·965	22·892	23·869	24·913	26·036	27·239	28·528	15
16	20·177	21·005	21·897	22·828	23·800	24·815	25·893	27·039	28·263	29·572	30·963	16
17	21·787	22·680	23·633	24·643	25·700	26·814	27·991	29·236	30·563	31·972	33·464	17
18	23·445	24·398	25·428	26·522	27·676	28·886	30·160	31·505	32·929	34·436	36·036	18
19	25·146	26·175	27·288	28·476	29·724	31·028	32·399	33·839	35·358	36·971	38·691	19
20	26·906	28·016	29·221	30·501	31·840	33·239	34·702	36·235	37·858	39·587	41·432	20
21	28·729	29·930	31·225	32·594	34·025	35·514	37·067	38·701	40·437	42·287	44·271	21
22	30·624	31·914	33·296	34·755	36·273	37·850	39·501	41·245	43·099	45·085	47·219	22
23	32·588	33·964	35·435	36·978	38·581	40·253	42·012	43·871	45·858	47·990	50·277	23
24	34·618	36·082	37·635	39·261	40·956	42·732	44·604	46·592	48·722	51·004	53·450	24
25	36·714	38·260	39·894	41·609	43·406	45·292	47·289	49·417	51·693	54·130	56·744	25
26	38·871	40·496	42·218	44·032	45·936	47·944	50·077	52·348	54·775	57·377	60·171	26
27	41·085	42·796	44·616	46·534	48·557	50·698	52·971	55·389	57·976	60·753	63·741	27
28	43·363	45·170	47·092	49·126	51·278	53·555	55·971	58·546	61·305	64·271	67·470	28
29	45·713	47·621	49·657	51·817	54·102	56·518	59·087	61·830	64·773	67·945	71·376	29
30	48·140	50·161	52·319	54·609	57·030	59·596	62·328	65·251	68·396	71·794	75·477	30
31	50·654	52·797	55·083	57·505	60·072	62·797	65·705	68·825	72·191	75·835	79·797	31
32	53·264	55·532	57·948	60·513	63·235	66·131	69·232	72·568	76·175	80·093	84·368	32
33	55·973	58·369	60·925	63·641	66·530	69·614	72·926	76·498	80·373	84·596	89·225	33
34	58·781	61·316	64·020	66·900	69·972	73·263	76·805	80·639	84·813	89·383	94·417	34
35	61·699	64·381	67·245	70·304	73·578	77·093	80·892	85·019	89·532	94·499	30	
										29		
36	64·733	67·573	70·613	73·870	77·363	81·129	85·214	89·674	94·576			
37	67·894	70·908	74·142	77·614	81·351	85·398	89·809	94·650	28			
38	71·196	74·401	77·847	81·558	85·570	89·935	94·720	27			51	
39	74·654	78·069	81·750	85·730	90·054	94·785	26			52		
40	78·286	81·933	85·878	90·164	94·847	25			53		89·982	13
					24					89·423	80·666	12
41	82·111	86·020	90·266	94·904				54	88·772	79·586	71·949	11
42	86·158	90·364	94·957	23			55					
43	90·459	95·007	22			56		88·007	78·330	70·383	63·749	10
44	95·057	21			57		87·093	76·854	68·561	61·724	55·996	9
	20			58		85·983	75·090	66·418	59·370	53·539	48·637	8
					84·603	72·947	63·860	56·601	50·680	45·769	41·628	7
			59	82·838	70·284	60·751	53·295	47·320	42·432	38·368	34·937	6
		60										
			80·496	66·878	56·887	49·277	43·307	38·510	34·576	31·303	28·531	5
4	..	77·234	62·358	51·945	44·283	38·429	33·826	30·118	27·077	24·540	22·383	4
3	..	56·062	45·387	37·897	32·367	28·132	24·795	22·108	19·898	18·048	16·470	3
2	..	36·254	29·422	24·615	21·057	18·324	16·175	14·440	13·006	11·805	10·778	2
1	..	17·618	14·328	12·008	10·284	8·962	7·922	7·078	6·380	5·795	5·289	1
		60	59	58	57	56	55	54	53	52	51	

H^M 3½ *per-cent—Death or* 65—(continued).

Duration.	31	32	33	34	35	36	37	38	39	40	Duration.
1	1·588	1·665	1·749	1·836	1·926	2·017	2·120	2·222	2·351	2·494	1
2	3·226	3·385	3·553	3·726	3·904	4·094	4·295	4·521	4·787	5·086	2
3	4·919	5·158	5·410	5·668	5·941	6·225	6·545	6·902	7·318	7·774	3
4	6·664	6·985	7·318	7·668	8·031	8·430	8·876	9·377	9·942	10·551	4
5	8·462	8·861	9·283	9·719	10·193	10·714	11·298	11·943	12·654	13·419	5
6	10·308	10·793	11·299	11·842	12·433	13·087	13·810	14·595	15·455	16·369	6
7	12·210	12·775	13·384	14·041	14·761	15·548	16·405	17·333	18·335	19·398	7
8	14·169	14·826	15·544	16·326	17·175	18·091	19·086	20·150	21·293	22·515	8
9	16·178	16·950	17·790	18·695	19·669	20·718	21·843	23·042	24·336	25·730	9
10	18·269	19·158	20·118	21·143	22·245	23·419	24·673	26·017	27·476	29·050	10
11	20·442	21·447	22·523	23·672	24·894	26·193	27·586	29·088	30·718	32·490	11
12	22·695	23·813	25·007	26·273	27·614	29·046	30·591	32·258	34·077	36·061	12
13	25·022	26·255	27·562	28·943	30·413	31·991	33·694	35·542	37·564	39·766	13
14	27·426	28·768	30·186	31·690	33·301	35·031	36·908	38·951	41·182	43·609	14
15	29·899	31·348	32·885	34·525	36·282	38·181	40·245	42·489	44·935	47·600	15
16	32·438	34·002	35·671	37·452	39·372	41·451	43·708	46·158	48·832	51·752	16
17	35·050	36·742	38·546	40·484	42·578	44·844	47·300	49·969	52·886	56·076	17
18	37·746	39·569	41·525	43·632	45·906	48·363	51·030	53·933	57·108	60·593	18
19	40·529	42·499	44·618	46·899	49·357	52·018	54·909	58·062	61·519	65·325	19
20	43·412	45·540	47·828	50·287	52·942	55·819	58·951	62·374	66·140	70·293	20
21	46·405	48·696	51·156	53·806	56·670	59·779	63·172	66·892	70·991	75·527	21
22	49·511	51·969	54·614	57·465	60·553	63·915	67·594	71·636	76·102	81·063	22
23	52·732	55·369	58·209	61·277	64·610	68·248	72·237	76·633	81·508	86·947	23
24	56·078	58·905	61·955	65·259	68·859	72·797	77·129	81·919	87·254	93·237	24
25	59·557	62·588	65·867	69·431	73·321	77·590	82·302	87·538	93·396	**40**	
									39		
26	63·182	66·435	69·966	73·811	78·021	82·659	87·802	93·543			
27	66·968	70·466	74·269	78·425	82·993	88·048	93·680	**38**			
28	70·935	74·697	78·802	83·306	88·278	93·807	**37**			**41**	
29	75·099	79·155	83·598	88·493	93·926	**36**			**42**		
30	79·486	83·871	88·694	94·038	**35**			**43**		93·064	23
				34					92·875	86·614	22
31	84·127	88·883	94·142				**44**	92·667	86·248	80·579	21
32	89·059	94·240	**33**			**45**					
33	94·331	**32**			**46**		92·439	85·847	80·048	74·901	20
	31			**47**		92·189	85·408	79·467	74·215	69·533	19
			48		91·913	84·924	78·829	73·464	68·701	64·438	18
				91·609	84·393	78·128	72·640	67·789	63·467	59·585	17
		49	91·272	83·806	77·356	71·734	66·789	62·402	58·481	54·952	16
	50										
		90·894	83·155	76·505	70·737	65·689	61·235	57·271	53·722	50·517	15
14	90·468	82·425	75·561	69·637	64·478	59·951	55·945	52·373	49·166	46·260	14
13	81·603	74·502	68·416	63·143	58·538	54·485	50·895	47·685	44·792	42·167	13
12	73·309	67·048	61·661	56·980	52·880	49·268	46·060	43·184	40·587	38·226	12
11	65·506	60·001	55·249	51·109	47·479	44·274	41·420	38·856	36·538	34·425	11
10	58·129	53·312	49·142	45·505	42·308	39·479	36·957	34·689	32·634	30·763	10
9	51·127	46·940	43·313	40·140	37·344	34·869	32·661	30·671	28·872	27·235	9
8	44·458	40·859	37·732	34·990	32·572	30·431	28·519	26·799	25·248	23·831	8
7	38·091	35·036	32·375	30·038	27·977	26·151	24·527	23·070	21·750	20·533	7
6	31·996	29·447	27·224	25·270	23·546	22·027	20·682	19·470	18·362	17·336	6
5	26·146	24·073	22·264	20·673	19·276	18·054	16·970	15·983	15·079	14·230	5
4	20·520	18·899	17·482	16·243	15·163	14·219	13·375	12·604	11·887	11·204	4
3	15·104	13·909	12·874	11·975	11·194	10·505	9·891	9·320	8·779	8·263	3
2	9·881	9·101	8·435	7·856	7·348	6·906	6·504	6·121	5·757	5·415	2
1	4·848	4·470	4·150	3·866	3·622	3·407	3·206	3·011	2·832	2·659	1
	50	49	48	47	46	45	44	43	42	41	

ORIGINAL TABLES.

ON THE CONSTRUCTION OF TABLES OF VALUES OF ENDOWMENT ASSURANCE POLICIES.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—In sending you, for publication, a further instalment of Tables of Endowment Assurance Policy-values according to the H^M Table, permit me, in the first place, to point out a slight typographical error in the paper which accompanied the former portion of the tables. On page 370, vol. xxii, the formula for the value of $\Sigma_n V_{\overline{xt}|}$ should read $n - \dfrac{\Sigma(1+{}_{|t-2}a_{x+1})}{1+{}_{|t-1}a_x}$, instead of $n - \dfrac{\Sigma(1+{}_{|t-2}a_{x+1})}{1+a_x}$, as there printed.

I may also take this opportunity of pointing out certain checks upon the accuracy of the preliminary stages of the work, which I have found it advisable to introduce. Referring to the table given on the lower part of the page just referred to, the accuracy of column (2) may be proved by observing that its *sum* must be equal to $S_{19} - S_{49} - 30N_{49}$. Column (3) may be checked by an independent calculation in logarithms, using seven-figure logarithms, at least as far as age 37, after which point six figures will be quite sufficient, as the annuity-values after that age consist of only five figures in all. The requisite logarithms or cologarithms of D_x to six places can be taken direct from the Institute Tables. The correctness of column (4) may be proved by constructing it from the last age backwards; then the number in this column opposite age 20 should be equal to the result of the direct summation of all the numbers in column (3) from age 21 onwards. The correctness of the reciprocals in column (5) can easily be proved by multiplying each one, as it comes into use on the machine, by the corresponding annuity-value, before going on to calculate the column of policy-values. Column (6) may be proved by addition, its sum being equal to the difference between the first and last numbers in column (3).

I am,
Yours, &c.,
D. CARMENT.

Sydney, N.S.W.,
7 September 1881.

JOURNAL

OF THE

INSTITUTE OF ACTUARIES

AND

ASSURANCE MAGAZINE.

On a Certain Method of Distributing the Surplus among the Assured, and the Construction of an Equitable Scale of Office Premiums with reference thereto. By H. W. MANLY, *Actuary to the Mutual Life Assurance Society, and Fellow of the Institute of Actuaries.*

[Read before the Institute, 30 Jan. 1882.]

IT is not my intention in this paper to travel over the same ground as that followed by Mr. Sprague in his paper in the 7th vol. of the *Journal*, p. 61, "On Certain Methods of Dividing the Surplus among the Assured in a Life Assurance Company; and on the Rates of Premium that should be Charged to Render them Equitable", and that by Mr. Tucker, in the 9th vol., p. 245, "On the Rates of Premiums Required to Provide certain Periodical Returns to the Assured", although the changes which have been introduced into the methods described by both writers, and the increased knowledge we possess of life assurance mortality statistics, would fully justify a revision and extension of their labours. Such a work, if undertaken, would fully repay the time spent upon it, and would prove a valuable addition to the information we already possess upon a very important subject.

Both the above-mentioned gentlemen enlarged upon what "H. A. S.", in the 8th vol. of the *Journal,* p. 167, describes as "The incongruity existing between the rates of premium charged at certain ages and the benefits accruing therefrom"; and they have shown that when the net cost of the assurances and bonuses is calculated upon a certain basis, the figures differ, often very materially, from those of the premiums charged. My paper to-night has reference not only to a method of distributing the surplus which is not mentioned in either of those papers, and, so far as I am aware, has never been described in any scientific work, but also to the construction of a set of office Tables which have been calculated throughout upon strict principles, and, moreover, are in actual use.

The office referred to was established nearly half a century ago upon the mutual system, and the founders started with the fundamental proposition that in such an institution it was necessary, before all things, to secure the due payment of the sum assured at the death of every member. The contract, therefore, made by the members with each other, was for the payment of certain fixed sums at death and nothing more, and the whole of the funds of the office were to be a guarantee for the due payment thereof; but, as the premiums charged had to be loaded to cover all possible expenses and contingencies, and were, therefore, admittedly larger than necessary to secure those sums, provision was made that at the death of every member, his share of the then existing surplus (if any) should be paid with the sum assured, the only condition being that he should have paid two full annual premiums. In order, then, that the relatives of each member should get his exact share of this surplus, it was necessary to know, first of all, the total actual surplus existing at his death. Frequent valuations were thus required, and it was provided that these should be made at the end of every year.

The surplus ascertained by these valuations, however, was not to be deemed a separate fund, but only the temporary result of a mathematical division of the whole fund. The appropriation was to last for a year only, for at the end of that period the two funds were to merge into one, and an entirely new appropriation be made entirely independent of the results of any former valuation. Thus no bonus was to be *guaranteed* except for one year, and the amount payable at death with the sum assured was to be the share of the surplus ascertained by the last yearly valuation only. The question then arose—how was the share of the deceased

member to be calculated? It was argued that each member contributed to the surplus every year a certain proportion of his premium, and that since those yearly contributions were left to fructify at compound interest, therefore the share to which each was entitled was proportionate to the premiums he had paid accumulated at compound interest; and as all the calculations had been made at 3 per-cent, the rate of accumulation was fixed at that rate. Thus, in order to determine the share of each member as he died, it was necessary, first, to make an annual valuation; second, to accumulate all the premiums paid on the existing policies from the date of their grant at 3 per-cent compound interest; and third, to apportion the surplus rateably to those accumulations. If a policy were payable by a limited number of premiums, it was only to share in proportion to the premium that would have been paid for the whole of life.

The principle of this method of distribution is probably unique, and I am not aware that there is another office giving a bonus at death in which the surplus is treated on the same principle. It is in reality the principle of the "London Life" and "Metropolitan", applied to the increase of the sum assured at death instead of the diminution of the future premiums during life.

Unhappily, in the early days of the institution, many offices in the anxiety to obtain increased new business, competed with each other in giving large bonuses, and the system in the office in question was materially altered by paying with the sum assured, not the member's ascertained share of the surplus, but that amount multiplied by the reciprocal of the value of a reversion at the age at which he died;—or, in other words, by paying in cash, if death occurred within a year, the amount which the deceased member's share would have purchased at the death of an average healthy life of his age. As no alteration was made in the method of calculating the reserves, the result was that the relatives of those who died early received considerably more than an equitable share of the surplus. The principles of equity upon which the whole system was professedly based, were thus altogether subverted; and the office, in order to return again to those principles, has recently commenced a new series, in which the bonuses are to be calculated upon the original plan pure and simple.

Now it is evident, if that plan is to be really equitable, that not only should the premiums be based on the rate of mortality existing among the members and the rate of interest realized on

the funds, but they should be prepared with special reference to the manner in which the members are to participate in the surplus. It was with the view, therefore, of inviting the public to join in a strictly equitable and mutual society, that the new scale of premiums was calculated.

When an actuary has to calculate a scale of premiums, he has to keep two things in view:—1st, the necessity of meeting competition; 2nd, the adjustment of the contributions to the benefits to be received.

Whenever competition in trade is unhampered by protective duties or by official restrictions, the tendency is always to reduce prices to the lowest level consistent with a fair return of profit. Prices will vary for the same kind of article, according to the quality of the material and style of manufacture; but they must all eventually follow the same economic law. In Life Assurance business, however, the competition is of another kind. Where the whole or the greater part of the profits are returned to the purchaser, the price is not, or ought not, to be very material, other conditions being equal. The attraction, therefore, which the competing offices first offered was not so much a variation in prices as a different form of benefit or a different mode of distributing profits; but it was soon found that the public as a class did not seek life assurance, and that instead of the public coming to the offices, the offices had to seek the public. The plan of employing agents and paying commission was first introduced; and the system has gradually developed into the employment of salaried agents, district superintendents, inspectors, paid canvassers, branch offices, &c., all of which means increased expenditure, to be paid eventually by the assured in the shape of reduced bonuses; and thus the competition in life assurance, instead of reducing the price to the purchaser, has really had the effect indirectly of increasing it. Any attempt, however, to materially increase the charge *directly* would probably meet with failure, for there is a price at which the most seductive canvasser could not induce the public to assure, and that price is probably not very much in excess of the highest premiums now charged by the offices. The minimum price we know to be the prime cost of the article, plus the expenses. The limits, therefore, within which any practicable scale of premiums must fall, are fairly defined.

At the same time, the actuary has to look to the scientific part of the question, and it is here that his skill is called into play. Certain benefits are to be offered, certain results have to be

attained, and he must fearlessly declare that to secure them, certain contributions must be obtained, and certain methods must be pursued. Nothing with him should be left to chance. As a scientific man, he should not dare to shelter himself behind the law of contract, and plead that because a man enters into a bargain with his eyes open, knowing that for a certain premium he would get certain benefits, therefore he ought not to complain afterwards if the terms were inequitable! He should, for the time being, be above the letter of the law, and endeavour to produce out of the materials at his command, the most equitable arrangement possible.

The first step in making a scale of premiums is to select the mortality table and rate of interest which are to form the basis of the calculations; and it is a well-established principle that these should correspond as closely as possible with the ascertained or expected experience of the office. The mortality experienced during 40 years by the office referred to in this paper was extracted, and it was found to follow so closely the Institute of Actuaries' H^M Table, that that table was unhesitatingly selected. The rate of interest realized by the office had always exceeded 4 per-cent, and as there seemed every prospect of that rate being realized for many years to come, it was adopted in the calculations.

The mortality table and rate of interest being determined, the prime cost of the insurance was easily ascertained, but then came the important question,—how was the prime cost of the bare sum assured to be increased in order to produce an equitable scale of premiums for the benefits above described and provide for expenses? The problem was somewhat unusual. In the first instance, the method of loading for expenses had to be determined,—(in itself a vexed question); then came the provision for bonus, which, be it remembered, is calculated on the premiums actually charged, accumulated at 3 per-cent compound interest; and, finally, the scale produced had to be within the limits of competition. The two additions had thus to be determined with reference to each other and the premium charged. To illustrate it symbolically, let P be the pure or prime cost premium, e the loading for expenses, B the loading for bonus, and X the office premium, at present unknown. Then we have—

$$P+e+B=X,$$

where B is wholly dependent upon X, as may also be a portion of

e, which may have to some extent particular reference to the premium charged, such as commission.

If the exact relations of e and B to X were known, the solution of the equation would not be very difficult, but as they have to be ascertained by trial, and as, moreover, the relation of B to X is complex, it is necessary, in order to solve the problem, to adopt the inverse process, and assume a value for X, whereby the value of P can be found for the assumed value of X, and the solution be completed by proportion.

The best value to adopt for easy manipulation is, of course, *unity*, and upon that basis a standard Table was calculated, for every age at entry, of the present values of a bonus of 100 per-cent of the premium 1 accumulated at 3 per-cent compound interest to the year of death. No bonus was to be payable if death occurred in the first year, and therefore the probability of paying one in that year had to be omitted from the calculations. This value I propose to call $\mathbb{B}$.

My friend, Mr. R. P. Hardy, has pointed out to me that this problem is a particular case of a very general one, namely:—To ascertain the value of an assurance of the accumulated amount of a life annuity in advance up to the end of the year of death. This is a problem which has doubtless puzzled many of us; but Mr. Hardy's solution is so ingenious, his demonstration is so clear, and the result is so striking, that I have begged the favor, which has been generously granted, to append it to this paper.

The manner in which I obtained my values may seem cumbersome when compared with Mr. Hardy's neat formula; but it so happened that every value tabulated in the process was required by me for another object, and therefore the labor was not thrown away.

The following is the manner in which the table was calculated:—

Let $a_{(2)}$, $a_{(3)}$, $a_{(4)}$, &c., represent the amount of 1 per annum, accumulated at 3 per-cent, at the end of 2, 3, 4, &c., years;

l, the number living at the age of the assured at the date of the policy;

${}^1d$, ${}^2d$, ${}^3d$, &c., the deaths occurring in the 2nd, 3rd, 4th, &c., years thereafter; and

v^2, v^3, v^4, &c., the present value of 1 payable at the end of 2, 3, 4, &c., years, at 4 per-cent interest.

Then the value of $\mathbb{B}$ will be

$$\frac{1}{l}\left(a_{(2)}.{}^{1}d.v^{2}+a_{(3)}.{}^{2}d.v^{3}+a_{(4)}.{}^{3}d.v^{4}+\&c.\right)$$

$$=\frac{1}{\mathrm{D}}\left(a_{(2)}{}^{1}\mathrm{C}+a_{(3)}{}^{2}\mathrm{C}+a_{(4)}{}^{3}\mathrm{C}+\&c.\right) \quad . \quad . \quad . \quad (1)$$

A table of these values may be easily computed with the assistance of the arithmometer, by placing the series of values within the bracket in vertical columns, in such a manner that the successive values in each horizontal line shall be capable of assuming the form P + QR.*

The following diagram will best illustrate the position in which the values of aC must be placed as they are calculated :—

Age at Entry.					
96	95	94	. . . . .	11	10
$a_{(2)}\,C_{97}$	$a_{(3)}\,C_{97}$ $a_{(2)}\,C_{96}$	$a_{(4)}\,C_{97}$ $a_{(3)}\,C_{96}$ $a_{(2)}\,C_{95}$	. . . .	$a_{(87)}\,C_{97}$ $a_{(86)}\,C_{96}$ $a_{(85)}\,C_{95}$ ⋮ $a_{(3)}\,C_{13}$ $a_{(2)}\,C_{12}$	$a_{(88)}\,C_{97}$ $a_{(87)}\,C_{96}$ $a_{(86)}\,C_{95}$ ⋮ $a_{(4)}\,C_{13}$ $a_{(3)}\,C_{12}$ $a_{(2)}\,C_{11}$
$\mathbb{S}_{96}$	$\mathbb{S}_{95}$	$\mathbb{S}_{94}$	. . . . .	$\mathbb{S}_{11}$	$\mathbb{S}_{10}$

$$\mathbb{B}_x=\frac{\mathbb{S}_x}{\mathrm{D}_x}.$$

Now the series of values in a table of the amount of an annuity of 1, are produced by the continuous summation of the amount of 1 at the end of 1, 2, 3, &c., years. Thus

$$a_{(2)}=1+(1+i)=2+i$$
$$a_{(3)}=1+(1+i)+(1+i)^{2}=a_{(2)}+(1+i)^{2}$$
$$a_{(4)}=1+(1+i)+(1+i)^{2}+(1+i)^{3}=a_{(3)}+(1+i)^{3}$$
&c. &c.

Hence

$$a_{(3)}\mathrm{C}_{97}=a_{(2)}\mathrm{C}_{97}+(1+i)^{2}\mathrm{C}_{97}$$
$$a_{(4)}\mathrm{C}_{97}=a_{(3)}\mathrm{C}_{97}+(1+i)^{3}\mathrm{C}_{97}$$
&c. &c.

* *See* Mr. Peter Gray's paper "*On the Arithmometer*", vol. xvii, page 249.

The series of values in each horizontal line are therefore obtained by placing the value of C common to that line on the face of the machine, and multiplying first by $2+i$ (the amount opposite year 2 in the column "Amount of 1 per annum" shown in any set of Interest Tables), which produces the value $a_{(2)}C$, and then by the successive values $(1+i)^2$, $(1+i)^3$, $(1+i)^4$, &c.,—that is, the series of values in the column "Amount of 1 for any number of years" to be found in any Table of Interest, commencing with year 2, which gives the values $a_{(3)}C$, $a_{(4)}C$, $a_{(5)}C$, &c. The values are easily checked at any point by multiplying the value of C by the proper amount of annuity.

The summation of each column is now easily accomplished, and the totals have only to be divided by D to produce the desired values. The necessities of the case for which my calculations were made required the annuities to be calculated at 3 per-cent, and the values of C at 4 per-cent, so that I do not think they would be of use to anyone but the office for which they were made. The following extract from the table will be quite sufficient for the purposes of this paper :—

Table $\mathbb{B}$.—*Showing the Present Value, at any given age, of the Amount of an Annuity of* 1, *commencing at that age and accumulated at* 3 *per-cent to the end of the year of death (no payment being made if death happens during the first year), calculated according to the Institute of Actuaries'* H^M *Table and* 4 *per-cent Interest.*

Age.	Value.	Age.	Value.
10	14·4665	45	12·2808
15	14·4295	50	11·4043
20	14·2705	55	10·3533
25	14·1492	60	9·1563
30	13·8888	65	7·8855
35	13·5117	70	6·5607
40	12·9907	75	5·2711

The value of $\mathbb{B}$ for all ages, being formed, the addition to the pure premium is found from the following equation :—

$$(P+L)(1+a)=P(1+a)+(P+L)r\mathbb{B}+E,$$

where L represents the whole addition for profit and expenses; r the rate of bonus, or proportion which the total accumulated premiums bears to the total ascertained surplus; and, consequently $(P+L)r\mathbb{B}$, the present value of the bonus expected to be

declared on the policy, based on the premium actually charged; and E the expenses, which may be wholly or partially dependent on or altogether independent of $(P+L)$. r and E had now to be found by trial; but as the society had flourished for many years, and had made large profits, r was already more or less fixed; and E was also known, the difficulty being how to distribute it to the best advantage.

It will be noticed that as the addition for bonus depended upon the premium charged, that addition increased as the premium increased, and, therefore, if any portion of the expenses were treated as a percentage, the premiums at the older ages would become almost prohibitory. A percentage for commission had consequently to be abandoned. A custom had, however, grown up to allow a brokerage of 1 per-cent on the sum assured, in lieu of an annual commission, and so that was adopted in lieu of a percentage. The other expenses of the office were found to be a little over 5*s*. per-cent per annum on the total sums assured, and thus the loading for expenses on a premium for the assurance of £100 was finally fixed at a single charge of £1, and an annual charge of 5*s*. Substituting these values for E, the equation becomes

$$(P+L)(1+a)=P(1+a)+(P+L)r\mathbb{B}+{\cdot}0025(1+a)+{\cdot}01 \quad . \quad . \quad (3)$$

from which we find

$$L=\frac{P.r.\mathbb{B}+{\cdot}0025(1+a)+{\cdot}01}{(1+a)-r\mathbb{B}}.$$

The rate of bonus being finally fixed at 25 per-cent of the accumulated premiums, r became $\cdot 25$, and the working formula

$$L=\frac{25P.\mathbb{B}+{\cdot}25(1+a)+1}{100(1+a)-25\mathbb{B}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The values of the pure premiums being multiplied by the values in Table $\mathbb{B}$ for each age, the computation of L became a simple mechanical operation, and this value being added to P, gave, of course, the office premium.

The following table will show the results of the calculations :—

Age. (1)	Pure Premium, H^M 4 per-cent P (2)	Loading L (3)	Loaded Premium P′ (4)	Percentage of Loading on: Pure Premium. (5)	Percentage of Loading on: Loaded Premium. (6)	Percentage of Difference between Loaded Premium and H^M 3 per-cent Premium on H^M 3 per-cent Premium. (7)	Analysis of Loading. Portion for Bonus $\frac{\cdot 25\,P' . \mathbb{B}}{1+a}$ (8)	Analysis of Loading. Portion for Commission and Expenses $\frac{1}{1+a}+\cdot 25$ (9)	Percentage on Pure Premium of: Loading for Bonus. (10)	Percentage on Pure Premium of: Loading for Expenses. (11)	Percentage on Loaded Premium of: Loading for Bonus. (12)	Percentage on Loaded Premium of: Loading for Expenses. (13)	Age. (14)
10	·8985	·5452	1·4437	60·7	37·8	35·7	·2476	·2976	27·6	33·1	17·2	20·6	10
15	1·0517	·5889	1·6406	56·0	35·9	33·7	·2899	·2990	27·6	28·4	17·7	18·2	15
20	1·2445	·6439	1·8884	51·7	34·1	32·3	·3430	·3009	27·6	24·1	18·2	15·9	20
25	1·4279	·6997	2·1276	49·0	32·9	30·9	·3970	·3027	27·8	21·2	18·7	14·2	25
30	1·6693	·7729	2·4422	46·3	31·7	29·9	·4677	·3052	28·0	18·3	19·2	12·5	30
35	1·9689	·8648	2·8337	43·9	30·5	29·2	·5567	·3081	28·3	15·6	19·6	10·9	35
40	2·3517	·9833	3·3350	41·8	29·5	28·8	·6713	·3120	28·5	13·3	20·1	9·4	40
45	2·8650	1·1430	4·0080	39·9	28·5	28·7	·8259	·3171	28·8	11·1	20·6	7·9	45
50	3·5415	1·3553	4·8968	38·3	27·7	28·8	1·0314	·3239	29·1	9·2	21·1	6·6	50
55	4·4577	1·6446	6·1023	36·9	27·0	29·2	1·3116	·3330	29·4	7·5	21·5	5·5	55
60	5·7150	2·0437	7·7587	35·8	26·3	29·6	1·6981	·3456	29·7	6·1	21·9	4·4	60
65	7·4274	2·5889	10·0163	34·9	25·8	30·0	2·2262	·3627	30·0	4·9	22·2	3·6	65
70	9·8657	3·3621	13·2278	34·1	25·4	30·4	2·9750	·3871	30·2	3·9	22·5	2·9	70
75	13·2988	4·4264	17·7252	33·3	25·0	30·5	4·0050	·4214	30·1	3·2	22·6	2·4	75

The figures in column (7) are very curious, as showing that almost identical results would have been obtained by loading the 3 per-cent pure premium with a constant percentage of 30 per-cent.

The values in columns (8) and (9) are absolutely correct according to actual calculations, there being only an occasional variation of unity in the last decimal place, thus showing the accuracy of the original computations. As a passing notice, it may be remarked that although the loading for bonus was calculated on the loaded premium, it yet happens to be nearly a constant percentage on the pure premiums. See col. (10).

The premiums for the whole of life having been determined, all the other forms of premium were calculated from them by the ordinary formulas, using the H^M 4 per-cent Table for the purpose. Thus, calling the office whole-life premium P′,

The single premium $= P'(1+a)$.

The premiums payable for a limited term of years t . . . $= \dfrac{P'(1+a)}{1+{}_{|t-1}a}$.

Half-premium for first five years, where the first portion was more than the corresponding term assurance increased 10 per-cent . $= \dfrac{P'(1+a)}{1+2a-{}_{|4}a}$.

Where the first portion was less than the term premium increased by 10 per-cent, the second half was calculated by the formula . $\dfrac{P'(1+a)-{}_{|5}P'(1+{}_{|4}a)\times 1{\cdot}1}{a-{}_{|4}a}$.

I have heard it stated in this room that those who follow the pure-premium system of valuation are not consistent, because they never carry it out in practice in calculating the single premiums, &c. This is true, however, only as regards the *three* per-cent purists who, from an old-fashioned idea that the scale of premiums should be calculated at 3 per-cent although the actual competitive rate is higher, are compelled, in order to avoid charging prohibitory rates, to have recourse to all manner of curious devices for calculating single and special premiums, by which the letter of the 3 per-cent law is adhered to, while its spirit is entirely lost sight of.

In the following table of premiums I claim to have produced an equitable scale for the peculiar form of distribution of profits

here described, and to have done so on the strictest scientific principles.

Scale of Office Premiums.

Age next Birthday.	Whole Life.	Single Premiums.	Limited Number of Payments. 10 Annual Payments.	Limited Number of Payments. 20 Annual Payments.	Limited Number of Payments. 30 Annual Payments.	Half Premiums first 5 Years. Annual Premium first 5 Years.	Half Premiums first 5 Years. Annual Premium for Remainder of Life.	Age next Birthday.
	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	
10	1 8 10	30 8 9	3 13 3	2 4 8	1 16 0	...	...	10
15	1 12 10	33 10 1	4 0 11	2 9 7	2 0 0	...	...	15
20	1 17 9	37 1 9	4 10 5	2 15 7	2 5 0	1 3 8	2 2 0	20
25	2 2 7	40 7 0	4 18 6	3 0 10	2 9 6	1 4 2	2 8 4	25
30	2 8 10	44 5 6	5 8 8	3 7 5	2 15 3	1 7 11	2 15 10	30
35	2 16 8	48 14 8	6 0 3	3 15 2	3 2 4	1 12 8	3 5 4	35
40	3 6 8	53 16 2	6 13 8	4 4 7	3 11 3	1 18 10	3 17 8	40
45	4 0 2	59 14 5	7 10 1	4 16 10	4 3 6	2 7 2	4 14 4	45
50	4 17 11	66 5 9	8 9 7	5 12 5	5 0 0	2 18 8	5 17 4	50
55	6 2 1	73 9 8	9 13 4	6 13 5	6 3 0	3 14 9	7 9 6	55
60	7 15 2	81 3 0	11 3 3	8 3 0	7 15 6	4 18 0	9 16 0	60
65	10 0 4	88 16 11	13 0 8	...	...	...	...	65

APPENDIX.

The following is the demonstration by Mr. R. P. Hardy referred to in the paper :—

Problem.—What is the present value of the amount to which an annuity will accumulate by the end of the year of death, the annuity falling due at the beginning of each year ?

This value is evidently $\Sigma v^n(1+i)\dfrac{(1+i)^n-1}{i}\,{}_{n-1|}q_x$

$$=\Sigma\frac{1+i}{i}\left\{v^n\cdot{}_{n-1|}q_x(1+i)^n-{}_{n-1|}q_x\right\}$$

$$=\frac{1+i}{i}\left\{\frac{\mathrm{C}_x(1+i)+\mathrm{C}_{x+1}(1+i)^2+\ldots\ldots-(\mathrm{C}_x+\mathrm{C}_{x+1}+\ldots\ldots)}{\mathrm{D}_x}\right\}$$

$$=\frac{1+i}{i}\left\{\frac{\mathrm{C}_x(1+i)+\mathrm{C}_{x+1}(1+i)^2+\ldots\ldots}{\mathrm{D}_x}\right\}-\frac{1+i}{i}\mathrm{A}_x \quad . \quad . \quad . \quad (1)$$

Now, when the rate of discount is the same as the rate of accumulation, the value becomes

$$\frac{1+i}{i}\left(\frac{d_x+d_{x+1}+\ldots\ldots\ldots}{l_x}-\mathrm{A}_x\right)$$

$$=\frac{1+i}{i}\left(1-\mathrm{A}_x\right)$$

$$=\frac{1+i}{i}\left[1-\left\{1-\frac{i}{1+i}(1+a_x)\right\}\right]$$

$$=1+a_x.$$

The truth of this will at once be seen from the following analysis :—

$$1+a_x=\frac{l_x+vl_{x+1}+v^2l_{x+2}+\ldots}{l_x}$$

$$l_x(1+a_x)=l_x+vl_{x+1}+v^2l_{x+2}+\ldots\ldots$$

Now, if we express the values on the right-hand side in terms of d_x, we have

$$\begin{array}{rlll} l_x= & d_x+ & d_{x+1}+ & d_{x+2}+\ldots\ldots \\ vl_{x+1}= & & vd_{x+1}+ & vd_{x+2}+\ldots\ldots \\ v^2l_{x+2}= & & & v^2d_{x+2}+\ldots\ldots \\ \vdots & & \vdots & \vdots \end{array}$$

Then, summing both sides, and remembering that the present value of an annuity-certain in advance is $(1+i)\dfrac{1-(1+i)^{-n}}{i}$, we have

$$l_x(1+a_x)=d_x(1+i)\frac{1-(1+i)^{-1}}{i}+d_{x+1}(1+i)\frac{1-(1+i)^{-2}}{i}$$
$$+d_{x+2}(1+i)\frac{1-(1+i)^{-3}}{i}+\&c.$$

Now, since the present value of the amount of an annuity-certain at the end of n years is the same as the present value of the same annuity for the same period, that is, $\dfrac{(1+i)^n-1}{i}\times(1+i)^{-n}$ $=\dfrac{1-(1+i)^{-n}}{i}$, we have

$$l_x(1+a_x)=vd_x(1+i)\frac{(1+i)-1}{i}+v^2d_{x+1}(1+i)\frac{(1+i)^2-1}{i}$$
$$+v^3d_{x+2}(1+i)\frac{(1+i)^3-1}{i}+\&c.,$$

and $$1+a_x=\Sigma v^n{}_{n-1|}q_x(1+i)\,\frac{(1+i)^n-1}{i},$$

the same function that we started with.

The above formula, however, holds good only when the rate of interest is the same for both accumulation and discount. When the rates are different, the solution is not so simple.

Let us call the rate of accumulation i'; then from (1) we have

$$\frac{1+i'}{i'}\left\{\frac{C_x(1+i')+C_{x+1}(1+i')^2+\ldots\ldots}{D_x}\right\}-\frac{1+i'}{i'}A_x.$$

Multiplying both numerator and denominator of the fraction within the brackets by $(1+i')^x$, we have

$$\frac{1+i'}{i'}\left\{\frac{C_x(1+i')^{x+1}+C_{x+1}(1+i')^{x+2}+\ldots\ldots}{D_x(1+i')^x}\right\}-\frac{1+i'}{i'}A_x \quad . \quad (2),$$

the solution of which can be simplified by constructing tables on the columnar method. Thus, if a complete series of $C_x(1+i')^{x+1}$ be formed and summed like the M column, and we call the new column T_x, and also form a column of $D_x(1+i')^x$, and denote it by Y_x, then (2) presents the form

$$\frac{1+i'}{i'}\left(\frac{T_x}{Y_x}-A_x\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3).$$

The modifications needed for the special case mentioned in the paper could be easily applied.

Since the text was written, it has been pointed out by Mr. King, and also by Mr. Sutton, that this formula is capable of still further reduction when the rate of discount, i, is greater than the rate of accumulation, i'. Thus, taking the value within the bracket in (2) and writing d_xv^{x+1} for C_x, we have

$$\frac{d_x\left(\frac{1+i'}{1+i}\right)^{x+1}+d_{x+1}\left(\frac{1+i'}{1+i}\right)^{x+2}+\ldots\ldots}{l_x\left(\frac{1+i'}{1+i}\right)^x}.$$

If, now, we let $\frac{1+i'}{1+i}=\frac{1}{1+I}$, so that $I=\frac{i-i'}{1+i'}$, the above value is seen to be the present value of 1 payable at the death of a life aged x, at the rate of interest I. Calling this A'_x, formula (2) becomes

$$\frac{1+i'}{i'}(A'_x-A_x) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4).$$

If the rate of discount, i, be less than the rate of accumulation, i', then I will be negative, in which case we may put $\frac{1+i'}{1+i}=1+\text{I}$, and then, in finding A'_x, we should have amounts to deal with instead of present values. Probably, in the latter case, formula (2) would be found the best for practical use.

Discussion.

The President (Mr. A. H. Bailey) concurred entirely with Mr. Manly in almost everything that he had said on the subject of competition. He believed that, as Mr. Manly says, the method of distribution mentioned in the early part of the paper is unique. He thought it was a modification of the system of the old Amicable. That office at its original inception, as was the fashion in those days, insured everybody from 15 to 45 for one year at a uniform annual premium of five guineas per-cent, and found that a tolerably profitable business. In the early part of this century a system was adopted of having annual valuations, and dividing the surplus amongst the claimants of that particular year; and the late Mr. Galloway, who was chief officer of the Amicable, a scientific man and a Fellow of the Royal Society, stoutly insisted that that was the only equitable method of dividing profits. The effect of course was to make the amount divided in any particular year dependent very much on the amount of claims in that year, so that if a man died in the year 1808, and another died in the year 1812, both having effected their policies in the same year, the amount received for the one claim might be very different from the amount received for the other; and the result was that the system was unpopular. He proceeded: Mr. Manly says, "When an actuary has to calculate a scale of premiums, he has to keep two things in view,—1st, the necessity of meeting competition, 2nd, the adjustment of the contributions to the benefits to be received." The paper deals almost exclusively with the second, whereas I have found, in practice, that the first is by a long way the most pressing; and that the two, unfortunately, are often incompatible. I do not think that, with the competition that now exists, the contributions can always be adjusted in a scientific manner to the benefits to be received; and therefore I somewhat object to Mr. Manly using his X as the unknown quantity. X must vary within tolerably narrow limits, and therefore the unknown quantity is rather B; and it is a very unknown quantity indeed, and I fear a diminishing quantity. I therefore slightly doubt the practical value of all the pains he has taken upon this matter. The public will not in the least degree appreciate or care for an adaptation of the premiums to the actual benefits they receive. Mr. Manly speaks of the premiums at the older ages becoming almost prohibitory. Well, that is a state of things which has really arisen. The first attempt to load pure premiums was, I believe, the work of Milne, who produced a table of office premiums based upon the Carlisle table with 4 per-cent interest, and loaded 40

per-cent. The effect of that 40 per-cent increase was to make the rate prohibitory at the older ages, and he had to make arbitrary reductions. That difficulty, I think, runs all through Mr. Manly's adaptations. The other point Mr. Hardy will tell us a good deal more about. It has interested both him and myself lately, because he and I with another gentleman have had to work away at that problem. Mr. Manly says, "This is a problem which has doubtless puzzled many of us; but Mr. Hardy's solution is so ingenious, his demonstration is so clear, and the result is so striking, that I have begged the favor, which has been generously granted, to append it to this paper." The case presented to us was that of a superannuation fund, where the premiums were to be returned with interest in the event of death or withdrawal before attaining a stipulated age. As the problem is put here, it is in a general form, and I think it had better not be confined to the present value of an annuity-certain, which was the particular case before us. The result is, I confess, very ingenious, and very much to Mr. Hardy's credit.

Mr. A. Baden—Mr. Manly's premiums at the higher ages are such as almost to exclude them from practice, for though the public, when they want to insure with profits, have an idea of entering into a little mild speculation, they also have an idea of doing it as cheaply as they can; and, however equitable a scale of premiums may be, they will go elsewhere rather than pay what they think too much. I cannot agree with you, Mr. President, that the two things are incompatible—the loading of a practical premium and something like a due regard to the manner in which the profits are distributed. I know at least one case of an office where the premiums are very fairly adjusted to the manner in which the profits are divided, and that too with the result that the scale of premiums brought out is eminently workable. The profits are a quinquennial tontine, the consequence of which is that, as the age advances, although the premium for the bonus assurance would be heavier, the chances of surviving the tontine being smaller, the addition is almost a constant quantity. The effect of that is that the premiums at the higher ages are really workable premiums. I should like to make one observation about this remark—"My friend, Mr. Hardy, has pointed out to me that this problem is a particular case of a very general one, namely, to ascertain the value of an assurance of the accumulated amount of a life annuity in advance up to the end of the year of death. This is a problem which has doubtless puzzled many of us"—and so forth. Well, the result brought out is that the present value of such an annuity is $1+a$. Now it appears to me that the result depends upon considerations which lie so very near to the surface, that it is almost a matter of course that it should be so. It must be almost clear that what we call the annual premium is really a sinking fund, which, improved at interest, shall, at the end of the year in which any life shall fail, amount to what we call the sum assured. A payment of 1 per annum, that is an annuity of 1, in advance, is an annual premium to secure a certain assurance; and of course the single premium for the same assurance must be $1+a$. An elaborate demonstration is given, which, however, is really not required in order to demonstrate so very simple a proposition as that the value of an accumulated annuity, the rate of interest being the same throughout, must be $1+a$.

Mr. G. KING, speaking of the concluding formulas of the paper, said:—The demonstration of Mr. Hardy is extremely elegant. He brings out first of all, the value of an annuity-due accumulated, where the rate of interest for discounting the payments is the same as the rate of interest at which the payments are accumulated, and then he works out more elaborately a formula for the case where the rates of interest are different. In the fourteenth volume of the *Journal*, p. 144, under the heading of "Practical Questions", there is an editorial paragraph where a similar problem is solved. There the question is, not the finding of the value of an accumulating annuity, but finding the present value of a fund invested at compound interest and accumulating during the existence of a given status. Of course the two questions are really one, because an annuity may be looked upon as simply the interest upon a fund invested. By slightly modifying the demonstration given in that volume, there comes out a formula similar to Mr. Hardy's, but which shows more clearly what the real principles are. If we assume with Mr. Manly that the rate of accumulation is i', and the rate at which the benefit is valued is i, we may look upon the annuity-due as the interest in advance upon a fund of $(1+i')\div i'$. That sum will produce an annuity-due of 1 for n years, and will itself remain intact at the end of the period. Therefore an assurance consisting of the accumulations of an annuity-due of 1 forborne, is equivalent to an assurance consisting of a sum of $(1+i')\div i'$ and its accumulations, less an assurance of the sum of $(1+i')\div i'$ itself. We therefore have, if A be the value at rate i of an assurance of the amount of an annuity-due of 1,

$$A=\Sigma\frac{1+i'}{i'}(1+i')^n\times(1+i)^{-n}{}_{n-1|}q_x-\frac{1+i'}{i'}\mathrm{A}_x$$

$$=\frac{1+i'}{i'}\left\{\Sigma\left(\frac{1+i'}{1+i}\right)^n{}_{n-1|}q_x-\mathrm{A}_x\right\},$$

where A_x is taken at rate i. If now we take

$$\frac{1+i'}{1+i}=\frac{1}{1+j}, \text{ so that } j=\frac{i-i'}{1+i'}, \text{ we have}$$

$$A=\frac{1+i'}{i'}\left\{\Sigma(1+j)^{-n}{}_{n-1|}q_x-\mathrm{A}_x\right\}$$

$$=\frac{1+i'}{i'}(\mathrm{A}''_x-\mathrm{A}_x),$$

where A''_n is taken at rate j. If $i'=i$, then $j=0$, and $\mathrm{A}''_x=1$, and we have

$$A=\frac{1+i}{i}(1-\mathrm{A}_x)$$

$$=\frac{1-\mathrm{A}_x}{d}$$

$$=1+a_x,$$

which agrees with Mr. Hardy's formula. If $i'=\cdot03$ and $i=\cdot04$, as in the paper, then the rate at which to calculate A'' is nearly $9\frac{3}{4}$ per

thousand. This is a special case of a very general problem—to find the present value of a fund either depending upon life, or for a term certain, accumulating, for instance in consols, at one rate of interest; the value to be found to pay a given rate of interest on the purchase-money. We sometimes come across such questions in reversionary transactions. Where money is accumulating at compound interest at one rate, we may always find the value at another rate, by merely changing suitably the rate of interest, and so dispensing with the complicated columns which according to the paper seem to be required. With regard to the equitable distribution of surplus, the present system of dividing profits in many companies is very unsatisfactory. To a great extent we rob Peter to pay Paul; and if Peter complain, we quiet him by saying that some time later we will find some one to rob for his benefit. Mr. Sprague and Mr. Tucker, in the *Journal*, vol. vii, p. 61, and vol. ix, p. 245, show how certain of the systems that have been adopted may be made equitable, so far as premiums charged are concerned. But besides constructing suitable tables of premiums, we must, in order to make many of the common systems of division equitable, also value in special ways. If at the beginning we divide among those who are then insured all the surplus that comes out under a net-premium valuation, we give them more than their share. Where there is to be an increasing reversionary bonus, the reserve required for bonus is analogous to the reserve required for the sums assured. The rate of mortality is an increasing one, and hence the reserve for the sums assured must be more than merely the future premiums. We must similarly, where we give an increasing bonus, maintain a special reserve in addition to the future loading. We must, in fact, keep back part of the present profits, in order to provide for this increasing reversionary bonus. This subject has not yet had sufficient attention among actuaries, and if they will adopt or perpetuate these systems without making such valuations, they will find, sooner or later, that they have made a great mistake. I know of one company only where this matter is properly attended to. There are two principles which, I think, ought always to be attended to in dividing profits. One is that profits ought to be divided among those who have contributed to them, and the second is that profits ought to be divided as nearly as possible in proportion to the contributions to them. These principles have not been sufficiently kept in mind; and I fear that the result will be some day to put certain companies in very great difficulty.

Mr. Sorley doubted whether Mr. Manly is entirely justified in claiming "to have produced an equitable scale for the peculiar form of distribution of profits here described, and to have done so on the strictest scientific principles." The rates of premium as ultimately fixed appear to have been deduced without explicit allowance being made for all the facts which ought to be considered in their adjustment. Mr. Manly, having found that the mortality experience of the company with which he was dealing, corresponded with that recorded by the Institute of Actuaries, at once proceeded to adopt the pure premiums of the H^M table as the basis for framing his premiums. Now the H^M table is deduced from the mortality of a mixed body of lives, selected at various ages, and therefore the H^M pure premiums

are not the true pure premiums at the date of assurance, but differ very considerably from them. Probably the most accurate pure premiums at date of assurance are to be found in Mr. Sprague's paper of 1876 (*Journal*, vol. xx, p. 95). From it we find that the true pure premium at age 20 is 7 per-cent greater than the H^M, while at age 75 it is so much as 20 per-cent less, and speaking generally, the analyzed pure premiums are higher than the H^M at the younger ages, and lower at the older ages. In Mr. Manly's office premiums there is, however, a set-off to this, more or less complete, in the fact that the premiums are loaded simply with a constant of 5*s.* per-cent at each age, and not, as is more usual, partly with a constant and partly with a percentage. As Mr. King has indicated, Mr. Manly's practical problem is not completely solved when a perfectly accurate scale of premiums has been constructed. It still remains to determine the proper reserve to be held on hand at the end of each valuation period, so that the surplus at each valuation may provide bonuses in something like the proportion contemplated in the construction of the premiums. It is pleasing to notice that Mr. Manly, in constructing premium rates for assurances other than ordinary whole of life, did so, not from the pure premiums for the particular form of assurance, but from the loaded office whole-life rates. When the bonuses on the special-class policies correspond to those on the ordinary policies, that is the true way—probably the only true way; for any other is almost certain to lead to incongruities. Mr. Manly quotes the title of an old contribution to the *Journal*, "The incongruity existing between the rates of premium charged at certain ages, and the benefits accruing therefrom." Possibly this incongruity may, to a large extent, have now disappeared; but any actuary who takes the trouble to examine the prospectuses of a number of life offices taken at random, may probably find one or more instances of an incongruity still existing between some of the special-class rates and the whole-life rates. If Mr. Manly's method were adopted, such a state of matters could not continue.

Mr. G. S. Crisford thought that the paper is valuable, because it draws attention to actual facts, rather than to theories. What we have really to deal with, both in valuation and distribution of profits, is the premiums which are charged by the companies as contained in the prospectuses. As Mr. Manly's system of apportioning profits and framing premiums is *actually in use*, his paper possesses a value which any theory, in itself and alone, does not possess. The paper, however, is not entirely consistent; the premiums are formed to allow a certain rate of bonus, and that has been very accurately done, but when he came to the question of expenses, instead of still confining himself to actual facts, he felt it necessary to make merely an assumption that a certain charge was made for expenses which, in the aggregate perhaps, might be pretty near, but which, when apportioned to different ages, is not in accordance with fact. If you look at the tables in column 11 and column 13, and suppose that the majority of lives which enter this office are of the middle ages, would that ratio of loading for expenses be sufficient to meet the actual charges? Suppose, for instance, it were a fact that the commission paid upon premiums was 10 per-cent for the first year, and 5 per-cent afterwards, the

commission alone would entirely absorb the whole of the loading for expenses. The result would be that it must of necessity upset the balance of loading which is left for the profit. It would be a very interesting and valuable thing if some of the members—the younger members, perhaps, who have more time—would devote themselves a little to an analysis of the actual premiums which are charged by various offices, comparing them with the pure premiums as brought out by our Institute tables, at various rates of interest, showing what the loading actually is, how much is required to pay expenses, and what is left for profit, and showing what the effect of the various methods of distribution would be upon that balance of loading as it actually exists. We have not so much to deal with tables of premiums formed accurately upon a particular theory; but in making our valuations we have sometimes to deal with premiums loaded without any consistent order. If we take the pure premium, which is a known quantity, and then take the actual charge for expenses, which is a known quantity, and then deduct these two from the premiums actually charged, we find some very curious scales of loading in the portion which is left for actual profit. If upon these actual facts we base our investigations and apply our theories, we shall arrive at some practical results which, with all the papers which we have had, would give us a very great deal of useful and valuable information.

Mr. Searle thought that Mr. Manly had failed of finding the solution of his problem, and that his premiums will not provide the bonus which he has put before himself as his object to provide. It is quite clear that, if Mr. Manly's facts work out according to his theories, his pure premiums will exactly provide for his risk. His loading for expenses should also exactly provide for them; but there will be a little wrong as regards this, because some of the loading will be spent in the first year. That, however, is only a small matter. But with regard to the bonus, supposing that we have a man who enters at any age, say 15, and who pays £100 in premiums; out of that the sum of £25 has to be set aside to be accumulated as the bonus ratio. We can take the 25 per-cent at the beginning instead of later; but it has the same effect. The man was to get a bonus equal to £25 accumulated at 3 per-cent. Now, for that £25 he has to pay 17·7 per-cent of his premium, that is £17. 14*s.*; and this £17. 14*s.* accumulated at 4 per-cent, is always to be equal to the £25 accumulated at 3 per-cent. Well, I take it that it is fairly obvious that it will not do it. It is true that there is a little adjustment, but that makes very little difference—that is to say, there is no bonus paid for the first year of assurance. The difference of that is, of course, that if there are l_x persons who pay that 17·7 each, that 17·7 will be divided between l_{x+1}, which is a slight decrease, and will slightly increase the 17·7, say to 17·8 or 17·9. Then, if 17·9 be accumulated at 4 per-cent as earned by the office, how is it possible that in three years' time that bonus of £25 accumulated at 3 per-cent can be obtained? It would provide less than this bonus in the earlier years, and a good bit more in the later years. If all members entered at the same age, and at the same time, it would be a perfectly equitable thing, because every man at the beginning would stand the same chance of getting the large bonuses and the small

ones; but inasmuch as they do not enter at the same time, it becomes decidedly inequitable, and we have to rob Peter to pay Paul. One man gets more than his 25 per-cent, and another gets less. Looking at the table, the top one pays 17·7 for his bonus, the bottom one pays 22·6. These are accumulated, say for three years only; then it is clear that the bottom one will have paid one-third part more at any rate than the top one, presuming that they pay the same amount of premium, £100, for a different amount of assurance. They will both be entitled to the same bonus, and they will have paid different amounts for it. But the amount expected will not be there in hand to pay them. The amount in hand will be somewhere between the two, and the difference will have to be made up from somewhere, or not be made up at all. Therefore, it seems that the problem has not been solved; and, in fact, cannot be solved.

Mr. R. P. Hardy wished to explain what he considered the simplest and most powerful way of looking at the question discussed by Mr. Manly. If all the payments made in respect of an annuity be left in the hands of the contractors, to bear interest at the same rate as was employed in discounting the values, then the l_x payments paid down will have to be returned in the order of d_x, d_{x+1}, &c.; and, if the same be left fructifying, they will themselves earn the further accretion of interest. Hence, the value for the first year's annuity payment is 1. By similar reasoning, the value of the second and third, &c., year's annuity payments will be $v\,.\,p_x$, $v^2\,.\,{}_2p_x$, &c., and so on—the sum total of which is $1+a_x$. The practical application of the principle involved in this question, which was recently before the President and another actuary with himself, was the reserve to be made for a return (upon death or in case of retirement from specified causes) of the contributions made by the members of a large superannuation fund, together with interest; and it was dealt with in the same way as the above. Having settled the rates of mortality and secession that were deemed applicable to the affairs of the fund, a column was formed showing the number left in the fund at each age out of a given radix. The decrement of this column was subdivided into that representing the number of deaths strictly, and the number of secessions from all causes; reducing the secessions to the number of effective claims (for those cases where a return was to be allowed), the total of these and the deaths, constituted a column showing the number and order of the claims for return of contributions and interest. By a simple arithmetical process, cognate to that for obtaining the value of $1+a_x$, the required value was obtained.

Mr. E. Justican wished to mention one or two points. First, as to the amounts of the accumulated annuities-certain. These would appear to be incorrectly stated, as, being payable in advance, the amounts at the end of 1, 2, 3, &c., years should stand thus—

$$a_{(1)}=1+i$$

$$a_{(2)}=a_{(1)}+(1+i)^2$$

$$a_{(3)}=a_{(2)}+(1+i)^3$$

&c., &c.,

the expressions given in the paper being each multiplied by the factor $(1+i)$. Then, as to the question, "To find the present value of

the amount to which an annuity-certain due will accumulate by the end of the year of death", an approximate value—sufficiently near for practical purposes—could be found from the formula

$$\frac{(1+a_x)4\%}{A_x(3\%)} \times A_x(4\%);$$

and when the rates of discount and accumulation are equal, the value becomes $1+a_x$, as stated in the paper. The following are values at the three ages, 35, 40, and 45; compared with those given in the paper, they are in each case slightly greater, as no correction has been applied to meet the deduction of the first year's risk.

Age.	Value given in paper.	Value by above formula.
35	13·51	13·56
40	12·99	13·01
45	12·28	12·31

Mr. H. W. Manly, in reply, said he believed that the system of the old Amicable was not so much in the minds of the founders of this office, as that of another office which reduces its premiums largely after a certain number of years. At any moment that office can revert to its original premiums; and so the intention here was that the payment which the office promised to make, should always be capable of being reduced to the simple sum assured. In fact, the object was to apply the system to increasing the sum assured at death, instead of reducing the premiums during life. With reference to the remark that in the premiums charged by the office, X is the known quantity, and B the unknown, that is generally true, as the premiums are really fixed by competition. In this office it happened to be just the other way. A slight variation in the rate of bonus made such a difference in the bonuses payable, that the actuary was bound to argue, "If we are not to get into the difficulty which the old Amicable fell into by the great variations in its bonuses from year to year, B must to a certain extent be fixed, and proper provision must be made for it." The system is so delicate, that a small decrease in the rate of bonus will cause the bonus upon an old policy to be actually less than the one previously declared. B, then, in this case was bound to be fixed; and when B is fixed, when you know beforehand the benefit which you are going to give to a man, you should charge him accordingly. If he will not pay the cost of that benefit, then you are very much better without him; otherwise you actually rob Peter to pay Paul, and you know it. On the other hand, when you have the power of distributing your profit by any method which you think best, then, of course, B is not so important in the composition of your premiums. At the time that the premiums in his table were calculated, none of those analyzed mortality tables which have been referred to, were published. He was perfectly well aware of the fact that the premium for a young life should be greater than that produced by the ordinary table, and that it was much less for older lives (for he had himself made investigations into the question), and therefore he felt less difficulty in loading the premium for expenses in the manner he had done. He believed that, practically, the one balances the other, as Mr. Sorley supposed they would.

The series of values of aC in table B were actually computed with the object of determining the proper reserve to be made for this special method of distribution, so that he did not leave out of consideration the important question of reserves. It was a very difficult problem to solve, and it took a great deal of time. It is not a thing which can be repeated every year; but nevertheless the first investigation afforded the means of determining what modification should be made in the actual results of the valuations produced by the ordinary methods. He thought that a portion of the incidental profits produced from the increased interest and surrender values, &c., might very fairly be devoted, as they are in some offices, to defraying expenses. Anyhow, notwithstanding the very small percentage of loading which has been added at the higher ages, the premiums become practically prohibitory. With reference to Mr. Searle's remarks, the 17 per-cent of the premium at age 15 accumulated at 4 per-cent is demonstrated to actually produce 25 per-cent of the premium accumulated at 3 per-cent by the year of death. In answer to Mr. Justican's question as to the calculation of the annuity, that is really a matter of office practice. With regard to this office, the premium is supposed to be paid, not on the 1st January, but on the 31st December; consequently, it has not to be accumulated for one year. He thought that Mr. Justican and Mr. Baden had both fallen foul of the rock upon which the late Mr. Peter Hardy and Mr. Wm. Morgan split.

Mr. JUSTICAN said he had stated that the amount to which 1 paid in advance would accumulate by the end of the year in which the life should fail is $\frac{1+a_x}{A_x}$. That is the amount which an annuity of 1 will insure. In other words, it is the amount to which the premiums will accumulate; and the annuity is here essentially a premium, call it what you like.

Mr. MANLY—Exactly so. Now, if you will refer to the controversy mentioned (see *Journal*, vol. vii, p. 1), you will find that, if the sum payable at death is the amount of the premiums paid, accumulated at compound interest, the average of those sums is not the sum assured, but something else. The problem is not quite so simple as Mr. Baden and Mr. Justican seem to imagine.

Mr. SEARLE feared he had not made himself quite clear. It is not a question of how many die every year. You have got to provide the bonuses for all who are alive as well as for those who are dead. You have got to allot a bonus of £25 for every £100 of premium that has been paid in; and 17·7 accumulated at 4 per-cent will not do it.

Mr. MANLY—You must remember that the bonus in this case is not paid until the man is dead.

Mr. SEARLE—Has it not to be allotted and announced?

Mr. MANLY—No, not allotted. It is announced, but only provisionally. Of course, if it were distributed, that would be another matter altogether. If the profits were distributed, then, of course, they could not be accumulated. But here the bonus is only paid on the man's death; and the consequence is that your 17 per-cent, put out at 4 per-cent interest, will accumulate to 25 per-cent by the year of death.

On the Valuation of Policies subject to Half-yearly and Quarterly Premiums.

[Being an Extract from a Joint Report by Messrs. THOS. B. SPRAGUE and GEORGE KING.]

[Read before the Institute, 27 Feb. 1882.]

WE will now state, with the necessary technical detail, our reasons for the opinion we have already expressed, that it is not permissible to add to the assets, or otherwise bring into the valuation as a credit, the present value of the extra charge made upon half-yearly and quarterly premiums in excess of the aliquot part of an annual premium. This extra charge is made for three different purposes, (1) to cover the cost and trouble of collecting premiums at frequent intervals; (2) to make good the loss of interest on the portion of the premium which is not paid at the beginning of the year; and (3) when the half-yearly or quarterly payments are really premiums, and not instalments of yearly premiums, to compensate for the loss of premium which will occur in some cases through the life assured dying before the full year's premium for the current policy-year has been paid. The proper amount of the extra to be charged for (2) and (3) is easily determined. Since the value of an annuity payable in advance is $1+a$, $\frac{3}{4}+a$, or $\frac{5}{8}+a$, according as it is payable yearly, half-yearly, or quarterly, the total yearly net premium for the age x is

$$\varpi=\frac{\mathrm{A}}{1+a},\quad \varpi^{(2)}=\frac{\mathrm{A}}{\cdot75+a},\quad \varpi^{(4)}=\frac{\mathrm{A}}{\cdot625+a};$$

according as it is payable yearly, half-yearly, or quarterly. Taking for example the ages 20, 40, 60, we have by the $\mathrm{H^M}$ table at 4 per-cent interest

Age	ϖ	$\varpi^{(2)}$	$\varpi^{(4)}$
20	1·245	1·261	1·269
40	2·352	2·389	2·407
60	5·715	5·855	5·929

These figures give us an average addition of 2·07 per-cent and 3·15 per-cent to the yearly premiums, whereas the additions actually made are 5 and $7\frac{1}{2}$ per-cent. The difference, amounting to 2·93 per-cent for half-yearly, and to 4·35 per-cent for quarterly premiums, is available to meet the extra expense mentioned in (1); and, so far as not required for this purpose, may be considered as direct gain. But there is no certainty that it will ever be received. The assured may discontinue his policy, or have the premium altered to yearly payments, and it would therefore not

be correct to take credit in any way in the valuation for this excess, and it can only be considered as direct gain when it has actually been received. In the same way, it would not be correct to bring into the valuation or treat as an asset, any addition that is made to the half-yearly or quarterly premiums to provide for the loss of interest at the rate actually earned upon the assets of the Society, so far as that rate exceeds the rate employed in the valuation.

There is no objection, however, to valuing the theoretical half-yearly and quarterly premiums as found above; but, if this is done, it will be necessary to value them by the appropriate annuities, and not by the same annuity as is properly employed when the premium is payable yearly. In a former report we gave the following formula* as the proper one to be used in all cases, whether the premium was payable yearly, half-yearly, or quarterly :—

$$A_{x+n+\frac{t}{12}} - (1 + a_{x+n+\frac{t}{12}})\varpi_{x+1} + \frac{m}{12}\varpi_{x+1}; \quad . \quad . \quad . \quad . \quad (A)$$

the age at entry being x and the duration of the policy n years and t months; and the next premium, yearly, half-yearly, or quarterly, being due in m months. This formula was recommended by us on the assumption that it was intended to ascertain the liability under each policy separately. In the valuation as actually made, all whole-life policies of the same age at entry, and of the same duration (as measured by complete quarters of a year), have been grouped together, whether the premium was payable yearly, half-yearly, or quarterly. It thus became necessary to value all the policies in each group by the same formula. It was assumed that in every case a full year's premium was due at the date of valuation; and the liability thus found was increased by $3\frac{1}{2}$ months' net premiums, because (1) the premiums were not all immediately due, and (2) some of them were payable half-yearly and quarterly instead of yearly. The formula (A) leads to the same result. If the premiums on a policy are payable yearly, the next premium may fall due at any time from one day to 12 months; and assuming the premiums to be uniformly distributed over the year, the addition to be made to the liability on this account will, on the average, be half-a-year's premium. In the case of policies subject to half-yearly premiums, the next half-yearly premium

* This formula is based on the supposition that the first year's premium is wholly absorbed by the first year's expenses and risk, so that at the end of the first year no reserve is made for the liability under the policy.

may be due at any time from one day to 6 months, and the average addition to the liability will therefore be 3 months' premium; and, similarly, in the case of quarterly premiums the average addition will be $\frac{1}{8}$th of a year's premium. If now we make the further assumption that policies subject to yearly, half-yearly, and quarterly premiums occur in equal numbers and amounts, then the average addition to be made all round will be $\frac{1}{3}(\frac{1}{2}+\frac{1}{4}+\frac{1}{8})=\frac{7}{24}$ of a year's premium, or $3\frac{1}{2}$ months' premiums.

If, instead of valuing the yearly net premium, ϖ, we bring into the calculation the larger premiums, $\varpi^{(2)}$, $\varpi^{(4)}$, formula (A) will require to be modified. Considering in the first instance, for the sake of simplicity, the case of a policy which has been exactly n years in force, the value when the premium is payable yearly, is

$$\mathrm{V}=\mathrm{A}_{x+n}-(1+a_{x+n})\varpi_{x+1};$$

but when the premiums are payable half-yearly or quarterly, the proper formulas will be

$$\mathrm{V}^{(2)}=\mathrm{A}_{x+n}-(\tfrac{3}{4}+a_{x+n})\varpi^{(2)}{}_{x+1},\quad \mathrm{V}^{(4)}=\mathrm{A}_{x+n}-(\tfrac{5}{8}+a_{x+n})\varpi^{(4)}{}_{x+1}.$$

The values as found by these latter formulas differ so little from the first, that it is usual to employ that formula in all cases; and this was the course, as already mentioned, that we formerly recommended. The values, however, are not exactly equal, and it is easy to see that the policy-value given by the second is greater than that given by the first, and the value given by the third greater than that given by the second. We have

$$\varpi_{x+1}=\frac{\mathrm{A}_{x+1}}{1+a_{x+1}},\quad \varpi^{(2)}{}_{x+1}=\frac{\mathrm{A}_{x+1}}{\frac{3}{4}+a_{x+1}};$$

$$\therefore \qquad (1+a_{x+n})\varpi_{x+1}=\frac{1+a_{x+n}}{1+a_{x+1}}\mathrm{A}_{x+1},$$

and $$(\tfrac{3}{4}+a_{x+n})\varpi^{(2)}{}_{x+1}=\frac{\frac{3}{4}+a_{x+n}}{\frac{3}{4}+a_{x+1}}\mathrm{A}_{x+1}.$$

Since the fractions multiplying A_{x+1} are both less than unity, and the numerator and denominator of the second are obtained by deducting the same quantity ($\frac{1}{4}$) from the numerator and denominator of the first, it follows from a well-known algebraical proposition, that the first fraction is greater than the second;

$$\therefore \quad (1+a_{x+n})\varpi_{x+1}>(\tfrac{3}{4}+a_{x+n})\varpi^{(2)}{}_{x+1};$$

and it follows that $\mathrm{V}^{(2)}>\mathrm{V}$. In the same way it may be shown that $\mathrm{V}^{(4)}>\mathrm{V}^{(2)}$.

We have thus proved that, when a policy has been in force for an exact number of years, the effect of bringing into the calculation the theoretically correct half-yearly and quarterly net premiums, is to increase the policy-values. A general idea of the amount of this increase (which is but small) may be formed from a consideration of the figures in the following table, which contains the values of the various quantities for ages at entry 20, 40, 60, n being 5.

x	A_{x+5}	$(1+a_{x+5})\varpi_{x+1}$	$(\frac{3}{4}+a_{x+5})\varpi^{(2)}{}_{x+1}$	$(\frac{5}{8}+a_{x+5})\varpi^{(4)}{}_{x+1}$	V	$V^{(2)}$	$V^{(4)}$
20	27·074	24·243	24·234	24·230	2·831	2·840	2·844
40	42·690	36·380	36·342	36·321	6·310	6·348	6·369
60	65·883	53·377	53·184	53·084	12·507	12·699	12·799

Passing on now to the more general case when the policy has not been an exact number of years in force, it is convenient to make the usual supposition that the policies have been on the average $n\frac{1}{2}$ years in force and that on the average the next yearly premium is due in 6 months' time, the next half-yearly premium in 3 months, and the next quarterly premium in 6 weeks. Then the formulas for the policy-values according to the principle actually adopted in the valuation will be

$$V = A_{x+n\frac{1}{2}} - (\cdot 5 + a_{x+n\frac{1}{2}})\varpi_{x+1} \text{ for yearly premiums.}$$
$$V^{(2)} = A_{x+n\frac{1}{2}} - (\cdot 75 + a_{x+n\frac{1}{2}})\varpi_{x+1} \text{ ,, half-yearly ,,}$$
$$V^{(4)} = A_{x+n\frac{1}{2}} - (\cdot 875 + a_{x+n\frac{1}{2}})\varpi_{x+1} \text{ ,, quarterly ,,}$$

But when we bring into the valuation the correct half-yearly and quarterly premiums, the proper formulas for the values of these premiums will be

$$(\cdot 25 + a_{x+n\frac{1}{2}} + \cdot 25)\varpi^{(2)}{}_{x+1} = (\cdot 5 + a_{x+n\frac{1}{2}})\varpi^{(2)}{}_{x+1}$$
$$(\cdot 375 + a_{x+n\frac{1}{2}} + \cdot 125)\varpi^{(4)}{}_{x+1} = (\cdot 5 + a_{x+n\frac{1}{2}})\varpi^{(4)}{}_{x+1}$$

and for the policy-values

$$U^{(2)} = A_{x+n\frac{1}{2}} - (\cdot 5 + a_{x+n\frac{1}{2}})\varpi^{(2)}{}_{x+1}, \quad U^{(4)} = A_{x+n\frac{1}{2}} - (\cdot 5 + a_{x+n\frac{1}{2}})\varpi^{(4)}{}_{x+1};$$

and we can show that $U^{(2)} > V^{(2)}$, $U^{(4)} > V^{(4)}$.

We have $$(\cdot 75 + a_{x+n\frac{1}{2}})\varpi_{x+1} = \frac{\cdot 75 + a_{x+n\frac{1}{2}}}{1 + a_{x+1}} A_{x+1},$$

and $$(\cdot 5 + a_{x+n\frac{1}{2}})\varpi^{(2)}{}_{x+1} = \frac{\cdot 5 + a_{x+n\frac{1}{2}}}{\cdot 75 + a_{x+1}} A_{x+1}.$$

Here the multipliers of A_{x+1} are both fractions less than unity, and the numerator and denominator of the latter fraction are obtained from those of the former by deducting the same quantity, ·25, from each. Therefore the second multiplier is less than the first, and $(\cdot 5+a)\varpi^{(2)}<(\cdot 75+a)\varpi$; and it follows that $U^{(2)}>V^{(2)}$. In the same way it may be proved that $U^{(4)}>V^{(4)}$; or in these cases, also, the effect of valuing the correct half-yearly and quarterly premiums, is to increase the policy-values.

The following table shows the values of the different quantities, by the H^M 4 per-cent tables, for policies which have been in force on the average $4\frac{1}{2}$ years, and were taken out at ages 20, 40, 60.

Half-yearly Premiums.

x	$A_{x+4\frac{1}{2}}$	$(\cdot 75+a_{x+4\frac{1}{2}})\varpi_{x+1}$	$(\cdot 5+a_{x+4\frac{1}{2}})\varpi^{(2)}{}_{x+1}$	$V^{(2)}$	$U^{(2)}$
20	26·786	24·019	24·007	2·767	2·779
40	42·188	36·090	36·046	6·098	6·142
60	65·279	52·818	52·611	12·461	12·668

Quarterly Premiums.

x	$A_{x+4\frac{1}{2}}$	$(\cdot 875+a_{x+4\frac{1}{2}})\varpi_{x+1}$	$(\cdot 5+a_{x+4\frac{1}{2}})\varpi^{(4)}{}_{x+1}$	$V^{(4)}$	$U^{(4)}$
20	26·786	24·179	24·164	2·607	2·622
40	42·188	36·395	36·336	5·793	5·852
60	65·279	53·570	53·284	11·709	11·995

It appears, therefore, contrary to what might have been anticipated, that if the larger half-yearly and quarterly premiums are brought into the valuation, the effect is not to diminish the liability, as might be supposed, but to increase it slightly.

Discussion.

The President (Mr. A. H. Bailey) said that the question considered in the paper was important, as quarterly premiums are on the increase. Half-yearly or quarterly premiums are (or ought to be) the equivalents to annual premiums; and therefore it is sufficient for practical purposes in a pure-premium valuation to substitute the corresponding annual premium. That was sufficiently accurate, and was the course he had always adopted himself.

Mr. Sutton said that bearing in mind the statement made that the company to which this report relates, makes a high rate of

interest upon its investments, he would have expected to see some reference made to the other end of the transaction, namely, to the loss of interest arising from the payment of the claim prior to the end of the year in which death takes place. As is well known, in using A, the assumption is made practically that payment of the claim will be made on the average half a year after death. There is, however, no actual necessity to take a year as the interval of time in these matters, and for consistency sake he should rather be inclined to the use of the general formula $A^{(m)}=P^{(m)}\mathbf{a}^{(m)}$. Theoretically, at all events, this would be better, and in the particular case before us it would apparently be practically better to use a formula which should give effect to the actual conditions of the policy as to payment of claim. The formulas used in this paper are admittedly only approximate, the correction applied being the same for all ages and rates of interest. He would have liked to see used the correction given by Mr. Woolhouse, which involves the age and rate of interest, namely,

$$a^{(m)}=a+\frac{m-1}{2m}-\frac{m^2-1}{12m^2}(\mu+\delta)\,.$$

If the Institute tables were again to be published, he would like to see the value of this correction tabulated so that it could be immediately used. On reading this paper, it occurred to him that the question raised had been long ago disposed of by Mr. Woolhouse in his paper "On an Improved Theory of Annuities and Assurances", in the *Journal*, vol. xv. In that most valuable paper Mr. Woolhouse has gone thoroughly into the matter, and arrives at the result that, whether the current premiums for valuation are payable yearly, half-yearly, quarterly, &c., the proper annuity-value to use is $\bar{a}$, that is, a continuous annuity, whose value is known to be approximately

$$a+\tfrac{1}{2}-\tfrac{1}{12}(\mu+\delta),\text{ or say }a+\tfrac{1}{2}.$$

On page 111 he says, ". . . . . We have only to further enunciate the principle here advanced, that the average present value of the current premiums of an assurance company at a given present age, the same being due at promiscuous periods after the date of valuation, is the same as that of a current annuity under similar circumstances, and in years' purchase is equal to the present value of a continuous annuity commencing immediately at the present age, and that this is accurately and rigidly true, whether the premiums be payable yearly or by instalments."

Mr. J. W. Gordon said that, remembering the third purpose which the additional premium paid half-yearly or quarterly serves, namely, that of compensating for the "loss of premium when the payments are really premiums and not instalments of yearly premiums", it can readily be seen why the result which has been brought out in the paper should arise. Translated from mathematical into colloquial language, it may be expressed in some such way as this. In determining the net quarterly premium, which at the age x will be equivalent to a given annual premium, we have to consider what is the mortality *within a year;* and as the term over which the premium is to be paid, will generally comprise many years, and does comprise

the whole term of life in the case under consideration; it is clear that the ratio of the quarterly to the annual premium will depend, not on the mortality in any particular year (q_x), but on the mean value of this function for the whole period. Now, starting from age 20, or any later age, mortality gets heavier as we proceed. Therefore, a larger provision must be made for proportions of premiums that will be lost in the uncompleted portions of the years of death; and it is only another way of saying this, to say that at the age $x+5$ the net quarterly premium which will be equivalent to a given annual premium, is greater than the net quarterly premium which will be equivalent to the same annual payment at the earlier age, x.

Mr. R. P. HARDY would have been glad to see the question considered with reference to those cases where a short annuity is required. There are some valuations in which those contracts are in very large and sometimes almost unmanageable numbers. He thought it desirable that points of this description, when they arise in practice, should be communicated to the Institute.

Mr. G. H. RYAN enquired whether in practice it is not more usual to charge what are generally called "instalment" premiums, than the theoretical half-yearly or quarterly premiums. The latter assume that the deaths in the year are equally spread over the year, and that it is an equal probability whether the assured will die in the first, the second, the third, or the fourth quarter. But this is an assumption which statistics do not confirm, for on referring to the Registrar-General's report for 1879, it will be seen that the mortality for the first three months of the year was ·62 per-cent; for the second three months ·52 per-cent; for the third three months ·41 per-cent; and for the fourth three months ·53 per-cent. That is to say, the mortality of the first quarter exceeds that of the second by about 20 per-cent; that of the third by about 50 per-cent; and that of the fourth by about 17 per-cent. In view of the heavy mortality of the first quarter shown by these figures, he thought it would not be quite correct or wise to adopt the theoretical half-yearly or quarterly premiums. But by charging the "instalment" premiums we should avoid the loss of the premium which the larger number of deaths in the early portion of the year would occasion. It may be a question for future consideration whether this heavy mortality in the first three months of the year would not require some small additional reserve in the valuation of annual policies.

The PRESIDENT—The assumption of a uniform distribution of deaths runs through all our calculations, and if we refined to the extent of allowing any weight to the undoubted fact that the mortality in the winter, or, say, in the first and fourth quarters of the year, is greater than in the summer, or in the second or third quarters, it might involve possibly the asking for two half-yearly premiums of unequal amount.

Mr. G. KING explained that the small paper presented by Mr. Sprague and himself was simply a fragment. It did not purport to be an exhaustive treatise on the subject, but was merely an extract from a business report, that had been submitted to the Institute because it deals with certain matters that are of practical importance.

It would have been interesting to discuss the subject from the point of view of limited premiums, as suggested by Mr. Hardy; but that had not been done in the report from which the paper was a verbatim extract; and the point was therefore necessarily left unnoticed in the paper. Mr. Sutton had suggested that, in dealing with half-yearly and quarterly premiums, we ought to include in the formulas the assurance payable at the end of the interval, instead of at the end of the year; but since we do not alter the date of the payment of a claim because we alter the mode of payment of the premium, it would not have been correct so to modify the formulas. The A that appears in the formulas may be taken to represent the value of the sum assured paid at the end of the year of death, or three months after proof of death, or at the moment of death—it does not in the least degree matter which; but whether the premium be paid yearly, half-yearly, or quarterly, the same value must be assigned to A throughout. He did not know what is the most common practice in dealing with half-yearly and quarterly premiums. With many of the old companies it is not a matter of any practical importance, because the majority of the premiums are receivable yearly, and the comparatively small number that are receivable otherwise will not affect the results. But certainly in some quarters—in the colonies, for instance—it is very common indeed to do a large amount of business at half-yearly and quarterly premiums—in some companies two-thirds of the whole business,—and, consequently it becomes a matter of importance to consider the question in connection with the valuation. The practical result of the paper is this—that if we take the ordinary annual premium P, and multiply it by the suitable annuity, that is to say $(\frac{1}{2}+a)$ for yearly premiums, and $(\frac{3}{4}+a)$ for half-yearly, and $(\frac{7}{8}+a)$ for quarterly, and value each policy separately, we get a result which is very close indeed to the result of multiplying the full premium, P, $P^{(2)}$, or $P^{(4)}$, as the case may be, in all cases by $(\frac{1}{2}+a)$, the reserve being only very slightly less. The practice of taking the yearly premium P in all cases, and valuing by $(\frac{1}{2}+a)$, is practically a very proper method to pursue where there is not a very large number of half-yearly and quarterly cases; but the effect of it is slightly to overstate the reserve necessary. We can look at the matter from a different point of view from that mentioned in the paper, that is to say, retrospectively. If we take the formula $(\frac{1}{2}+a)$, that practically assumes that a whole year's premium was received six months ago; whereas, in the case of a half-yearly policy, only half a premium was received six months ago, and it is an even chance whether the other half has been received or not. Therefore, on the average, we have received three quarters of a year's premium, and we can very fairly add to the value of our premiums one quarter of the premium income under half-yearly premium policies, and consequently reduce our liability to the same amount. Similarly we can add to the value of the premiums five-eighths of the premium income under quarterly premium policies. Where, however, the number of policies on half-yearly and quarterly premiums is not very great, that correction is really unnecessary; and if we omit it, we are on the safe side by somewhat overstating our liability. That is the course pursued by most of the oldest and

strongest companies, while other companies, quite correctly from a theoretical point of view, and quite safely from a practical point of view, insert the equivalent annual premium $P^{(2)}$ or $P^{(4)}$. As regards the unequal distribution of deaths over the year, we do not refer to the calendar year, but to the year of life; and consequently it is not a matter of the slightest consequence to us how the deaths happen in the calendar year. We must look at the year of life.

NOTE BY MR. SPRAGUE.

Having regard to the fact that the President thinks the paper goes into unnecessary refinements, while other speakers wish further refinements introduced, it is perhaps fair to conclude that, in the practical method of valuation suggested by Mr. King and myself, the happy mean has been attained. The common method of valuation in London is to assume that the next yearly premium will, on the average, fall due in six months' time, and to add ·5 to the tabular annuity. If this method is pursued, the proper course in the case of half-yearly and quarterly premiums is, not to substitute the equivalent yearly premium, but to value the theoretical half-yearly and quarterly premiums; and if we value the equivalent yearly premium, we get too large a reserve. It is, however, well known that the assumption that the premiums on the average fall due in six months' time from the valuation, is at variance with the facts; and it would therefore not be suitable to value the premiums by the continuous annuity, as suggested by Mr. Sutton. Some offices introduce a correction by assuming that the next yearly premium falls due, not in six months' time on an average, but in seven or eight months' time. An assumption of this sort may answer sufficiently well when almost all the premiums are payable yearly, but will, I think, be found quite unsuitable when there are a great many half-yearly and quarterly premiums. In my opinion, a better plan is to value all the policies as if the next premium were immediately due, and to make for each policy an additional reserve equal to the proportion of the premium for the time that will elapse before the next premium falls due. This is the principle that was adopted in the report by Mr. King and myself. This plan being followed, it will greatly facilitate the valuation if the same annuity, $1+a_{x+n}$, can be used whether the premiums are payable yearly, half-yearly, or quarterly; and in the preceding paper it is shown that the error resulting from this course is very trifling. I have since carried the investigation a step further, and obtained a remarkably neat formula for the amount of the error. Assuming ϖ_x to be the annual net premium valued, and the age at valuation to be $x+n$, then the value of a policy on which the premium is payable yearly is $A_{x+n}-(1+a_{x+n})\varpi_x$. If the premium is payable half-yearly, the value will be $A_{x+n}-(\cdot75+a_{x+n})\varpi_x^{(2)}$. The difference between these values is

$$(1+a_{x+n})\varpi_x-(\cdot75+a_{x+n})\varpi_x^{(2)}=(1+a_{x+n})\varpi_x-\frac{\cdot75+a_{x+n}}{\cdot75+a_x}(1+a_x)\varpi_x$$
$$=\mathrm{A}_x\left(\frac{1+a_{x+n}}{1+a_x}-\frac{\cdot75+a_{x+n}}{\cdot75+a_x}\right)$$
$$=\frac{\mathrm{A}_x}{4}\cdot\frac{a_x-a_{x+n}}{(1+a_x)(\cdot75+a_x)}$$
$$=\tfrac{1}{4}\varpi_x^{(2)}\cdot{}_n\mathrm{V}_x\,;$$

that is to say, the reserve when the theoretical half-yearly premium is valued, is greater than the reserve when the yearly premium is valued, by a sum equal to the value of a policy for a half of one half-yearly premium. This result is obviously correct. In accepting half-yearly premiums we run the risk, as compared with the yearly payments, of losing the second half-yearly premium in the year of death; or, on the usual assumption of a uniform distribution of deaths, we lose one half-yearly premium in every other case; that is to say, on the average we lose half a half-year's premium in every case, and we require to have in hand a sufficient sum to compensate for this loss.

I take this opportunity of publishing another neat result which I have lately arrived at. If x is the age at entry, and we value the net premium for age $x+1$, thus assuming that the first year's premium will be absorbed by the first year's expenses and risk, in accordance with the principle which I have on various occasions recommended, the result of this will be that in the first insurance year we make a reserve for the policy exactly equal to the risk of death in the unexpired portion of the year. First, take the case of a policy just effected; then the value, on the principle indicated, will be

$$\mathrm{A}_x-a_x\varpi_{x+1}=v-(1-v)a_x-a_x\,\varpi_{x+1}$$
$$=v-a_x(1-v+\varpi_{x+1})$$
$$=v-\frac{a_x}{1+a_{x+1}}$$
$$=v-vp_x=vq_x\,,$$

which is the single premium for a term insurance of one year. Next suppose that a portion of the first year, represented by the fraction t, has elapsed; then the value of the policy will be

$$\mathrm{A}_{x+t}-{}_{1-t|}\mathbf{a}_{x+t}\cdot\varpi_{x+1}\,.$$

Now $\quad \mathrm{A}_{x+t}=(1-t)\mathrm{A}_x+t\,\mathrm{A}_{x+1}$, approximately,

and $\quad {}_{1-t|}\mathbf{a}_{x+t}=(1-t)a_x+t(1+a_{x+1})\,;$

whence $\quad {}_{1-t|}\mathbf{a}_{x+t}\cdot\varpi_{x+1}=(1-t)a_x\varpi_{x+1}+t\,\mathrm{A}_{x+1},$

and $\quad \mathrm{A}_{x+t}-{}_{1-t|}\mathbf{a}_{x+t}\cdot\varpi_{x+1}=(1-t)(\mathrm{A}_x-a_x\varpi_{x+1})$
$$=(1-t)vq_x\,,$$

which is the proportionate part of the year's risk for the unexpired time, $1-t$.

On the Rate of Interest in Annuities-Certain. By George F. Hardy, F.I.A.

[Read before the Institute, 27 Feb. 1882.]

The problem of finding the rate of interest corresponding to a given annuity-value, has engaged considerable attention at various times; and many formulas have been proposed for obtaining approximate solutions by the aid of existing tables of annuities. Most of these will be found in Mr. Sutton's paper on the *Rate of Interest yielded by Foreign Loans* (*J.I.A.*, xix, 77). Of the 5 formulas there given, (2) is probably the most generally useful, (3), (4), and especially (5) (Prof. De Morgan's formula), involving a good deal of calculation. The object of the present short paper is to suggest 2 new formulas, which give very accurate results, and at the same time require but little work in their practical application.

Let a be the value of an annuity for n years certain, at a rate of interest i, (which it is required to find), and let a_1, a_2, a_3, be the tabulated annuity-values for the same term at the rates i_1, i_1+h, i_1+2h, where a lies between a_1 and a_2, and consequently i between i_1 and i_1+h. Put $i=i_1+\rho$; then if Δ, Δ^2, represent the successive differences of a_1, a_2, &c., we shall have approximately

$$\rho=h\left\{\frac{\Delta-\frac{1}{2}\Delta^2}{a-a_1}+\frac{\frac{1}{2}\Delta^2}{\Delta-\frac{1}{2}\Delta^2}\right\}^{-1} \quad . \quad . \quad . \quad . \quad (1)$$

This formula is obtained as follows:—

We have, stopping at second differences,

$$a=a_1+\frac{\rho}{h}(\Delta-\tfrac{1}{2}\Delta^2)+\frac{\rho^2}{h^2}\frac{\Delta^2}{2} \quad . \quad . \quad . \quad . \quad (2)$$

$$=a_1+\frac{\rho}{h}(\Delta-\tfrac{1}{2}\Delta^2)\left[1+\frac{\rho}{h}\frac{\frac{1}{2}\Delta^2}{\Delta-\frac{1}{2}\Delta^2}\right] \quad . \quad . \quad . \quad . \quad (3)$$

As the quantity $\frac{\rho}{h}\frac{\frac{1}{2}\Delta^2}{\Delta-\frac{1}{2}\Delta^2}$ is a small fraction, we may write approximately, for the expression in the brackets [ ], $\left(1-\frac{\rho}{h}\frac{\frac{1}{2}\Delta^2}{\Delta-\frac{1}{2}\Delta^2}\right)^{-1}$; that is,

$$a=a_1+\frac{\frac{\rho}{h}(\Delta-\frac{1}{2}\Delta^2)}{1-\frac{\rho}{h}\frac{\frac{1}{2}\Delta^2}{\Delta-\frac{1}{2}\Delta^2}} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

This last expression is, in fact, more exact than (2), since, in expanding (4), new terms appear, which approximately equal those suppressed in (2).* From (4) we get

$$(a-a_1)\left(1-\frac{\rho}{h}\,\frac{\tfrac{1}{2}\Delta^2}{\Delta-\tfrac{1}{2}\Delta^2}\right)=\frac{\rho}{h}(\Delta-\tfrac{1}{2}\Delta^2)\,;$$

whence

$$\frac{\rho}{h}=\frac{a-a_1}{\Delta-\tfrac{1}{2}\Delta^2+\dfrac{(a-a_1)\tfrac{1}{2}\Delta^2}{\Delta-\tfrac{1}{2}\Delta^2}}\quad .\ .\ .\ .\ (5)$$

$$=\left\{\frac{\Delta-\tfrac{1}{2}\Delta^2}{a-a_1}+\frac{\tfrac{1}{2}\Delta^2}{\Delta-\tfrac{1}{2}\Delta^2}\right\}^{-1}\quad .\ .\ .\ (6)$$

from which we at once get (1), which is substituted for (5) as being a more convenient working formula.

We might have obtained (1) as follows: neglecting the 2nd power of $\frac{\rho}{h}$ in (2), we should have approximately

$$\frac{\rho}{h}=\frac{a-a_1}{\Delta-\tfrac{1}{2}\Delta^2}\,;\qquad\therefore\quad\frac{\rho^2}{h^2}\,\frac{\Delta^2}{2}=\frac{\rho}{h}\left(\frac{a-a_1}{\Delta-\tfrac{1}{2}\Delta^2}\right)\frac{\Delta^2}{2}.$$

Substituting this value in (2), we have

$$a-a_1=\frac{\rho}{h}\left\{\Delta-\tfrac{1}{2}\Delta^2+\frac{a-a_1}{\Delta-\tfrac{1}{2}\Delta^2}\,\frac{\Delta^2}{2}\right\},$$

which agrees with (5), and therefore with (1).

* The suppressed terms in (2) are $\frac{\rho}{h}\left(\frac{\rho}{h}-1\right)\left(\frac{\rho}{h}-2\right)\frac{\Delta^3}{6}$ + smaller terms; the new terms in (4) are $\left(\frac{\rho}{h}\right)^2\frac{(\Delta^2)^2}{4(\Delta-\frac{1}{2}\Delta^2)}$ + smaller terms. Considering only the leading terms, their average values (found by integrating with respect to $\frac{\rho}{h}$, since this fraction may have any value from 0 to 1) will be $\frac{\Delta^3}{18}$ and $\frac{(\Delta^2)^2}{12(\Delta-\frac{1}{2}\Delta^2)}$ respectively. Now the quantities $\frac{\Delta^3}{h^3}$, $\frac{\Delta^2}{h^2}$, $\frac{\Delta-\frac{1}{2}\Delta^2}{h}$, are very nearly equal to the first three differential coefficients of a, that is, of $\frac{1-(1+x)^{-n}}{x}$. If, therefore, we develop a in ascending powers of x and differentiate, we shall find $(\Delta-\frac{1}{2}\Delta^2)$ $=-h\frac{n+1}{2}\left(1-2\frac{(n+2)}{3}x+\text{\&c.}\right)$; $\Delta^2=h^2\frac{(n+1)(n+2)}{3}\left(1-3\frac{(n+3)}{4}x+\text{\&c.}\right)$; $\Delta^3=-h^3\frac{(n+1)\ldots(n+3)}{4}\left(1-4\frac{(n+4)}{5}x+\text{\&c.}\right)$; from which we find $\frac{\Delta^3}{18}=-\frac{h^3(n+1)(n+2)(n+3)}{72}\left(1-\frac{4n+16}{5}x+\text{\&c.}\right)$ and $\frac{(\Delta^2)^2}{12(\Delta-\frac{1}{2}\Delta^2)}$ $=-\frac{h^3(n+1)(n+2)(n+2)}{54}\left(1-\frac{5n+19}{6}x+\text{\&c.}\right)$; the latter quantity being approximately equal to $\frac{4}{3}$ of the former. Hence we may say that, roughly speaking, the new terms in (4) supply the place of those rejected in (2).

It will be sufficient to give one or two examples to show the accuracy of the formula in practice. Where great exactness is required, it will be better to operate with the logs or with the reciprocals of the annuity-values, as the differences with these functions diminish more rapidly; but with Rance's Tables, where the values are tabulated for every $\frac{1}{4}$ per-cent, we obtain results sufficiently exact for all practical purposes. In practice, in fact, (1) may be worked out with sufficient accuracy by Crelle's Tables, as an example of which process we may take the following:—

A. An annuity-certain for 30 years is bought for 19 years' purchase; what rate of interest is made on the investment?

Here $$a=19{\cdot}0000;\ i={\cdot}03+\rho;\ h={\cdot}0025;$$

		Δ	Δ^2
at 3 per-cent	$a_1=19{\cdot}6004$	$-{\cdot}6185$	$+{\cdot}0286$
,, $3\frac{1}{4}$,,	$a_2=18{\cdot}9819$	$-{\cdot}5899$	
,, $3\frac{1}{2}$,,	$a_3=18{\cdot}3920$		

$$a-a_1=-{\cdot}6004;\ \tfrac{1}{2}\Delta^2={\cdot}0143;\ \Delta-\tfrac{1}{2}\Delta^2=-{\cdot}6328;$$

$$\therefore\quad \rho={\cdot}0025\left(\frac{6328}{6004}-\frac{143}{6328}\right)^{-1}$$

$$={\cdot}0025(1{\cdot}054-{\cdot}023)^{-1}$$

$$=\frac{{\cdot}0025}{1{\cdot}031}={\cdot}002425;$$

whence $$i={\cdot}032425,$$

which is correct to the last figure. All the above working was done by "Crelle", and every figure used is put down.

B. As another example, we will take that discussed by Mr. Baden in his remarks upon Mr. Sutton's paper. Given $a=11{\cdot}99051$; term $=30$ years; to find i.

Taking the values of the annuities at $7\frac{1}{4}$, $7\frac{1}{2}$, and $7\frac{3}{4}$ per-cent, from "Rance", we find

$a=11{\cdot}99051$	Δ	Δ^2
$a_1=12{\cdot}10366$	$-{\cdot}29327$	$+{\cdot}01149$
$a_2=11{\cdot}81039$	$-{\cdot}28178$	
$a_3=11{\cdot}52861$		

$$h={\cdot}0025;\ a-a_1=-{\cdot}11315;\ \tfrac{1}{2}\Delta^2={\cdot}00575;\ \Delta-\tfrac{1}{2}\Delta^2=-{\cdot}29902;$$

$$\rho={\cdot}0025\left\{\frac{29902}{11315}-\frac{575}{29902}\right\}^{-1}.$$

11315)29902	2·6426	29902)575	·0192	2·6426	
22630		299		– ·0192	
7272		276		2·6234)·002500	·000953 = ρ
6789		269		2361	
483		7		139	
453				131	
30				8	
23					
7					

$$i_1 = \cdot 072500$$
$$\rho = \cdot 000953$$
$$\therefore \quad i = \cdot 073453,$$

which, again, is correct to the last figure, the true value of i being ·07345253. In this case the whole of the working is set out in full, but the result might have been obtained with equal accuracy by "Crelle".

If no tables are available which give annuity-values for every quarter per-cent, we get equally good results by working with the logs or reciprocals at intervals of $\frac{1}{2}$ per-cent. Thus, taking example A, with the values of $\frac{1}{a}$ given in Chisholm's "Commutation Tables", for 3, $3\frac{1}{2}$, and 4 per-cent, we have

$$\frac{1}{a} = \frac{1}{19} = \cdot 0526316$$

		Δ	Δ^2
$\frac{1}{a_1}$	$= \cdot 0510193$	$+ \cdot 0033520$	$+ \cdot 0001068$
$\frac{1}{a_2}$	$= \cdot 0543713$	$\cdot 0034588$	
$\frac{1}{a_3}$	$= \cdot 0578301$		

$h = \cdot 005$; $\frac{1}{a} - \frac{1}{a_1} = \cdot 0016123$; $\frac{1}{2}\Delta^2 = \cdot 0000534$; $\Delta - \frac{1}{2}\Delta^2 = \cdot 003296$; whence

$$\rho = \cdot 005 \left\{ \frac{32986}{16123} + \frac{534}{32986} \right\}^{-1}$$
$$= \cdot 005(2\cdot 0621)^{-1} = \cdot 0024248$$
$$i_1 = \cdot 0300000$$
$$\therefore \quad i = \cdot 0324248,$$

which result differs from the truth by 4 in the 7th place. If we had worked with the logs instead of the reciprocals of the annuity-values, our result would have been in error by only 1 in the 7th place of decimals, or about $\frac{1}{100}$th of a penny per-cent. Even with intervals of 1 per-cent in the tabulated values, this method gives

fair results. Thus in our example B, when the reciprocals are used for intervals of 1 per-cent, $i = {\cdot}073450$, or by logs, $i = {\cdot}073455$, which results are respectively less and greater than the truth by the quantity ·0000025; that is, there would be an error in the rate per-cent of $\frac{1}{16}$th of a penny.

As a final test, we may apply the formula to the example quoted by Mr. Sutton from De Morgan, namely, $a = 9{\cdot}5233704$; term $= 100$ years; assuming that we have only the values tabulated to 10 per-cent, we get (taking the values at 8, 9, and 10 per-cent, and working with the reciprocals), $i = {\cdot}10499992$, a result almost identical with that brought out by Prof. De Morgan's very elaborate formula.

It will be seen that the greatest error in any of the examples discussed does not amount to more than a fraction of a farthing per-cent, an exactness probably quite sufficient for all ordinary requirements. The formula may obviously be employed in the solution of any similar inverse problem; for example, to find the rate of interest returned to the purchaser of a Debenture Bond. In this case we have to solve the equation

$$i_1 - i - \frac{\rho}{a} = 0,$$

where i_1 is the nominal rate of interest, i the true rate, ρ the premium paid for the bond (a negative quantity if bought at a discount), and a the value of an annuity for the given term of n years at the rate i. Thus, let $i_1 = {\cdot}06$; $\rho = {\cdot}075$; $n = 20$ (the number of years at the end of which the bond is redeemable at par).

We have,

$$\text{at } 4\% \ (i_1 - i) - \frac{\rho}{a} = {\cdot}02 - {\cdot}075\ ({\cdot}07358175) = +{\cdot}01448137 = u_1$$

$$,,\ 5\% \qquad = {\cdot}01 - {\cdot}075\ ({\cdot}08024259) = +{\cdot}00398181 = u_2$$

$$,,\ 6\% \qquad = \quad -{\cdot}075\ ({\cdot}08718456) = -{\cdot}00653884 = u_3$$

Here $i = {\cdot}04 + \rho$; $u - u_1 = -{\cdot}01448137$; $\frac{1}{2}\Delta^2 = -{\cdot}00001055$; $\Delta - \frac{1}{2}\Delta^2 = -{\cdot}01048901$;

$$\therefore \quad \rho = {\cdot}01 \left(\frac{1048901}{1448137} + \frac{1055}{1048901}\right)^{-1}$$

$$= {\cdot}01378710;$$

whence $\qquad i = {\cdot}05378710$, a result which differs from the truth by unity in the 8th place.

Generally speaking, however, it is sufficient in these problems to use the ordinary interpolation by first differences; thus we get, in the above example, from u_2 and u_3,

$$i = \cdot 053785,$$

which is true to the 5th place. With rates at intervals of a quarter per-cent, the results by this latter method are, perhaps, as exact as can be required for any purpose. Thus, taking the values of $\frac{1}{a}$ for $5\frac{1}{4}$ and $5\frac{1}{2}$ per-cent, in the above example we find $i = \cdot 05378690$, which though not so exact as the result given by the formula (1), yet differs by only $\frac{1}{200}$th of a penny per-cent from the true rate.

I will now add another formula which gives on the whole better results than the preceding, especially when working with the values of the annuities themselves, or their logs. If we assume that the successive differences of a_1, a_2, &c., form a geometrical progression (which will be found to be approximately true), we shall have

$$a = a_1 + \frac{\rho}{h}\Delta + \frac{\rho}{h}\left(\frac{\rho}{h} - 1\right)\frac{\Delta^2}{2} + \frac{\rho}{h}\left(\frac{\rho}{h} - 1\right)\left(\frac{\rho}{h} - 2\right)\frac{\Delta^3}{6} + \&c. \quad . \quad . \quad . \quad (6)$$

(putting $\frac{\Delta^2}{\Delta} = \kappa$)

$$= a_1 + \frac{\rho}{h}\Delta + \frac{\rho}{h}\left(\frac{\rho}{h} - 1\right)\frac{\kappa\Delta}{2} + \frac{\rho}{h}\left(\frac{\rho}{h} - 1\right)\left(\frac{\rho}{h} - 2\right)\frac{\kappa\Delta^2}{6} + \&c. \quad . \quad . \quad (7)$$

$$= a_1 + \frac{(1+\kappa)^{\frac{\rho}{h}} - 1}{\kappa}\Delta \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

$$\therefore \quad \frac{(a - a_1)\kappa}{\Delta} = (1+\kappa)^{\frac{\rho}{h}} - 1\,;$$

whence

$$\frac{\rho}{h} = \frac{\log\left(\frac{a - a_1}{\Delta}\kappa + 1\right)}{\log(1+\kappa)}$$

$$= \frac{\log\left\{\frac{a - a_1}{(\Delta)^2}\Delta^2 + 1\right\}}{\log\left(\frac{\Delta^2}{\Delta} + 1\right)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Substitute for the logarithms the approximate values $\log(1+z) = \frac{2z}{2+z}$, and the above expression will be found to reduce to the following:—

$$\rho = h \cdot \frac{\Delta + \frac{1}{2}\Delta^2}{\frac{(\Delta)^2}{a - a_1} + \frac{1}{2}\Delta^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

where Δ, Δ^2, a, a_1, ρ, have the same meanings as in formula (1). If we apply (10) to example A, we get (taking the values to an extra figure)

$$\Delta = -\cdot 61852, \ \tfrac{1}{2}\Delta^2 = \cdot 01432$$

$$a - a_1 = -\cdot 60044;$$

whence $$\rho = \cdot 0025 \left\{ \frac{\cdot 60420}{\frac{(\cdot 61852)^2}{\cdot 60044} - \cdot 01432} \right\}$$

$\log \cdot 61852 = 9\cdot 79135$

$\log (\cdot 61852)^2 = 9\cdot 58271$

$\log \cdot 60044 = 9\cdot 77847$

$9\cdot 80424 = \log \ \cdot 63715$

$- \quad 1432$

$\cdot 62283$

$\log \cdot 60420 = 9\cdot 78118$

$-\log \cdot 62283 = 9\cdot 79437$

$9\cdot 98681$

$+\log \cdot 0025 = 7\cdot 39794$

$\log \cdot 0024252 = 7\cdot 38475$

$\therefore \quad \rho = \cdot 0024252$

and $\quad i = \cdot 0324252$

which is true to the last figure.

In the same way we get for the value of i, in example B, $i = \cdot 0734528$, with an error of 3 in the last place.

It may not be generally known that Inwood's Tables, published by Lockwood and Co., give the cologs of the values of annuities-certain for all durations to 100 years, for every $\frac{1}{8}$th per-cent below 6 per-cent, and for other intervals, up to 12 per-cent. From the closeness of the intervals, the results given by ordinary interpolation by 1st differences are very exact. The book is, I believe, published at 8*s*.

DISCUSSION.

The PRESIDENT (Mr. A. H. Bailey) remarked that the rate of interest in annuities-certain is a question of importance. He had not had time to read Mr. Hardy's paper carefully, and would therefore only say that Mr. Hardy did not seem to know Colonel Oakes's tables, which give the values of annuities-certain for every $\frac{1}{8}$ per-cent, and which are therefore very applicable to the problems considered in the paper.

Mr. SUTTON suggested that it would be desirable to explain how the formula (No. 2) is obtained. It is hardly self-evident, and remembering the trouble involved in dealing with formulas for inter-

polation when he conducted the Institute class, he thought it desirable that the intermediate steps should be inserted. The formula, as it stands, is identical, as far as it goes, with Mr. Woolhouse's general formula (*J.I.A.* xi, 68)

$$V_x = V_0 + x\left(a_0 + \frac{x}{2}b_0\right) + \frac{x(x^2-1)}{2.3}\left(c_0 + \frac{x}{4}d_0\right) + \&c.$$

Neglecting central differences beyond the first two, this becomes

$$V_x = V_0 + x\left(a_0 + \frac{x}{2}b_0\right).$$

Putting $\rho \div h$ for x, since the interval in the above is 1 instead of h, and substituting the values a_1 for V_0, and a for V_x, and substituting for a_0 and b_0 their equivalents $\Delta - \frac{1}{2}\Delta^2$ and Δ^2 respectively, the result is Mr. Hardy's formula. The formula he had himself used in the paper referred to by Mr. Hardy, when put into a similar shape to Mr. Hardy's, is

$$\rho = \frac{a - a_1}{\dfrac{da_1}{di} + \dfrac{1}{2}\dfrac{a-a_1}{\dfrac{da_1}{di}} \cdot \dfrac{d^2a_1}{di^2}}.$$

If, in this formula, instead of $\dfrac{da_1}{di}$ and $\dfrac{d^2a_1}{di^2}$ we substitute their approximate values, as given by Mr. Woolhouse, namely,

$$\frac{da_1}{di} = a_0 = \frac{\Delta - \frac{1}{2}\Delta^2}{h}, \quad \frac{d^2a_1}{di^2} = b_0 = \frac{\Delta^2}{h^2},$$

we get the formula proposed by Mr. Hardy. In Mr. Woolhouse's remarkable paper on General Numerical Solution, of which the paper before us is only a particular case, the very example used by Mr. Hardy is taken as a numerical illustration. With regard to Mr. Hardy's second working formula (No. 10), which he says gives better results, he was not sure that Mr. Hardy had used the simplest method of deducing it. Acting upon the approximate result

$$\log_e(1+z) = \frac{2z}{2+z} = \left(z - \frac{z^2}{2} + \&c.\right),$$

where z is very small, the formula might be obtained directly from his other formula (No. 5). He did not remember to have ever seen Crelle's tables referred to in any paper read before the Institute; but he considered them most valuable for the practical labors of an actuary.

Mr. G. F. Hardy, in reply, said he was aware of the existence of Col. Oakes's tables, but Rance's were so generally used that he preferred to assume that the annuity-values were only available, as in these tables, at intervals of $\frac{1}{4}$ per-cent. Of course, where they are tabulated for intervals of $\frac{1}{8}$ per-cent, the calculations are much shorter. He mentioned Inwood's tables, thinking that it should be

known that so unpretentious a book contains such a valuable series of interest tables. Mr. Sutton, who obtains formula No. 2 from Mr. Woolhouse's formula of central differences, does not consider it self-evident. He himself certainly had taken it to be so, obtaining it by a very slight change from the ordinary formula in finite differences

$$a_x = a_0 + x\Delta + \frac{x(x-1)}{2}\Delta^2 + \&c.$$

Collecting the coefficients of x and x^2, we immediately obtain

$$a_x = a_0 + x(\Delta - \tfrac{1}{2}\Delta^2) + x^2\frac{\Delta^2}{2}.$$

It is true that formula (1) is equivalent to that given in Mr. Sutton's paper, only that the differential coefficients which appear in the latter are represented in the former by approximate values; but formula (1) is deduced by an entirely different process, namely, by making use of the successive differences of the function dealt with, which can easily be obtained from the tables, in lieu of the differential coefficients, the values of which must be calculated. This method, moreover, gives better results than if the differential coefficients are used; indeed, the first part of the paper goes to show that the errors involved in the two approximations which are made, to a certain extent neutralize one another. In the first place, of course, we make use of an approximation by stopping at second differences; in the second place, we adopt another approximation, by writing an expression of the form $(1+z)$, where z is a small fraction, in what is taken to be an equivalent form, $(1-z)^{-1}$. Although, however, we have these two successive approximations, the new terms which appear by the use of the second, nearly represent those which are suppressed by the employment of the first, so that practically the formula approximately takes account of the third differences as well as the second. The object of the other formula given—(No. 10)—is to approximately represent all the successive differences. If the theory were correct that these formed a geometrical progression, this formula would be absolutely accurate (setting aside the approximations to the values of the logarithms).

An Improved Method of Approximating to the Value of Annuities involving Three Lives. By George F. Hardy, F.I.A.

[Read before the Institute, 27 Feb. 1882.]

The ordinary method of approximating to three-life annuities is as follows:—Let the lives be aged respectively x, y, z, (x being the youngest). Find an age w such that at the given rate of interest $a_w = a_{yz}$, then $a_{xyz} = a_{xw}$ nearly. Baily diminishes the value thus found by ·05, but I cannot say if this correction is

usually made.* It will generally be found to be too small if the lives are very young, but much too large at the older ages.

It has been shown that the above method (neglecting now the correction referred to) is strictly accurate in any mortality table where the mortality conforms to Mr. Gompertz's hypothesis, and where consequently the force of mortality can be represented throughout by the expression

$$\mu_x = \mathrm{B}q^x$$

(B and q being constants). The reason for this is, that the force of mortality for two joint lives, x, $x+t$, may be represented by a similar expression

$$\mu_{x\,.\,x+t} = \mathrm{B}(q^x + q^{x+t}) = \mathrm{B}q^x(q^t + 1) = \mathrm{B}q^{t'+x}$$

(where $q^{t'} = q^t + 1$).

It follows from this that if we have

$$\mu_w = \mu_{yz},$$

we have also $$\mu_{w+t} = \mu_{y+t\,.\,z+t}\,;$$

and, the force of mortality in the joint status being equal to that for the single life, throughout the table, the two annuities are identical, not only as to their total value, but also as to the value of each successive payment.

The force of mortality in all known tables, however, can be very much better represented, as in Mr. Makeham's formula, by the expression

$$\mu_x = \mathrm{A} + \mathrm{B}q^x.$$

The introduction of the constant A into the expression, however, renders it impossible to represent the various payments of a joint-

* We have obtained the following information as to the practice on this point of 24 offices in London and Edinburgh. Seven offices, in applying Simpson's rule, make the correction of $-{\cdot}05$; and one other makes it only when the lives upon which the annuity depends, are over 60 years of age. Twelve offices use Simpson's rule without the correction. Two use the following modification of Simpson's rule, devised by Mr. Meikle:—

$$a_{xyz} = a_{yz}\sqrt{\left(\frac{a_{xr}\,.\,a_{x(r-1)}}{a_r\,.\,a_{r-1}}\right)}$$

where $a_{r-1} > a_w > a_r$, r being the integral age next above w. (This formula is given and explained in a paper on Joint Life Annuities, by Mr. J. J. McLauchlan, in the *Transactions of the Actuarial Society of Edinburgh*, New Series, No. 2.) One office has adopted the tables appended to Messrs. King and Hardy's paper on the Practical Application of Mr. Makeham's Formula to the Graduation of Mortality Tables (*J.I.A.*, xxii, 191). One office calculates the values of annuities on three joint lives by means of Mr. Woolhouse's formula (D) (*J.I.A.*, xi, 321).

The correction of $-{\cdot}05$, although attributed to Baily by D. Jones, appears to have been originally suggested by Price. See his *Observations on Reversionary Payments*, 7th edition, vol. ii, p. 366.—Ed. *J.I.A.*

life annuity by those of any single-life annuity at the same rate of interest. In other words, we have, with Mr. Gompertz's formula,

$$-\frac{1}{D_w}\frac{d}{dw}D_w=\log_e(1+i)+Bq^w$$

$$-\frac{1}{D_{y.y+t}}\frac{d}{dy}D_{y.y+t}=\log_e(1+i)+Bq^y(q^t+1).$$

Put $q^{t'}=(q^t+1)$ and $y+t'=w$, and the latter expression reduces exactly to the former, and we have

$$\frac{1}{D_w}\frac{d}{dw}D_w=\frac{1}{D_{y.y+t}}\frac{d}{dy}D_{y.y+t},$$

and consequently for all values of n,

$$\frac{D_{w+n}}{D_w}=\frac{D_{y+n.y+t+n}}{D_{y.y+t}}.$$

With Mr. Makeham's formula, however, we get

$$-\frac{1}{D_w}\frac{d}{dw}D_w=\log_e(1+i)+A+Bq^w$$

$$-\frac{1}{D_{y.y+t}}\frac{d}{dy}D_{y.y+t}=\log_e(1+i)+2A+B(q^y+q^{y+t})$$

and this latter expression can no longer be reduced to the form of that for the single life at the same rate of interest; but, as Mr. Woolhouse has shown, this may be done if we take a new rate of interest, i', for the single-life annuity, such that

$$\log_e(1+i')=\log_e(1+i)+A.$$

In order, therefore, to avoid the error involved in the ordinary method for obtaining the value of a_{xyz}, or at least to considerably diminish it, we should proceed as follows. Suppose i the given rate of interest; let $i'=i+A$, which will practically satisfy the equation $\log_e(1+i')=\log_e(1+i)+A$. Let a_x, &c., denote annuities at the rate i; a'_x, &c., annuities on the same statuses at the rate i'. Then, to find the value of a_{xyz}, let $a'_w=a_{yz}$, and $a'_{xw}=a_{xyz}$, nearly.

The value of the constant, A, differs of course in various tables of mortality, but it will be almost always somewhat over ·005; hence there will always be increased accuracy if we take the annuities a'_w, a'_{xw}, at an addition of $\frac{1}{2}$ per-cent over the original rate of interest. I find for the values of A in the following tables, approximately,

H^M	$A = \cdot 00620$
English . . .	$= \cdot 00750$
Carlisle . . .	$= \cdot 00750$
Davies's Equitable .	$= \cdot 00666$

Hence, if the above rule is followed, it may be expected to reduce the error by the ordinary method to about one-fifth in the case of H^M Table, one-fourth for Davies's Equitable, and one-third for the English and Carlisle. If, in the case of the two last, the annuities a'_w, a'_{xw} be taken at an advance of $\frac{3}{4}$ per-cent on the original rate of interest, the error in the final value for a_{xyz} may be expected practically to disappear, and will certainly be very small as compared with the error arising from the usual mode of approximation.

I will now give one or two examples of the effect, in practice, of simply raising the rate at which a'_w and a'_{xw} are taken by $\frac{1}{2}$ per-cent, and then show how we may approximate to the results we should obtain by raising the rate by the true value of the constant A.

To find the value of $a_{20.20.30}$ (H^M 3 per-cent) :—

By the ordinary method we find

$$a_{20.30} = 17\cdot3149 = a_{39\cdot526}\,; \quad a_{20.39\cdot526} = 15\cdot5276\,;$$

$$\therefore \quad a_{20.20.30} = 15\cdot5276, \text{ or, if diminished by Baily's constant } \cdot05, \\ = 15\cdot4776.$$

By the method proposed,

$$a_{20.30} = 17\cdot3149 = a'_{35\cdot041} \text{ (at } 3\tfrac{1}{2} \text{ per-cent)}\,; \quad a'_{20.35\cdot041} = 15\cdot4194$$

$$\therefore \quad a_{20.20.30} = 15\cdot4194.$$

The true value of the required annuity at 3 per-cent is 15·3912. Hence, by the ordinary method, there is an error of no less than +·1364, which, by the proposed alteration, is reduced to +·0282, about one-fifth of the former quantity. It will be seen at once that the amount of work is precisely the same in each case; all the steps corresponding.

As another illustration, we may find the value of $a_{30.40.50}$ (H^M 3 per-cent).

The ordinary rule gives

$$a_{40.50} = 11\cdot8177 = a_{55\cdot747} \qquad a_{30.55\cdot747} = 10\cdot8114\,;$$

$$\therefore \quad a_{30.40.50} = 10\cdot8114.$$

By the method proposed,

$$a_{40.50} = 11\cdot8177 = a'_{54\cdot200} \text{ (at } 3\tfrac{1}{2} \text{ per-cent)}\,; \quad a'_{30.54\cdot200} = 10\cdot7924\,;$$

$$\therefore \quad a_{30.40.50} = 10\cdot7924.$$

The true value in this case is 10·7840, so that the errors by the two methods are respectively $+·0274$ and $+·0084$. In the present instance Baily's correction would not much improve the result.

It will be obvious that the results just obtained may be made more accurate by the following simple method. What we have done is equivalent to assuming the value of A to be ·005; its true value we know to be ·0062. If, therefore, we find the value of the required annuities on the assumption that A=·0100 (that is, by taking a'_w and a'_{xw} at 4 per-cent instead of $3\frac{1}{2}$ per-cent) and then add to our former result one-fourth of the difference between it and the result thus obtained, we shall practically have a result equivalent to that which would have been obtained by assuming A=·00625 (that is, by taking a'_w, &c., at $3\frac{5}{8}$ per-cent). These remarks refer to the H^M Table. In the case of the Carlisle and English, one-half of the difference between the results (using 4 per-cent and $3\frac{1}{2}$ per-cent respectively for a'_w, &c.,) must be added to the latter result; and for the Equitable Experience, one-third of the difference must be added.

As illustrations, let us take the examples just given. In finding the value of $a_{20.20.30}$, we obtained by taking $a_{20.30}$ at 3 per-cent $=a'_{35·041}$, at $3\frac{1}{2}$ per-cent, $a_{20.20.30}=15·4194$. By using 4 per-cent annuities for a'_w, a'_{xw}, we get

$$a_{20.30}=17·3149, \text{ as before,}$$

$$=a'_{28·935}$$

$$\therefore \quad a_{20.20.30}=a'_{20.28·935}=15·2755$$

$$\text{By } 3\tfrac{1}{2} \text{ per-cent annuities}=15·4194$$

$$\text{Difference} -\ ·1439$$

$3\frac{1}{2}$ per-cent result	15·4194
$\frac{1}{4}$th difference—	·0360
	15·3834
True value	15·3912
Error= —	·0078

Supposing that the values of the joint annuities at $3\frac{1}{2}$ per-cent had not been tabulated, we should have proceeded thus:—

Value by ordinary process (all annuities taken at 3 per-cent)	=15·5276
Value a'_w, &c., at 4 per-cent	=15·2755
Difference=	·2521
Value a'_w, &c., at $3\frac{5}{8}$ per-cent	$=15·5276-\frac{5}{8}(·2521)$
	=15·5276
	— ·1576
$a_{20.20.30}$	=15·3700

Taking the second example, we find

$$
\begin{array}{llr}
a'_w \text{ and } a'_{xw} \text{ at } 3\tfrac{1}{2} \text{ per-cent, } & a_{30.40.50} & =10{\cdot}7924 \\
\quad ,, \qquad ,, \quad \text{at } 4 \qquad ,, & ,, & =10{\cdot}7628 \\
& \text{Difference} = - & {\cdot}0296
\end{array}
$$

$$\therefore \quad a_{30.40.50} = 10{\cdot}7924 - \tfrac{1}{4}({\cdot}0296) = 10{\cdot}7850$$

$$\text{True value} = 10{\cdot}7840$$

$$\text{Error} = \ {\cdot}0010$$

Or, when the joint annuities are not tabulated at $\frac{1}{2}$ per-cents,

$$
\begin{array}{llr}
\text{All annuities at 3 per-cent, } & a_{30.40.50} & =10{\cdot}8114 \\
a'_w \text{ and } a'_{xw} \text{ at } 4 \qquad ,, & ,, & =10{\cdot}7628 \\
& \text{Difference} = & {\cdot}0486
\end{array}
$$

$$\therefore \quad a_{30.40.50} = 10{\cdot}8114 - \tfrac{5}{8}({\cdot}0486) = 10{\cdot}7810$$

$$\text{Error} = - \ {\cdot}0030$$

The above remarks assume throughout that the mortality table dealt with conforms nearly to Mr. Makeham's formula, and that a constant value of A may be adopted throughout the table. In the case of the H^{M} Table this supposition is very nearly true. The method may however be varied so as to become quite general, and this may be done in several ways. For example, let us have, as before, $a'_w = a_{yz}$. One necessary condition, in order that a'_{xw} should be exactly equivalent to a_{xyz}, would be that (for all values of t) a'_{w+t} should equal $a_{y+t.z+t}$. Since this condition cannot be exactly met, we may take an average value of t (say for example the nearest integer in a_{xyz}), and if a rate cannot be found at which we have both

$$a'_w = a_{yz},$$

$$a'_{w+t} = a_{y+t.z+t},$$

we may assume (the first equation being satisfied) that the error in the second equation will be proportional to the error in the final result found by putting $a_{xyz} = a'_{xw}$. Thus we should proceed as follows:—

To find $a_{20.20.30}$ we have, as before,

$$a_{20.30} = 17{\cdot}3149 = a'_{35{\cdot}041} \text{ (at } 3\tfrac{1}{2} \text{ per-cent)}$$

$$= a''_{28{\cdot}935} \text{ (at 4 per-cent).}$$

Now (taking 17 as our value of t), we have

$$^{17}a_{20.30}=a_{37.47}=12{\cdot}7499,$$

$$\text{at } 3\tfrac{1}{2} \text{ per-cent } a'_{52{\cdot}041}=12{\cdot}5330=a_{37.47}-{\cdot}2169,$$

$$\text{at } 4 \text{ per-cent } a''_{45{\cdot}935}=13{\cdot}6523=a_{37.47}+{\cdot}9024.$$

And, assuming that the errors in the above equations are proportionate to the errors in the resulting values of $a_{20.20.30}$, as obtained from $a'_{20.35{\cdot}041}$ (at $3\frac{1}{2}$ per-cent) and $a''_{20.28{\cdot}935}$ (at 4 per-cent), we should have as our final result

$$a_{20.20.30}=a'_{20.35{\cdot}041}-\frac{{\cdot}2169}{{\cdot}2169+{\cdot}9024}(a'_{20.35{\cdot}041}-a''_{20.28{\cdot}935})$$

$$=15{\cdot}4194-{\cdot}194\,({\cdot}1439)$$

$$=15{\cdot}3915\,;$$

the true value being 15·3912, a difference of only ·0003. Applying the same process to find $a_{30.40.50}$, we get the value 10·7839 with an error of only ·0001.

The above method is somewhat laborious; and, in the case of the Carlisle Table especially, where the graduation is very bad, we usually have errors affecting the 2nd place of decimals. We have, however, another necessary condition, in order that a_{xw} should be the exact equivalent of a_{xyz}: this is, that we should have

$$\left.\begin{aligned} a_{yz}&=a_w \\ \mathrm{I}a_{yz}&=\mathrm{I}a_w \\ \mathrm{I}^2a_{yz}&=\mathrm{I}^2a_w \\ &\&c. \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (1)$$

in other words, we must have the successive orders of increasing annuities equal for both statuses. This will be seen if we expand the values of a_{xyz} and a_{xw} in terms of these annuities

$$a_{xyz}=a_{yz}+k'\mathrm{I}a_{yz}+k''\mathrm{I}^2a_{yz}+\&c. \quad . \quad . \quad . \quad . \quad (2)$$

$$a_{xw}=a_w+k'\mathrm{I}a_w+k''\mathrm{I}^2a_w+\&c. \quad . \quad . \quad . \quad . \quad (3)$$

where k', k'', &c., are the successive differential coefficients of D_x divided by that quantity. Generally, it will be a sufficient approximation, if we can make $a_{yz}=a_w$ and $\mathrm{I}a_{yz}=\mathrm{I}a_w$; or if this again cannot be done exactly, except by using a fractional rate of interest for which values are not tabulated, we can always, by the following simple contrivance, find the approximate ratio of $\mathrm{I}a_{yz}$ and $\mathrm{I}a_w$.

Supposing we have two annuities, a_w, a'_w, at different rates of

interest $i, i',$; then putting δ for the value of $\log_e(1+i')-\log_e(1+i)$, we shall have

$$a_w = a'_w + \delta \mathrm{I} a_w + \&c. \quad . \quad . \quad . \quad . \quad . \quad (4)$$

also
$$a_{yz} = a'_{yz} + \delta \mathrm{I} a_{yz} + \&c. \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Hence
$$\frac{\mathrm{I}a_w}{\mathrm{I}a_{yz}} = \frac{a_w - a'_w}{a_{yz} - a'_{yz}}, \text{ nearly} \quad . \quad . \quad . \quad . \quad (6)$$

But if we neglect the terms involving k'', &c., in equations (2) and (3), as we have the corresponding terms in equations (4) and (5), we get

$$\frac{\mathrm{I}a_w}{\mathrm{I}a_{yz}} = \frac{a_w - a_{xw}}{a_{yz} - a_{xyz}}; \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

and finally, from equations (6) and (7), we have

$$a_{xy} - a_{xyz} = \frac{a_{yz} - a'_{yz}}{a_w - a'_w}(a_w - a_{xw})$$

$$\therefore \quad a_{xyz} = a_{xy} - \frac{a_{yz} - a'_{yz}}{a_w - a'_w}(a_w - a_{xw}) \quad . \quad . \quad . \quad (8)$$

By means of this formula we can avoid all fractional ages, the working being very simple. The point to be attended to is that we shall take a_w at such a rate of interest that we have, as nearly as possible, $a_w = a_{xy}$, and $a_w - a'_w = a_{xy} - a'_{xy}$: this, by ensuring that the first two of the set of equations (1) are satisfied, makes the approximation to the remaining conditions, $\mathrm{I}^2 a_{xy} = \mathrm{I}^2 a_w$, &c., more close.

I close this paper with a few examples of the application of this last formula (8). Taking, again, $a_{20.20.30}$ H$^{\mathrm{M}}$ 3 per-cent, we have

at 3 %, $a_{20.30} = 17{\cdot}3149$; at $3\frac{1}{2}$ %, $a_{35} = 17{\cdot}3245$ and $a_{20.35} = 15{\cdot}4263$
,, 4 ,, $a'_{20.30} = 15{\cdot}1533$; ,, $4\frac{1}{2}$,, $a'_{35} = 15{\cdot}1862$

$$\left.\begin{array}{l}\text{Difference for } 1\ \%\\ = a_{20.30} - a'_{20.30}\end{array}\right\} = 2{\cdot}1616 \qquad a_{35} - a'_{35} = 2{\cdot}1383$$

$$\therefore \quad a_{20.20.30} = a_{20.30} - \frac{2{\cdot}1616}{2{\cdot}1383}(\overset{3\frac{1}{2}\%}{a_{35}} - \overset{3\frac{1}{2}\%}{a_{20.35}})$$

$$= 17{\cdot}3149 - 1{\cdot}019(1{\cdot}8982)$$

$$= 15{\cdot}3958;$$

being in excess of the true value by ·0046. Here, and in other cases, having found a_{yz} and $a_{yz} - a'_{yz}$, we must see by inspection at what rate of interest a_w must be taken, so that the required

conditions ($a_w = a_{yz}$ and $a_w - a'_w = a_{yz} - a'_{yz}$) may be most nearly fulfilled. It will generally be found that a_w should be taken at a rate $\frac{1}{2}$ per-cent greater than a_{yz}.

As a further example of this process, we will find the value of $a_{40.40.40}$ (Government Annuity Tables, Males, 5 per-cent). Here, as the values at $5\frac{1}{2}$ per-cent are not available, a_w must be taken at 5 per-cent, and we shall have

$$\begin{array}{llll}
\text{at 5 per-cent,} & a_{40.40} = 10{\cdot}611\,; & \text{at 5 per-cent,} & a_{51} = 10{\cdot}750 \\
\text{,, 6 ,,} & a'_{40.40} = 9{\cdot}712\,; & \text{,, 6 ,,} & a'_{51} = 9{\cdot}850 \\
\hline
 & a_{40.40} - a'_{40.40} = {\cdot}899 & & a_{51} - a'_{51} = {\cdot}900
\end{array}$$

$$\begin{aligned}
\therefore\quad a_{40.40.40} &= a_{40.40} - \frac{899}{900}(a_{51} - a_{40.51}) \\
&= 10{\cdot}611 - {\cdot}999\,(1{\cdot}551) \\
&= 9{\cdot}062,
\end{aligned}$$

the true value being 9·052; a difference of ·010.

Applying the same process to find $a_{50.50.50}$ by the same table, at 5 per-cent, we obtain the value 6·954; in excess of the truth by ·003. Again, for $a_{30.32.34}$ (Carlisle 3 per-cent) taking a_w in this case $= a_{40}$ at 4 per-cent (which most nearly satisfies the required conditions), we find the value 12·960, the true value being 12·984. For $a_{20.22.26}$, by same table and rate of interest, we see by inspection that we must take $a_w = a_{35}$ at $3\frac{1}{2}$ per-cent, which gives for the value required 14·953, the true value being 14·938. As a final example we will work out, by the same process, the value of $a_{30.40.50}$ by the H^{M} Table at 3 per-cent.

Here we have,

$$\begin{array}{llll}
\text{at } 3\,\%,\ a_{40.50} = 11{\cdot}8177\,; & \text{at } 3\tfrac{1}{2}\,\%,\ a_{54} = 11{\cdot}8849\,; & a_{54} - a_{30.54} = 1{\cdot}0416 \\
\text{,, 4 ,, } a_{40.50} = 10{\cdot}7394\,; & \text{,, } 4\tfrac{1}{2} \text{ ,, } a_{54} = 10{\cdot}8557 & \\
\hline
\left.\begin{array}{l}\text{Difference for in-}\\ \text{crease of } 1\,\%\end{array}\right\} = 1{\cdot}0283 & 1{\cdot}0292 &
\end{array}$$

$$\begin{aligned}
\therefore\quad a_{30.40.50} &= a_{40.50} - \frac{1{\cdot}0283}{1{\cdot}0292}(a_{54} - a_{30.54}) \\
&= 11{\cdot}8177 - 1{\cdot}0407 \\
&= 10{\cdot}7770
\end{aligned}$$

the true value being 10·7840; a difference of ·0070.

It will be convenient, in conclusion, to tabulate the results arrived at (1) by the use of the ordinary method; (2) by the same

method with Baily's correction; (3) by taking the equivalent single-life annuity, a_w, at an advance of $\frac{1}{2}$ per-cent in the rate of interest; (4) by means of equation (8).

Annuity required.	Errors by under-mentioned Methods.			
	Ordinary Method.	Ordinary Method, with Baily's Correction.	Equivalent Single-Life Annuity at $\frac{1}{2}$% higher Rate.	By Equation (8).
(1)	(2)	(3)	(4)	(5)
H^M 3%, $a_{20.20.30}$	+·136	+·086	+·028	+·005
,, $a_{30.40.50}$	+·027	−·023	+·008	−·007
Carlisle 3%, $a_{20.22.26}$	+·170	+·120	+·055	+·015
,, ,, $a_{30.32.34}$	+·098	+·048	+·087	−·024
Gov. Males 5%, $a_{40.40.40}$	+·052	−·002	+·022	+·010
,, ,, $a_{50.50.50}$	+·026	−·024	+·010	+·003
Average error . .	+·085	+·035	+·035	·000
Do., irrespective of sign	·085	·051	·035	·011

A comparison of the 2nd and 5th columns will show that equation (8) will enable us, with scarcely any additional labour, generally to reduce the error by the ordinary method to about one-eighth of its amount. It will be seen that by far the largest errors in columns (4) and (5) are those belonging to the Carlisle Table, this being no doubt due to its irregularities. If these results are excluded, we get an average error in column (4) of only ·017, and in column (5) of only ·006.

Discussion.

The President (Mr. A. H. Bailey) remarked that the paper was an excellent intellectual exercise, but in practice actuaries do not trouble themselves in the values of annuities with the accuracy of the third place of decimals.

Mr. Sutton had not been quite able to ascertain where Mr. Hardy gets his value of A from in the expression for the value of μ. He thought that it would be desirable, particularly as regards the H^M Table, that there should be some description of how it is obtained. He called attention to this point because he had referred to Mr. Woolhouse's various values, and it does not agree with any one of them.* He was not sure that Mr. Hardy

* Mr. Hardy states that the value of A used by him is obtained from the constants given in the joint paper by Messrs. King and Hardy, on Mr. Makeham's formula (*Journal*, vol. xxii, p. 191).—Ed. *J.I.A.*

had correctly represented the common practice hitherto in approximating to three-life annuities. His own plan, certainly, would have been in the cases given to take, not the exact age to three decimal places, but the year older. For instance, in the first example the annuity on the two older joint lives, 20 and 30, is found to be equal to the annuity on a single life, age 39·526. Well, he should certainly in practice have taken that as 40. He would like again to call attention to Mr. Woolhouse's convenient formula (*J.I.A.*, xi, 321)

$$A = m(V_m + V_{2m} + \ldots) + \frac{m-1}{2} - \frac{m^2-1}{12}(\mu+\delta).$$

The values of V_m, V_{2m}, ... can be rapidly calculated, and the formula gives generally results very near the truth; and he was not at all sure that he could not calculate a joint-life annuity by it almost as quickly as by Mr. Hardy's second method of approximation. He considered this short paper of Mr. Hardy's to be one of the most practically valuable which the Institute has had read before it for some time past, and the more it is read and gone into by persons who are in the habit of making similar calculations, the more they will appreciate it.

Mr. R. P. Hardy was much pleased with the methods and results of the paper. These are not (with submission to the President) merely intellectual exercises, but practical aids of essential service, for instance, if an estate has to be cut up for division under the Court of Chancery. If one were put into the witness-box to justify calculations made by such formulas, it would be easy to show that they had been done correctly, and so satisfy the technical requirements of the Court. Calculations by means of Mr. Woolhouse's formula can be made with very great rapidity; but if you had eight or nine of them to complete before ten o'clock in the morning, you would be very thankful if you could reduce the work as shown in the paper.

Mr. Bumsted asked Mr. Hardy whether he had applied his formula to the case of annuities on old lives—say, to a joint annuity on lives of 70, 80, and 90, and whether the difference would come out greater than it does in the cases in the paper, which relate only to very young lives—20, 30, and 40, and 30, 40, and 50.

The President, in asking Mr. Hardy to reply, said he hoped he would not be misunderstood when he said that the paper is an intellectual exercise. But it was strongly impressed on his mind that all our calculations of the values of life annuities are approximations, and that, therefore, errors of ·001 in one case and ·003 in another, need not be regarded.

Mr. Hardy, in his reply, said that the first part of his paper was intended to show that by a very small alteration in the ordinary method of proceeding, and one which does not involve the slightest additional work, the errors by the ordinary method, which in some cases may be considerable, may be reduced to about one-fourth or one-fifth of their amount. It is not always a question of errors in the third place of decimals, as will be seen by the first example given, where the ordinary process produces an error of a unit in the first

place of decimals. If we had to solve a question in which the final result was not the three-life annuity, but the difference between it and the joint-life annuity (which might happen in the case of a survivorship), the error might amount to as much as 5 per-cent of the whole amount. The more exact methods described involve certainly some amount of calculation; still they will be found useful where it is desired to obtain results as accurate as possible, without going through the longer calculations involved in Mr. Woolhouse's formula. The whole of the work can be readily gone through in about five or six minutes. Mr. Sutton considers that it is quite sufficient in practice to take the equivalent life at the next higher age, and that if the age came out, for instance, as in the first example, 39·526, he would take age 40. This would involve very little error; but it evidently would be a rather rough process for arriving at our result to deal in the same way with the equivalent life aged 35·04, by calling this 36; while if we took the next younger age, 35, we should be still further increasing the error due to the ordinary method of approximation.

On the Influence of Selection on the Mortality from different Classes of Diseases amongst Assured Lives. By William R. Dovey, F.F.A., *Actuarial Assistant, Standard Life Assurance Company, London.*

UPON the selection of lives depends to a large extent the financial success of a life assurance company, as the acceptance of under-average risks at ordinary rates may be as prejudicial to the well-being of an office as an imprudent investment of its funds. The opposition of interests also suggests the necessity for caution, and calls for the constant exercise of sound judgment. The care taken by life offices to exclude inferior risks, invariably has the effect of reducing the number of deaths in the early years of assurance considerably below those indicated in the tables upon which their calculations are based. The extent and duration of this favourable influence have been repeatedly investigated, and the subject is now well understood. In this paper it is proposed to extend the enquiry to the partial death-rates from various classes of diseases, and thus to ascertain the strength of this self-protecting power in different directions. A somewhat wider meaning will be attached to the term "selection" in the following pages than has usually been associated with it by actuarial writers. It will be regarded as including not only the effects of the medical examination, but also those other selecting causes which impart to the mortality of assured

lives the characteristics it is seen to possess when compared with that of the general population.

The death-rates of assured lives from different groups of diseases in the earlier years of risk, have been calculated by Mr. Meikle, and published in his *Observations on the Rate of Mortality of Assured Lives, as experienced by* 10 *Assurance Companies in Scotland.** With the exception of a column relating exclusively to diseases of women, his figures are here reproduced.

Table (A).

Rate of Mortality from various Causes of Death during each year of Assurance.

Year of Assurance.	Causes of Death.							
	Zymotic Diseases.	Uncertain Seat.	Tubercular Diseases.	Brain and Nerves.	Heart and Blood-vessels.	Lungs.	Digestive Organs.	Urinary Organs.
0	·001629	·000088	·000374	·000528	·000198	·000506	·000704	·000088
1	·001684	·000303	·000981	·000969	·000436	·000848	·000884	·000157
2	·001958	·000493	·001915	·001383	·000657	·001164	·001109	·000164
3	·001788	·000566	·002109	·001712	·000764	·001024	·001467	·000290
4	·001894	·000734	·002031	·002099	·000990	·001536	·001297	·000393
5	·002214	·000889	·001968	·002251	·000738	·001230	·001419	·000530
6	·002060	·000812	·002351	·002518	·001228	·001810	·001540	·000416
7	·002337	·000917	·002154	·002566	·001100	·001512	·001764	·000321
8	·001982	·000584	·001956	·003150	·001422	·001600	·001778	·000483
9	·002067	·001047	·002010	·003199	·001274	·002067	·001727	·000566
10	·002097	·000826	·002383	·003558	·001874	·002256	·002224	·000540
11	·001875	·001154	·001983	·003786	·002055	·002560	·002488	...
General average probability of death from each of the causes of death . .	·002129	·000812	·001749	·002973	·001485	·001938	·001821	·000534

In order to arrive at the *computed* rates, with which the above may be compared so as to elicit the effects of selection, two preliminary operations were necessary, the first being to analyze the mortality which prevails in an unselected population, and the second to ascertain the distribution, as regards age, of a body of assured lives passing through years 0, 1, 2, &c., of assurance. With reference to the first, I should naturally have preferred using the statistics embodied in the reports of the Registrar-General of England, as they afford the widest basis attainable for such calculations. The system of nomenclature of diseases, however, adopted by Dr. Farr, did not lend itself to the purpose in hand, and I therefore availed myself of the reports of the Registrar-

* Messrs. Blackwood and Sons; Edinburgh, 1872.

General of Scotland, and more particularly of the appendix to the *Eleventh Detailed Annual Report,* which contains summaries of the deaths registered in Scotland during the 10 years, 1855-1864 inclusive.* From this source the number of deaths caused by the various groups of diseases, at different periods of life, were obtained.

TABLE (B).

Deaths in Scotland, 1855-1864, *Males.—Specified Causes.*

Ages.	Zymotic Diseases.	Uncertain Seat.	Tubercular Diseases.	Brain and Nerves.	Heart Diseases.	Lung Diseases.	Digestive Organs.	Urinary Organs.	Various minor Causes.	Violent Deaths.
0– 4	44,757	949	15,413	6,439	285	19,038	12,082	237	18,339	2,283
5– 9	8,387	339	3,652	1,028	183	1,189	528	132	373	763
10–14	2,278	222	2,435	639	223	390	445	101	290	936
15–24	4,101	385	10,760	1,241	768	1,465	1,053	340	449	2,455
25–34	2,475	457	7,914	1,405	980	1,747	1,124	411	318	2,282
35–44	2,202	719	4,950	1,923	1,434	2,334	1,552	449	377	2,075
45–54	2,241	1,221	3,473	2,378	1,825	3,441	1,976	540	526	1,731
55–64	2,214	1,798	2,319	3,505	2,545	4,431	2,437	839	1,361	1,309
65–74	2,148	1,858	910	4,226	2,648	4,419	2,333	1,192	6,047	745
75 & uprds.	1,724	1,370	193	3,380	1,491	3,091	1,209	1,147	16,709	412

In addition to the above, there were 510 cases, attributed to various diseases, in which the age at death was not recorded. From enquiries made, it appears probable that these refer mainly to deaths during infancy. In the absence, however, of any definite guide, and as this residuum is too small to affect the computed rates afterwards obtained for assured lives, except perhaps occasionally in the sixth decimal place, no attempt was made to spread the number pertaining to each group over the different ages.

The years of life applicable to this decade have been calculated by Mr. Meikle, and will be found on page 2 of his work already mentioned. The figures there given relate to the total deaths, including those from unspecified causes, which are not embraced in my calculations. A slight adjustment was therefore necessary, and this was effected by diminishing the years of life, in the ratio of the total deaths to the deaths from specified causes. The numbers are here shown, as arrived at by Mr. Meikle, and as adjusted for the purposes of this investigation.

* Similar tables have been published for the decade 1861-70, but the statistics of the earlier period were preferred, as they are contemporaneous with those relating to assured lives.

Male Population, Scotland, 1855-64.

Ages.	Years of Life.	Years of Life, adjusted.
0– 4	2,087,650	1,881,154
5– 9	1,826,902	1,745,892
10–14	1,649,364	1,567,250
15–24	2,781,028	2,683,783
25–34	1,928,120	1,854,476
35–44	1,520,779	1,465,228
45–54	1,179,542	1,129,353
55–64	817,927	776,635
65–74	421,351	419,155
75 & upwards	190,124	188,949

Having now brought together the years of life, and the corresponding deaths at various ages from the different groups of diseases, I proceeded to derive the probabilities of dying, and the results are collected in Table (C), (p. 289).

So far as I am aware, the mortality of a general population has not hitherto been represented in this analyzed form. Attention may, therefore, be drawn to the fact that the partial death-rates bear a strong resemblance to the rate of mortality from all causes combined, as regards the periods of life at which they are most destructive. The rate applicable to each group of diseases will be seen to be very fatal during infancy, diminishing to a minimum in boyhood, and thereafter (with one exception) increasing in strength to the end of life. It is worthy of notice that these remarks also apply to the class of violent deaths. Only in the case of tubercular diseases is a departure found from the general curve to which the rest conform. This group possesses the same features as the others, until the decennial period 25–34, when the maximum is attained, and thereafter the rate decreases as the age increases.

The second preliminary process was, as already mentioned, to group according to ages, a body of assured lives passing through each year of assurance 0–9 inclusive. The materials for this purpose were taken from pages 249–253 of the *Mortality Experience of* 20 *Offices,* published by the Institute of Actuaries. Having separated the lives in each of these years of assurance into quinquennial groups of ages, the proportion was then calculated, which the number in each group bore to the total number at all ages. The following example (p. 290) shows how the figures for year 0 in the H[M] Table were obtained.

Table (C).

Probabilities of dying from Different Causes, at the undermentioned Ages, derived from the Scottish Registrar-General's Returns for the Period 1855–1864.—*Males.*

Ages.	(1) Zymotic Diseases.	(2) Diseases of Uncertain Seat.	(3) Tubercular Diseases.	(4) Diseases of Brain and Nervous System.	(5) Diseases of Heart and Blood-vessels.	(6) Lung Diseases.	(7) Diseases of Digestive Organs.	(8) Diseases of Urinary Organs.	(9) Various minor Classes of Diseases.	(10) Violent Deaths.	Ages.
0– 4	·023793	·000505	·008194	·003423	·000152	·010121	·006423	·000126	·009749	·001214	0– 4
5– 9	·004804	·000194	·002092	·000589	·000105	·000681	·000302	·000076	·000214	·000437	5– 9
10–14	·001454	·000142	·001554	·000408	·000142	·000249	·000284	·000064	·000185	·000597	10–14
15–24	·001528	·000144	·004009	·000462	·000286	·000546	·000392	·000127	·000167	·000915	15–24
25–34	·001335	·000246	·004268	·000758	·000529	·000942	·000606	·000222	·000172	·001231	25–34
35–44	·001503	·000491	·003378	·001312	·000979	·001593	·001059	·000306	·000257	·001416	35–44
45–54	·001984	·001081	·003075	·002106	·001616	·003047	·001750	·000478	·000466	·001533	45–54
55–64	·002851	·002315	·002986	·004513	·003277	·005705	·003138	·001080	·001752	·001686	55–64
65–74	·005125	·004433	·002171	·010082	·006318	·010543	·005566	·002844	·014427	·001777	65–74
75 & upwards	·009125	·007251	·001022	·017890	·007892	·016360	·006399	·006071	·088436	·002181	75 & upwards

Ages.	Nos. exposed to Risk.	Logs of Nos. exposed to Risk.	Logs in preceding Column minus 4·80785.	Proportion exposed to Risk.
Under 10	235	2·37107	$\bar{3}$·56322	·00366
10–14	348	2·54158	$\bar{3}$·73373	·00542
15–19	1411·5	3·14968	$\bar{2}$·34183	·02197
20–24	7279	3·86207	$\bar{1}$·05422	·11330
25–29	12776	4·10639	$\bar{1}$·29854	·19886
30–34	12781·5	4·10658	$\bar{1}$·29873	·19894
35–39	10503·5	4·02133	$\bar{1}$·21348	·16349
40–44	7486	3·87425	$\bar{1}$·06640	·11652
45–49	5095·5	3·70719	$\bar{2}$·89934	·07931
50–54	3160	3·49969	$\bar{2}$·69184	·04919
55–59	1823	3·26079	$\bar{2}$·45294	·02837
60–64	874	2·94151	$\bar{2}$·13366	·01360
65–69	346	2·53908	$\bar{3}$·73123	·00538
70–74	104·5	2·01912	$\bar{3}$·21127	·00163
75–79	17	1·23045	$\bar{4}$·42260	·00026
80 & above	6·5	·81291	$\bar{4}$·00506	·00010
	64247	4·80785	...	1·00000

In this way Table (D) (p. 291) was computed, showing, out of 100,000 lives passing through each year of assurance, how many will be in each of the successive quinquennial periods of life.

By means of Tables (C) and (D) we are able to compute the expected rate of mortality in each year of assurance for each of the specified groups of diseases. The precise manner in which they were arrived at, will be seen by glancing over the following example of the calculations:—

Expected Deaths.—Zymotic Diseases: Year 0 of Assurance.

Ages.	Proportion of Assured Lives at each Age.	Logs of Nos. in previous Column.	Log of Probability of Dying.	Sum of Logs in two preceding Columns.	Computed Rate of Mortality.
Under 10	·00366	$\bar{3}$·56348	$\bar{2}$·17497	$\bar{5}$·73845	·00005476
10–14	·00542	$\bar{3}$·73400	$\bar{3}$·16256	$\bar{6}$·89656	·00000788
15–24	·13527	$\bar{1}$·13120	$\bar{3}$·18412	$\bar{4}$·31532	·00020669
25–34	·39780	$\bar{1}$·59966	$\bar{3}$·12548	$\bar{4}$·72514	·00053106
35–44	·28001	$\bar{1}$·44717	$\bar{3}$·17696	$\bar{4}$·62413	·00042085
45–54	·12850	$\bar{1}$·10890	$\bar{3}$·29754	$\bar{4}$·40644	·00025494
55–64	·04197	$\bar{2}$·62294	$\bar{3}$·45500	$\bar{4}$·07794	·00011966
65–74	·00701	$\bar{3}$·84572	$\bar{3}$·70969	$\bar{5}$·55541	·00003593
75 & upwards	·00036	$\bar{4}$·55630	$\bar{3}$·96023	$\bar{6}$·51653	·00000329
	1·00000	...	...	...	·00163506

TABLE (D).

H^M.—*Number of Lives exposed to Risk in each Quinquennial Period of Life to* 100,000, *at all Ages, for each Year of Assurance,* 0–9 *inclusive.*

Ages.	Year 0.	Year 1.	Year 2.	Year 3.	Year 4.	Year 5.	Year 6.	Year 7.	Year 8.	Year 9.	Ages.
Under 10	366	291	220	175	136	111	82	65	43	23	Under 10
10–14	542	441	383	365	326	283	218	165	125	107	10–14
15–19	2,197	1,549	1,187	878	712	561	459	380	352	323	15–19
20–24	11,330	8,561	6,005	3,971	2,408	1,660	1,224	935	688	546	20–24
25–29	19,886	18,406	16,650	14,398	11,991	9,326	6,790	4,757	3,177	1,934	25–29
30–34	19,894	20,564	20,653	20,478	19,911	18,905	17,169	15,345	13,180	10,909	30–34
35–39	16,349	17,132	18,352	19,286	20,202	20,557	21,164	20,909	20,478	19,795	35–39
40–44	11,652	12,886	13,923	15,184	16,166	17,301	18,053	19,339	20,140	20,925	40–44
45–49	7,931	8,721	9,566	10,354	11,320	12,341	13,610	14,640	15,950	16,809	45–49
50–54	4,919	5,596	6,325	7,039	7,680	8,486	9,309	10,054	10,812	11,781	50–54
55–59	2,837	3,191	3,564	4,107	4,747	5,355	5,928	6,609	7,352	8,007	55–59
60–64	1,360	1,714	1,999	2,304	2,639	3,041	3,423	3,730	4,190	4,814	60–64
65–69	538	690	859	1,046	1,229	1,403	1,711	1,986	2,252	2,541	65–69
70–74	163	207	237	326	403	523	668	842	940	1,095	70–74
75–79	26	38	65	74	107	125	162	190	254	309	75–79
80 & upwards	10	13	12	15	23	22	30	54	67	82	80 & upwards
	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	

In this way the computed rates were arrived at in each of the tables which follow, and I now proceed to examine the effects of selection under the different classes of disease.

Table (E).—*Zymotic Diseases.*

Years of Assurance.	Actual Rates of Mortality.	Computed Rates of Mortality.	Actual Deaths to 100 computed.
0	·001629	·001635	99·63
1	·001684	·001635	103·00
2	·001958	·001668	117·38
3	·001788	·001696	105·42
4	·001894	·001728	109·61
5	·002214	·001755	126·15
6	·002060	·001811	113·75
7	·002337	·001857	125·85
8	·001982	·001906	103·99
9	·002067	·001956	105·67

By the comparison thus made, it will be observed that the actual mortality from this group of diseases exceeds the expected in each year of assurance as far as the table extends, with the exception of year 0, when the two rates are nearly equal. The comparatively heavy death-rate to which assured lives are subject may be in a very large measure explained by the fact that the inhabitants of towns are much more liable to these diseases than persons living in the country. Thus, during the 10 years 1861–1870, the average annual proportion of deaths from zymotic diseases to every million persons was, in the

Insular	Districts of Scotland	2,886
Mainland Rural	do.	4,120
Town	do.	6,669
Scotland (general average)		5,132

It is exceedingly probable that the proportion of assured lives residing in towns is larger than that of the entire population; and assuming this to be the case, we have a sufficient reason for the mortality amongst the former being heavier than that derived from the death registers of the nation. Diseases of this nature are controllable to a large extent by means of sanitary precautions, and communities are thereby enabled to exercise a measure of self-protection. With the individual, however, personal healthiness affords little security; for the soundest constitution may at any time succumb to an attack of fever or dysentery. As life offices attract mainly those who reside in centres of population,

where the risk of infection is much greater than in rural districts, we may conclude that, as far as the mortality from zymotic diseases is concerned, selection—using the term in a wide sense—has a negative value.

TABLE (F).—*Diseases of Uncertain Seat.*

Years of Assurance.	Actual Rates of Mortality.	Computed Rates of Mortality.	Actual Deaths to 100 computed.	Averages of consecutive Years.
0	·000088	·000528	16·67	
1	·000303	·000570	53·16	
2	·000493	·000617	79·91	82·4
3	·000566	·000667	84·86	
4	·000734	·000721	101·80	108·0
5	·000889	·000778	114·27	
6	·000812	·000838	96·90	99·5
7	·000917	·000899	102·00	
8	·000584	·000963	60·64	81·2
9	·001047	·001028	101·85	

The comparatively narrow basis from which the figures relating to assured lives are derived, occasions irregularities which do not appear in the statistics drawn from the larger numbers of the male population of Scotland. The actual deaths increase from 16·67 of the computed number, in year 0, to 84·86 in year 3; and thereafter, with two exceptions, cease to compare favourably with the estimated mortality. To this table and those which follow, I have appended the averages of consecutive years whenever these appeared to be instructive. The great majority of the deaths in this group of diseases, are attributed to cancer and dropsy; and as death from the former cause is preceded by an illness of a protracted nature, we may infer that selection strongly influences the deaths therefrom in the early years of assurance.

Dropsy has ceased to be regarded as a primary cause of death, but it still figures largely as such in the reports of the Registrar-General. The mortality amongst the general population from diseases of uncertain seat, is thus overstated, by deaths being included in it that would be more correctly attributed to those organic diseases of which dropsy is frequently a symptom. On the other hand, the mortality from diseases of the heart, and of the digestive and urinary organs, which are sometimes accompanied by dropsy, is understated. The same is true, to a lesser extent, of the corresponding statistics of assured lives, which were calculated from the experience of Scotch offices from 1815 to

31 December 1863. During the greater part of this period, dropsy was regarded as being a sufficient designation of the cause of death in the certificates required by life assurance offices in proof of death, though now it is seldom met with in these documents.

Table (G).—*Tubercular Diseases.*

Years of Assurance.	Actual Rates of Mortality.	Computed Rates of Mortality.	Actual Deaths to 100 computed.	Averages of consecutive Years.
0	·000374	·003750	9·97	
1	·000981	·003672	26·72	39·5
2	·001915	·003668	52·21	
3	·002109	·003619	58·27	57·6
4	·002031	·003569	56·91	
5	·001968	·003518	55·94	61·9
6	·002351	·003460	67·79	
7	·002154	·003408	63·20	60·7
8	·001956	·003356	58·28	
9	·002010	·003306	60·93	

This group deserves particular attention, not only on account of the important disease it includes (namely, consumption), but also because it is here that life assurance companies expect to derive especial benefit from the professional skill of their medical officers. The comparison of the actual with the expected rate of mortality, shows that selection reduces the mortality during the first eighteen months to a very large extent. From the second to the sixth years of assurance, the actual deaths vary from 52·21 to 67·79 per-cent of the computed. The ratio does not show a tendency to increase in the later years of assurance.

The very low death-rate from tubercular diseases amongst newly-assured lives, is no doubt owing to the medical examination detecting the signs of incipient disease, and eliminating from the applicants those who are clearly liable to an early death from consumption. When the immediate effects of the medical examination have, however, passed away, there remains a more permanent, though less strongly marked influence, in the reduction of the mortality for a long series of years—to about two-thirds of the rate which obtains amongst the general population of the country.

Leaving for a few moments the consideration of the rates of mortality relating to the years of assurance, we may observe the effects of selection, by contrasting the death-rates applicable to decennial periods of life. The figures in the following table have

been brought together in order to illustrate this aspect of the subject :—

TABLE (H).

Annual Number of Deaths from Consumption to 10,000 *Men living at each Group of Ages.*

Ages.	British Lives.		American Lives.	
	10 Assurance Companies.	Population of Scotland.	New York Mutual.	City of New York.
20–29	23	46	24	70
30–39	21	35	20	71
40–49	16	31	17	66
50–59	13	30	14	84
60–69	8	27	18	110
70–79	6	17	30	151

The rates of mortality of English and American assured lives, are very similar during the first four decades. After age 60, a marked deviation takes place—the British rate diminishing, and the American increasing. This feature is also presented by the figures drawn from the unselected lives. I am unable to discover any cause for this difference, and can only assume that it arises from the dissimilar climatic and other conditions which prevail in the two countries.

The above figures also clearly indicate, that consumption is *not* pre-eminently a disease of youth. The erroneousness of the popular opinion on the subject has been demonstrated by several writers. The American statistics agree in attributing a very heavy rate of mortality to consumption at the advanced ages of life. In this country, on the contrary, the probability of dying from consumption diminishes as the age increases, but not so rapidly as is commonly supposed. In Scotland, during the decade 1861–1870, consumption was the dominant cause of death *at all ages between* 15 *and* 60. On account of its causing about one-third of the deaths which occur between the ages of 20 and 30, it is much noticed as a cause of death amongst young persons. At later ages consumption becomes less and less conspicuous, mainly on account of the increased strength of other diseases. An examination of the columns in Table (C), leads to the conclusion that, instead of consumption being chiefly a disease of youth, it is distinguished as being more evenly destructive at all ages from boyhood to senility, than is the case with any other class of diseases.

When consumption has caused one or more deaths among the near relations of a proposer, the circumstance is invariably regarded as affecting very materially the eligibility of the life for assurance. In enquiring into this subject, it is surprising how little statistical evidence, of a satisfactory nature, can be brought forward in support of the opinion generally entertained as to the very hereditary nature of this disease.

Several physicians have collected statistics of patients under treatment for consumption in hospitals and other similar institutions. Their tables are, however, deprived of much of their value —as promoting the solution of the general question—on account of their being framed from the statements of poor people, who are often very ignorant of the precise causes of death of their relations. The reports which have been published by medical officers of life assurance institutions, generally refer to this subject; but the materials at their disposal have been usually too limited to do justice to the ability employed in their interpretation. The report by the medical officers of the Mutual Life Insurance Company of New York (issued in 1877) contains the results of a very elaborate investigation into their consumptive risks. The following table is there given, comparing the family histories of 1,031 persons who died of consumption, with the family histories of an equal number who died of other diseases:—

Table (J).

	Parent Consumptive.			Brothers or Sisters.	Total with Family Taint.
	Fathers.	Mothers.	Total.		
Consumptives . .	45	56	101	93	194
Percentage .	...	...	9·79	9·02	18·81
Non-Consumptive .	20	32	52	50	102
Percentage .	...	...	5·04	4·85	9·89

It thus appears, that of those who died from consumption, 18·81 per-cent were members of families in which consumption had made an appearance at the time the proposal papers were filled up. Of those dying from other diseases, only 9·89 per-cent belonged to families which exhibited a consumptive taint. The experience of this office may therefore be regarded as sustaining, at least with respect to American lives, the view usually entertained as to the hereditary character of consumption. The

mortality from this disease in the United States is, however, so dissimilar to the rate which prevails in England, that there is good reason to doubt whether the experience of the New York Mutual corresponds to that of British offices. No enquiry into the hereditary nature of consumption, based on a large number of lives, has yet been made by statistical writers in this country, so far as I have been able to ascertain; and there are no data sufficiently extensive or exact to guide those who have not had long experience, as to the weight which should be attached to the statements, frequently made by proposers, to the effect that a parent or a brother or sister has died of consumption. With the view of supplying to some extent the want of information on the subject, I have brought together, in the following table, the results of an examination of the proposal papers relating to 818 persons, whose deaths are recorded in the obituary of a large assurance company. Cards were used similar to those employed in collecting statistics for the New Experience Table, and the particulars were first filled in relating to 409 lives who had died of consumption. An equal number of cards were then completed respecting persons who had died of various other diseases, taken from the pages of the current obituary in the order in which the deaths were intimated to the office.

TABLE (K).

Comparison of the Family Histories of 409 Persons dying of Consumption with those of an Equal Number dying of other Diseases.

	Parents.	Brothers and Sisters.	Totals.	Percentages.
Consumptives . .	10	52	62	15·2
Non-Consumptives .	5	39	44	10·8

Out of 409 consumptives, there were 62 or 15·2 per-cent, who had lost a parent or a brother or sister from consumption; and out of the same number of non-consumptives, there were 44 or 10·8 per-cent.

These figures show that consumption has a decided tendency to develop itself in the healthy members of families in which it has already appeared, while they discountenance the extreme opinions occasionally entertained. Though the table does not present a new aspect of the subject, it is not on that account

devoid of value, as the collection and interpretation of statistics on this and other points of assurance practice tend to diminish the existing diversity of opinion regarding them, and promote the general acceptance of well-grounded views. By associating, also, a definite significance with those defects of personal or family history which are brought to light in the proposal papers, a decision as to the eligibility of a life for assurance is arrived at with facility and confidence.

While liability to consumption depends to a large extent on personal health and inherited constitution, there appears good reason to believe that its ravages are strongly influenced by occupations and habits of life. As the materials were at hand, the lives under observation were roughly classified according to the avocations of the assured, with the following result:—

Table (L).

Distribution according to Occupation of 409 Lives who died of Consumption, and of an Equal Number who died of other Diseases.

Occupations.	Consumptives.	Non-Consumptives.
Clerks	47	10
Farmers	29	46
Professional Men, Manufacturers, &c.	130	167
Tradesmen	91	92
Artizans	30	23
Miscellaneous	82	71
Totals	409	409

The number of clerks amongst those dying of consumption is very large, and, when compared with the small number found in the other column, indicates that a heavy mortality from this disease prevails amongst those who are thus designated. The figures relating to farmers are very favourable, and contrast strongly with those of the class just referred to. This is accounted for by the consideration that a farmer's life is free from the sedentary habits and other conditions which give rise to a high death-rate from tubercular diseases amongst those who are employed as clerks. The smaller number of deaths in the first column—amongst professional men—may be explained by the circumstance that the average age at the time of insuring is higher in this group than in the others; and as the chance of dying of consumption

gradually diminishes after the period of adolescence, a light mortality in this class might have been expected.

The average duration of the policies was as follows:—

409 lives dying of consumption . .	8 years 9 months.
409 lives dying of other diseases . .	13 „ 3 „

Although the average duration of the first group of risks is short, it is not on this account to be inferred (as is sometimes done in comparisons of this kind) that the mortality has been unduly heavy and greater than that provided for in the ordinary premiums charged. Fuller consideration points to an opposite conclusion, as on referring to Table (G) it will be seen that, in the united experience of 10 offices (to which the company whose consumptive risks are now being examined was one of the largest contributors), the actual deaths were fewer than the computed, and that a considerable profit must have accrued from this source. It is clearly incorrect to assume that the mortality from any disease has been abnormally high, unless it can be shown to have exceeded its accustomed limits as a factor in the general mortality.

TABLE (M).—*Diseases of Brain and the Nervous System.*

Years of Assurance.	Actual Rates of Mortality.	Computed Rates of Mortality.	Actual Deaths to 100 computed.	Averages of consecutive Years.
0	·000528	·001279	41·28	
1	·000969	·001362	71·14	
2	·001383	·001456	94·99	102·6
3	·001712	·001554	110·17	
4	·002099	·001659	126·52	127·1
5	·002251	·001762	127·75	
6	·002518	·001855	135·74	131·8
7	·002566	·002006	127·92	
8	·003150	·002127	148·09	145·0
9	·003199	·002254	141·93	

On comparing the actual with the computed deaths in this class, it is seen that after the second year of assurance selection ceases to exert its favourable influence on the mortality of assured lives. Thereafter the death-rate experienced is heavier than that which prevails amongst the general population; and the unfavourable difference increases in a marked manner with the length of time the lives have been under observation. On referring to Table (C), it will be seen that the probabilities of dying under this head increase very rapidly with age—a large proportion of those who succumb

to these diseases being persons passing through the last few decades of life. The fact that the assuring classes are more subject to these diseases, appears on reflection to be so natural a conclusion that statistics are hardly required to prove it. The bulk of the population is comprised of those who gain their livelihood by manual labour, while a large proportion of the insured follow avocations which require mental rather than physical exertion. The professional man, also, has responsibilities and anxieties connected with his daily work, to which the artizan is a stranger, and which must render him less able to withstand the attacks of diseases of the brain and the nervous system. The experience of the New York Mutual is different in this respect from that of Scotch offices. The rates of mortality they experience from diseases of this nature, are less than those which prevail amongst the inhabitants of New York, and are considerably below those derived from British assured lives. This point will be briefly referred to afterwards.

Table (N).—*Heart Diseases.*

Years of Assurance.	Actual Rates of Mortality.	Computed Rates of Mortality.	Actual Deaths to 100 computed.	Averages of consecutive Years.
0	·000198	·000917	21·59	
1	·000436	·000980	44·49	
2	·000657	·001049	62·63	65·4
3	·000764	·001121	68·16	
4	·000990	·001197	82·72	70·4
5	·000738	·001272	58·02	
6	·001228	·001358	89·83	83·1
7	·001100	·001440	76·38	
8	·001422	·001525	93·24	86·1
9	·001274	·001612	79·03	

In this group of diseases the influence of selection is strongly manifested, and appears to extend as far as we have been able to contrast the actual and the computed rates of mortality. In later years of assurance, there is good reason to believe that the mortality of assured lives from heart disease is higher than that of the general population; for Mr. Meikle, in his *Observations on the Mortality of Assured Lives*, has shown that, after age 45, the proportion of deaths from heart diseases to the total deaths, is greater among policyholders than among the general male population. Below age 45, the proportion is in favour of the assured. On referring to Table (D) it will be seen that, in the ninth year of assurance, of

the total lives at risk, 54·6 are under age 45, so that the heavier mortality to which assured lives above that age are subject will not, until after the first ten years of assurance, perceptibly weigh down the rate.

TABLE (P).—*Lung Diseases.*

Years of Assurance.	Actual Rates of Mortality.	Computed Rates of Mortality.	Actual Deaths to 100 computed.	Averages of consecutive Years.
0	·000506	·001628	31·08	
1	·000848	·001732	48·96	
2	·001164	·001848	62·99	57·5
3	·001024	·001970	51·98	
4	·001536	·002098	73·21	64·2
5	·001230	·002228	55·21	
6	·001810	·002378	76·12	68·0
7	·001512	·002522	59·95	
8	·001600	·002674	59·84	66·5
9	·002067	·002827	73·12	

The main diseases in this group are bronchitis, pleurisy, pneumonia, and asthma—consumption being included in the class of tubercular diseases. In year 0 of assurance the actual deaths are, roughly speaking, about one-third of the computed; in year 1, about one-half; and in year 2, about two-thirds. During the following years the ratios show irregularities, which are to a great extent lost sight of in the column of averages. The two rates do not appear to approach each other in the later years, and it is probable that the influence of selection in reducing the mortality from this class of diseases, extends for a much longer period than the first decade, if indeed it does not endure to the last years of assurance. Though no direct statistical information can be obtained on the point, it is very probable that these diseases are not so prevalent amongst the upper and middle classes as amongst the labouring population, and especially those who work in the open air and are exposed to the inclemency of the weather. In considering the peculiarities which are traceable in the mortality of assured lives, it is desirable to bear in mind that the medical examination, if it is the chief, is not the only determining influence, and that many features may be correctly ascribed to the conditions and modes of life which prevail in those ranks of society from which the policyholders have been drawn.

TABLE (Q).—*Diseases of the Digestive Organs.*

Years of Assurance.	Actual Rates of Mortality.	Computed Rates of Mortality.	Actual Deaths to 100 computed.	Averages of consecutive Years.
0	·000704	·001003	70·19	
1	·000884	·001059	83·47	
2	·001109	·001123	98·75	
3	·001467	·001189	123·38	108·0
4	·001297	·001259	103·02	
5	·001419	·001328	106·85	
6	·001540	·001408	109·38	
7	·001764	·001483	118·95	111·9
8	·001778	·001561	113·90	
9	·001727	·001641	105·24	

In this important class of diseases, selection has only a transient influence. In year 3 of assurance, the actual deaths rise above the computed, and in subsequent years the mortality experienced is unfavourable. The figures before us strengthen the opinion that diseases of the stomach and liver attack most readily those who, while following avocations of a sedentary character, are able to command the luxuries of the table. Professional and business men form the majority of assured lives, but are in a minority in the population; and the influence of their mode of life will account for much of the unfavourable mortality experienced under this head by life offices.

TABLE (R).—*Diseases of the Urinary Organs.*

Years of Assurance.	Actual Rates of Mortality.	Computed Rates of Mortality.	Actual Deaths to 100 computed.	Averages of consecutive Years.
0	·000088	·000321	27·41	
1	·000157	·000341	46·04	
2	·000164	·000363	45·18	
3	·000290	·000386	75·13	
4	·000393	·000411	95·62	
5	·000530	·000436	121·56	102·2
6	·000416	·000465	89·46	
7	·000321	·000495	64·85	
8	·000483	·000525	92·00	86·1
9	·000566	·000557	101·62	

Selection reduces the actual rate of mortality very considerably during the first three years of assurance; but, on account of the irregularities in the figures representing the experience of the offices, the measure of its force in the later years is somewhat uncertain. The ratio of the actual to the computed deaths attains its highest point in year 5, and after declining to year 7, again

rises. From the column of averages, it would appear that the influence of selection is not exhausted at the end of the decade.

Gout is a predisposing cause to disorders of the bladder and kidneys; and as the returns of the Registrar-General show that it is becoming more and more frequent, an increasing number of deaths amongst assured lives may be anticipated from diseases of the urinary organs.

The remaining classes of diseases are unimportant, and are not of a nature to be affected by the medical examination. The number of deaths which they caused among the persons assured in the 10 Scotch offices previously referred to, were too few to form satisfactory data for determining the probabilities of dying, and they were not computed. The materials are consequently not in existence, which would enable me to contrast the actual and computed rates of mortality from these causes of death, were it thought desirable to do so. In order, however, to complete as far as possible the preceding series of tables, the computed rates of mortality from these minor groups of diseases, and the class of violent deaths combined, were calculated. The results are here given:—

TABLE (S).—*Minor Groups of Diseases and Violent Deaths.*

Years of Assurance.	Computed Rates of Mortality.	Years of Assurance.	Computed Rates of Mortality.
0	·001748	5	·002213
1	·001815	6	·002356
2	·001913	7	·002503
3	·002004	8	·002664
4	·002117	9	·002829

The effects of selection as regards the partial death-rates having now been investigated, it appeared desirable, before leaving the subject, to contrast in a final table the computed and the actual rates of mortality from all causes. To effect this, the figures given in the last table were added to the corresponding rates relating to zymotic diseases, tubercular diseases, and the other groups, thus obtaining in a single column the computed rates from all causes in the successive years of assurance. The actual rates were calculated from the statistics on pages 249–253 of the volume published by the Institute of Actuaries, containing the mortality experience of 20 offices. They differ slightly from those yielded by the numbers at risk and deaths, tabulated by Mr. Sprague in his paper "On the Rate of Mortality among Assured Lives as influenced by the duration of the Assurance" (*J.I.A.*,

xv, 338), as the figures for ages under 15 and above 75 were not included in his calculations.

Table (T).—*Deaths from all Causes.*

Years of Assurance.	Actual Rates of Mortality.	Computed Rates of Mortality.	Actual Deaths to 100 computed.	Averages of consecutive Years.
0	·004592	·012809	35·85	
1	·007622	·013166	57·90	
2	·009894	·013705	72·19	
3	·011498	·014206	80·94	
4	·013161	·014759	89·17	87·4
5	·013079	·015290	85·54	
6	·014254	·015929	89·49	
7	·014893	·016613	89·65	
8	·015342	·017301	88·68	89·6
9	·016296	·018010	90·49	

In the first five years, the proportion of actual deaths to the computed, rapidly rises; but, during the remaining years, the ratios remain almost stationary. As the unfavourable influence of the withdrawal of healthy persons by surrendering their policies, is strongly felt after the first 10 years of assurance, it may be assumed that, had the table extended further, the actual rates would have approached nearer the computed.

The sum of the effects of selection under these various divisions, gives to the mortality of assured lives those characteristics which are observed on comparing the rates experienced, with those derived from a community of unselected lives. As when two rays of light, emitted by dissimilar bodies, are passed through the spectrum, the distinctive qualities of each are seen to proceed from a different combination of the primary colours; so in separating into those factors which medical science has defined, the mortality of two communities, the features of each will be found to arise from the different intensities of the various causes of death.

In the course of our remarks, we have referred to some of the differences which exist in the rates of mortality of assured lives in this country and America. Under the head of Diseases of the Brain and Nervous System, it was mentioned that the death-rates from such disorders in Scotch offices, was considerably higher than that found to prevail amongst the members of the Mutual Life Assurance Society of New York. This difference in favour of the American lives is not an exceptional case, and will be found to characterize their general mortality experience. In the actuarial report of this office, published in 1874, it is shown that between

the ages of 27 and 63, the probabilities of dying do not in any instance exceed 80 per-cent of the Institute H^M.

By contrasting the mortality in various years of assurance a similar result is obtained, except in the first year of assurance, when there is a slight difference in favour of the English offices. The singularly favourable experience of this office, is thus referred to in the report:—" The only explanation which suggests itself, is " that the care here exercised in the selection of risks, has been " greater than in the English offices, and that its effects were not " manifested in the first year of assurance, when the deaths were " mainly due to accidents and to the sudden and irresistible " attacks of disease, but subsequently they became more and more " apparent, when the strong constitutions, correct habits of life, " and a favourable family history, presented a higher vitality to " resist the several forces which destroy life." On comparing the experience of this office with those of the Scotch companies, as regards the mortality under different classes of diseases, difficulties arise in the way of accepting the above explanation as satisfactorily meeting the circumstances of the case. Thus, if the lives accepted by the *New York Mutual* were more carefully selected than those in British offices, the most marked differences would be found in the mortality from those diseases in which selection has most influence—pre-eminently in consumption. On contrasting, however, the numbers dying from this disease with the corresponding deaths in the Scotch offices, as calculated by Mr. Meikle, it is found that the American office has experienced the higher rate of mortality. This is shown in the following table:—

TABLE (U).

Number of Lives dying of Consumption, out of 10,000 *exposed to risk, in each of the undermentioned years of Assurance.*

Years of Assurance.	Scotch Offices.	New York Mutual.
0	4	7
1	10	17
2	19	21
3	21	25
4	20	24
5–9	21	28

On the other hand, in the mortality arising from diseases of the brain and nervous systems, over which the highest medical skill exercised in the selection of proposers has only a limited influence, the death-rate experienced by the American office is considerably below that deduced from the records of the Scotch

companies. Considering the high position held by the medical schools of this country, and that assurance offices here avail themselves of the best talent of the profession, it appears very unlikely that their services will yield results so inferior to those of American physicians. I am rather disposed to refer the very favourable experience of the *New York Mutual* to other causes.

By glancing over the various tables showing the effects of selection, it is at once apparent that it affects the mortality under the several groups of diseases very differently. In some instances, its influence is strong and comparatively brief; in others, it is less pronounced, but more prolonged. Its effects are most conspicuous in the case of tubercular diseases, and in this direction the medical advisers of life assurance institutions are very successful in protecting the interests of the offices for which they act. Zymotic diseases, on the contrary, are felt with at least equal severity in the chosen ranks of the assured as amongst the miscellaneous communities of the population. Of the other classes of diseases, it may be remarked that the differences between the actual and the computed rates of mortality, appear to arise less from the medical examination, than from the influence of the habits and conditions of life which prevail amongst those grades of society to which the majority of the assured belong.

Valuations of Policies in Bankrupt Life Insurance Companies.

[From the *Spectator* of New York.]

THE question as to how policies shall be valued for the purpose of distributing the funds of a bankrupt life insurance company, is one of considerable interest, and has given rise to some discussion and met with different solutions in different cases. The usual method adopted has been to compute the net values of such policies, by the legal standard of valuation, as of the date of the appointment of the receiver. It is contended, however, that this is not correct, for the reason that the statutes do not fix the standard of valuation for any purpose except to enable the Insurance Superintendent to judge of the solvency or insolvency of the company. When he has reported the company insolvent, and the courts have dissolved it, the policyholders appear in a different attitude; not as a mass of creditors, requiring a reserve fund for their protection, but as a number of separate and individual claimants, each having a claim for damages for the breach of his own contract. The court is then obliged to determine what is the proper measure for damages to each claimant; and, in doing

so, must necessarily consider what is the contract in each case that has been broken by the failure of the company.

It is contended that the net method of valuation ignores the actual contract, because it charges the policyholders only with net premiums as receivable *in futuro,* instead of the actual premium stated in the policy and which the policyholder has contracted to pay. Hence it was that in the distribution of the funds held by the Insurance Department of New York for the Widows and Orphans Benefit Life and several other companies, Justice Larned, of the Supreme Court, adopted the rule that the policyholder should be credited with the value of his insurance as a single premium and charged with the value of the gross premiums stated in his policy. The effect of this method was, of course, to wipe out the value of most of the running policies, and to create some curious anomalies. For example, if two persons had taken policies of the same kind, age, and amount, one upon the stock plan with a low rate of premium, and the other on the participating plan with a higher rate of premium, the non-participating policy, the value of whose future premiums was small, might have a value; and the other, with a large value in future premiums, would have none; so that the one who had paid the most would get the least. Again, a person stopping payment and taking a paid-up policy just before the failure, would have a good value; whereas, if he paid another premium, so as to carry his policy beyond the failure, its value would be nothing.

This conclusion of the matter is justified by Justice Larned, in an opinion printed in the *Journal of the Institute of Actuaries* for January 1880, page 132. The reason is, that the method looks not at the value accumulated by past payments upon the policy, but at the future value of the contract to the policyholders. It must be evident, however, that if we are to take the gross premiums named in the policy, we should also take a gross or loaded premium for the value of the insurance, or the single premium from which the value of the future premiums is deducted.

Another gross method suggested by the legal mind is more reasonable. We refer to the rule laid down by Justice Bradley, of the United States Supreme Court, in the case of Statham *v.* New York Life Insurance Company, reported 5 Big. Life and Acc. R. 607. He takes the premium stated in the policy, and deducts it from the gross premium chargeable at the advanced age, by the same table of rates as used in fixing the premium in the policy. This difference is the excess which the policyholder will have to pay in the future, in order to get the same kind of

policy in another company. The value of this difference considered as an annuity, is therefore the loss to him from the breach of his policy. This makes a homogeneous computation, both quantities taken being gross premiums, while it satisfies the legal mind with facts in place of assumptions. It produces, however, larger values than the net method. We believe it has been adopted by the courts of Virginia.

In the State of New York, however, the net method seems to be fixed upon and settled by the Court of Appeals in the matter of the Security Life Insurance and Annuity Company, reported 78 New York, 114. It is the most practical and equitable mode of valuation that can be adopted. As the companies are compelled to keep reserves on that basis, it results that the net reserve on each policy represents the measure of the interest of that policy in the common funds. The company is judged by that standard, and if it fall short thereof, it is wound up. The same valuation by which its insolvency is decreed, should govern the distribution of the fund. Otherwise a company found insolvent by the net method of valuation, might, by distributing its funds by the gross method suggested by Judge Larned, exhibit a large surplus after paying those policyholders whose claims had value. Questions would then arise: Why was the company dissolved? To whom does the surplus belong? It might be a good scheme for stockholders, who could thus derive a handsome benefit from their own mismanagement, while the policyholders who had created the fund footed the bills.

[The interesting question discussed in the above article has on various occasions been considered in this *Journal*: *see* Mr. Sprague's paper *On the Liquidation and Reconstruction of an Insolvent Life Insurance Company,* xvi, 229; Mr. Bunyon *On the Valuation of Claims upon Current Policies in the Liquidation of a Life Office, with reference to the Decisions in Bell's and Lancaster's cases,* xvii, 1; *On Valuation of Policies for proof in Liquidation,* xviii, 32; *On the Liquidation of an Insolvent Life Office,* xx, 281; and Mr. Pitcairn's letter on the same subject, xv, 385. We are inclined to agree with the writer in the *Spectator* that the same valuation by which the insolvency of an office is decreed, should govern the distribution of the fund; but we are of opinion that the proper conclusion to be drawn from this position, is that a net-premium valuation is not the proper means to determine whether a life office is solvent or not.—ED. *J.I.A.*]

JOURNAL

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On the Rates of Mortality in Victoria, and on the Construction of Mortality Tables from Census Returns by the Graphical Method of Graduation. By ARTHUR FRAS. BURRIDGE, *of the Equity and Law Life Assurance Society, and Fellow of the Institute of Actuaries.*

[Read before the Institute, 27 March 1882.]

THE records of the Colony of Victoria afford a field for scientific enquiry in many directions, and questions of interest to the physician, the economist, or the actuary, are to be found therein. Its separate existence is but of thirty years' duration, as it was on 1 July 1851 that the Queen's proclamation was issued erecting it into a separate colony, to be called after her Majesty's name.

Many social and economic questions concerning the colony have already been considered; and no longer ago than 1879, the present Government Statist, Mr. Henry Heylyn Hayter, read an exhaustive paper before the Statistical Society. The object of the present enquiry, however, has, so far as I am aware, not received previous attention; and as it is one of strictly professional and scientific interest, it is hoped that it may be appropriately considered by the members of this Institute.

That object is to deduce from the registered vital statistics of the colony a mortality table exhibiting the actual rate of mortality which has been experienced; and to compare that table with other standard tables, in order to note the effect of the different conditions of life prevalent in Victoria. For it is to be expected that a colony, whose geographical extent is only slightly inferior to that of Great Britain, whose difference of temperature between summer and winter varies less than that of Lisbon or the favoured Riviera, and whose population is far greater than any other Australasian colony, would possess special characteristics calculated to mark their effect on the rate of mortality of the people.

The Government returns are well known for their extent and completeness; and a study of these models, for such they are, will show the value of the method of registration introduced by the late Registrar-General, Mr. W. H. Archer, and continued by the present Government Statist. In addition to the ordinary official returns, an annual digest of the statistics of the colony has been published by the Government Statist since 1873, entitled *Victorian Year Book,* containing a large quantity of interesting matter. We have, therefore, ample materials for the work in hand, and the best testimony to their value seems to be, that by their assistance the subject can be readily pursued by an independent enquirer.

The last Census in the colony was taken on 3 April 1881; but when these calculations were commenced, the approximate returns given in Table A only had been issued. Without a detailed

Table A.—*Statement of the Population at the Period of each Enumeration from* 1836 *to* 1881. (*From the "Approximate Returns" of Census* 1881.)

Date of Enumeration.	Population.			Increase per-cent on previous Census.
	Males.	Females.	Total.	
25 May 1836	142	35	177	
8 Nov. 1836	186	38	224	26·6
12 Sept. 1838	3,080	431	3,511	1467·5
2 March 1841	8,274	3,464	11,738	234·3
2 March 1846	20,184	12,695	32,879	180·1
26 April 1854	155,887	80,911	236,798	206·2
29 March 1857	264,334	146,432	410,766	73·5
7 April 1861	328,651	211,671	540,322	31·5
2 April 1871	401,050	330,478	731,528	35·4
3 April 1881	450,286	408,296	858,582	
	[452,083	410,263	862,346]	17·9

The figures in brackets were subsequently given as the result of the tabular enumeration, and, it is stated, may be accepted as final.

enumeration of the population at each age, it was felt that these figures were not applicable; for in such a rapidly increasing population, any assumption as to age based on the returns of a former Census, would certainly be erroneous.

In these circumstances, recourse was had to the previous Census of 2 April 1871; and from this and the deaths of the same year, the Mortality Table now presented was deduced. As the required population was that living in the middle of the year, the enumerated males and females were respectively increased by 1·0048 and 1·0108.* The population thus increased, and the deaths, are given for quinquennial groups of ages in the following table.

[TABLE B, *see next page.*]

It may be objected that the results of a single year are an insufficient basis on which to found a representative mortality table, and that, unless that year happens to be an average one, the results are misleading. It is replied, that, although the results thus obtained do not measure with perfect accuracy the average mortality, yet such results undoubtedly show the rate for the year in question, and, to a great extent, reveal the general rate of mortality.

The year 1871 is a convenient one, because when the full returns of the recent Census are available, an interesting comparison can be made of the rate of mortality in the two census years, and of the rate that prevailed throughout the intervening period.

Before passing on to explain the method employed to deduce the rate of mortality for each age, it will be useful to consider the special features to be found in the returns of deaths for several years, both in Victoria and other parts of Australasia.

* These corrections were found by comparing the censuses of 1861 and 1871, which showed the following annual rates of increase (the interval between the censuses being 9·986 years):—Males, 1·02014; Females, 1·04562. The interval between 2 April and 1 July is about ·24 of a year; and the corrections become

$$\text{Males, } (1{\cdot}02014)^{\cdot 24} = 1{\cdot}0048;$$
$$\text{Females, } (1{\cdot}04562)^{\cdot 24} = 1{\cdot}01076.$$

TABLE B.—*Census Returns and Deaths—Victoria*, 1871.

Ages.	Males enumerated, 1871.	Mean Population, Males, 1871.	Deaths of Males, 1871.	Deaths per-cent of mean Male Population.		HM μ_x	Females enumerated, 1871.	Mean Population, Females, 1871.	Deaths of Females, 1871.	Deaths per-cent of mean Female Population.		HF μ_x	Ages.
				Victoria.	English No. 1.	For central age.				Victoria.	English No. 1.	For central age.	
		=1·0048 × (2)		=(4) ÷ (3)				=1·0108 × (8)		=(10) ÷ (9)			
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
0– 5	58,937	59,220	2,464	4·161	6·84	...	57,751	58,375	2,115	3·624	5·86	...	0– 5
5–10	53,549	53,806	213	·396	·96	...	52,954	53,526	192	·359	·92	...	5–10
10–15	43,012	43,218	122	·282	·51	·00366	42,573	43,033	93	·216	·55	·00468	10–15
15–20	26,264	26,390	102	·387	·72	·00357	28,292	28,598	102	·357	·80	·00709	15–20
20–25	24,004	24,119	143	·593	·92	·00681	25,418	25,693	122	·475	·88	·00763	20–25
25–30	28,875	29,014	205	·707	·99	·00682	25,395	25,669	168	·655	1·02	·01166	25–30
30–35	33,257	33,417	255	·763	·96	·00804	24,100	24,360	184	·755	1·00	·01171	30–35
35–40	38,405	38,589	369	·956	1·24	·00933	23,625	23,880	227	·951	1·29	·01205	35–40
40–45	34,876	35,043	413	1·179	1·21	·01067	17,945	18,139	178	·981	1·16	·01293	40–45
45–50	22,091	22,197	332	1·496	1·70	·01341	11,368	11,490	140	1·219	1·51	·01392	45–50
50–55	16,572	16,652	318	1·910	1·85	·01725	8,528	8,620	125	1·450	1·57	·01541	50–55
55–60	8,738	8,780	235	2·677	2·86	·02348	4,784	4,836	83	1·716	2·54	·01965	55–60
60–65	6,335	6,365	196	3·079	3·40	·03387	3,779	3,820	99	2·592	2·92	·02751	60–65
65–70	3,053	3,068	156	5·085	5·71	·04937	1,891	1,911	91	4·762	5·18	·04264	65–70
70–75	2,004	2,014	150	7·448	7·34	·07398	1,321	1,335	66	4·944	6·61	·0676	70–75
75–80	670	673	81	12·036	12·59	·1168	458	463	39	8·424	11·72	·1130	75–80
80–	408	410	62	15·122	...	...	296	299	43	14·38	...	...	80–

The following table, C, shows the death-rate for the ten years, 1871 to 1880. I have estimated the population for this table upon the approximate returns of the Census of 1881, and it will be found that the rate for Victoria thus derived does not coincide with the rate given in Table D (col. 2). The latter table has been extracted from the *Victorian Year Book*, 1879-80; and since the estimated population there employed was in excess of the actual population, as taken at the last census, the death-rate deduced therefrom was too light.

TABLE C.—*Annual Death-Rate of Victoria*, 1871 *to* 1880. (*Mean Population estimated from Approximate Returns of Census* 1881.)

Year.	Estimated Mean Population.		Number of Deaths.		Deaths per 1,000 of the Mean Population.		
	Males.	Females.	Males.	Females.	Males.	Females.	Total.
1871	402,975	334,047	5,845	4,072	14·5	12·3	13·5
1872	407,668	341,186	6,308	4,523	15·5	13·3	14·5
1873	412,416	348,477	6,565	4,936	15·9	14·2	15·1
1874	417,220	355,924	6,994	5,228	16·8	14·7	15·8
1875	422,080	363,530	8,563	6,724	20·3	18·5	19·5
1876	426,995	371,300	7,716	5,845	18·1	16·1	17·0
1877	431,968	379,234	7,345	5,431	17·0	14·3	15·8
1878	437,000	387,339	7,179	5,523	16·4	14·3	15·4
1879	442,090	395,616	7,033	5,087	15·9	12·9	14·5
1880	446,445	403,898	6,610	5,042	14·8	12·5	13·7
Means	...	...	...	...	16·5	14·3	15·5

The results will be seen to be very remarkable. Beginning with a low rate in 1871, the death-rate rapidly increases each year to a maximum in 1875, after which it diminishes with equal rapidity, until finally, in 1880, it reduces again to the rate for the year 1871. It is further found that these extraordinary fluctuations in the death-rate were not confined to Victoria, but prevailed with curious coincidence throughout Australasia, as shown in Table D (p. 314).

So far as I have been able to find, the only explanation which has been offered of these fluctuations is that, "in 1875, and to a certain extent in the preceding and succeeding year, the death-rate was swelled by epidemics of measles and scarlatina"; but it must be felt that these causes are totally inadequate to produce such astonishing results, and attention is now drawn to them in the hope that some light may be thrown on the subject.

Table D.—*Annual Death-Rate of Australasia for the Years* 1871 *to* 1879. (*Extracted from Victorian Year Book*, 1879–80.)

Year. (1)	Victoria. (2)	New South Wales. (3)	Queensland. (4)	South Australia. (5)	Western Australia. (6)	Tasmania. (7)	New Zealand. (8)
1871	13·43	12·54	14·83	12·87	...	13·38	10·13
1872	14·23	14·11	14·97	15·33	14·02	13·79	11·68
1873	14·74	13·84	16·06	13·48	16·24	14·52	12·67
1874	15·30	15·12	17·98	17·05	18·74	16·21	13·05
1875	18·76	18·09	23·80	19·45	17·88	20·	15·92
1876	16·33	18·11	18·82	16·28	14·18	16·54	12·66
1877	15·03	15·28	17·29	13·99	15·70	19·17	11·47
1878	14·62	15·88	20·41	15·44	14·07	15·66	11·01
1879	13·64	...	...	...	...	...	...
Means	15·12	15·37	18·02	15·49	15·83	16·16	12·32

Another point for consideration is the proportion of deaths from various causes, as this cannot fail to have an important bearing on the rate of mortality. In order to give a general idea of the relation which the principal causes of death bear to each other in Australasia, the following table has been compiled from the returns of different colonies.

Table E.—Causes of Death.—*Table showing the Proportion of Deaths from each Class of Disease to the whole number registered.*

	England and Wales. Average of 10 years.	Victoria. 1873.	Victoria. Average of 16 years.	New South Wales. 1873.	South Australia. 1875.	Tasmania. 1873. "An Average Year."	New Zealand. 1875.
Zymotic Class ...	22·74	25·45	30·47	21·03	21·	15·95	30·04
Constitutional Class	18·75	13·95	13·06	13·61	20·	16·35	12·2
Local Class ...	38·88	38·64	32·01	38·44	37·	38·63	35·1
Developmental Class	16·12	14·07	15·20	17·34	18·	21·87	13·45
Violent Deaths ...	3·51	7·89	9·26	8·8	4·	7·2	7·83
Not specified ...	...	...	...	0·78	...	...	1·38
	100·00	100·00	100·00	100·00	100·	100·00	100·00

Explanation of Classification.

Zymotic Class.—Diseases that are epidemic, endemic or contagious; such as small-pox, measles, diphtheria, typhoid fever, cholera, &c.

Constitutional Class.—Gout, cancer, scrofula, phthisis, hydrocephalus.

Local Class.—Apoplexy, paralysis, diseases of the nervous system, diseases of the circulatory system, bronchitis, pneumonia, asthma.

Developmental Class.—Infantile diseases, teething, premature birth, &c.; diseases of adults, atrophy and debility, old age, &c.

Violent Deaths.—These include accidents of all kinds; the principal being fractures, contusions, &c., burns and scalds, drowning and suffocation; and, lastly, judicial hanging.

The returns of the causes of death are given in great detail in the *Year Book,* and the following summary extracted therefrom gives a comparison of the average annual rate in Victoria and England and Wales for each class of deaths.

TABLE F.—*Average number of Annual Deaths per* 100,000 *of Mean Population.*

	VICTORIA (Average of 25½ years, 1853 to 1878).	ENGLAND AND WALES (Average of 25 years, 1850 to 1874).
Zymotic diseases . . .	513·84	503·87
Constitutional diseases . .	218·25	420·54
Local diseases	552·65	849·97
Developmental diseases . .	235·27	355·15
Violent deaths. . . .	137·65	76·13
	1657·66	2205·66

Although it is beyond the province of this paper to discuss the various classes of disease, yet the mortality from phthisis, classed among the constitutional diseases, cannot be altogether passed over.

The general opinion on the climate of Australasia respecting consumption is very favourable, and it may cause some surprise to find how many deaths are registered under that disease. It might be thought that a considerable number of these deaths would arise from the immigration of phthisical invalids, who sought prolongation of life in a climate possessing many favourable conditions to the consumptive patient; but it is shown in the following table that, for the years under observation, the proportion of such cases was exceedingly small.

Taking the mean for the ten years, 1864 to 1873, we find (*Year Book,* 1873) that the deaths from phthisis bore the relation of 11·96 to 10,000 of the population, and for the 19 years, 1861 to 1879 (*Year Book,* 1879–80), the figures are, for Melbourne and suburbs, 21·97, and for extra-metropolitan districts, 8·57, or a general rate of 15·27.

The death-rate from phthisis in Melbourne is found to closely approximate to that of the United Kingdom, which in 1875 was 22·24; in 1876, 21·45; and in 1877, 21·01 per 10,000 persons living; but the rate for the whole of Victoria is considerably lower.

The following table shows the deaths from phthisis in Victoria during the four years, 1876 to 1879, and the period of residence in Australasia (*Year Book,* 1879–80).

Period of Residence in Australasia.	Total deaths of Phthisis in Victoria in 4 years.	Proportion per-cent.
Under 1 month . . .	19	·47
1 month to 1 year . .	86	2·15
1 year to 2 years . .	55	1·38
2 years to 5 years . .	137	3·43
5 years and upwards . .	2,595	64·91
Born there . . .	1,106	27·66
	3,998	100·00

Immigration and Emigration.

An abstract is given in Table G of the returns for the years 1870 to 1879, showing the net gain to the colony under this head. In these returns, persons above 12 years of age are classified as adults, those between 1 year and 12 years as children, and those under 1 year as infants. It will be seen that the number of emigrant infants was each year in excess of the immigrants, amounting in the 10 years to 2,128; and this fact points to the tendency of families to quit the colony. In other words, Victoria exports infants.

The returns were not available for use in the calculations, inasmuch as they only record the migration by sea. Some proportion of the immigrants arriving at a Victorian port would proceed up country, and, passing away overland, would be lost to the colony.

Table G.—Abstract of Migration Returns.—*Table showing the* excess *of Immigration over Emigration in Victoria for the years* 1870 *to* 1879.

Year. (1)	Males. (2)	Females. (3)	Adults. (4)	Children. (5)	Infants. (6)
1870	6,974	4,493	10,087	1,391	− 11
1871	5,100	3,282	7,453	1,054	−125
1872	371	1,381	1,624	382	−254
1873	2,470	696	3,626	−180	−280
1874	2,452	915	3,841	−169	−305
1875	2,869	533	4,018	−322	−324
1876	3,272	548	4,699	−563	−316
1877	5,351	1,902	7,327	19	− 93
1878	4,002	774	4,965	55	−244
1879	4,549	623	5,258	40	−126
	37,410	15,147	52,928	+1,707	−2,128

[The minus sign denotes excess of Emigrants over Immigrants.]

On the Construction of the Mortality Tables.

The method adopted to deduce the results in the mortality table from the recorded data was that employed by Milne in the construction of the Carlisle Table, and I have described it as minutely as possible, in order that anyone may test the correctness of the results.

Children under Five Years of Age.

In order to find the mortality of children from ages 0 to 5 in 1871, the births for the six years, 1866 to 1871, and the deaths for 1871, were required. They are given in the following table:—

Table H.—*Showing the Births for the six years*, 1866 *to* 1871; *and the Deaths for* 1871, *from ages* 0 *to* 4.

Year.	Births.		Age.		Deaths in 1871.	
	Males.	Females.			Males.	Females.
1866	12,670	12,340	Under 1 year	Under 1 month	619	450
1867	13,093	12,515		1 to 3 months	258	240
1868	13,841	13,402		3 „ 6 „	324	281
1869	13,225	12,815		6 „ 12 „	509	433
1870	13,997	13,154	1 to 2 years		463	413
1871	14,000	13,382	2 „ 3 „		135	136
			3 „ 4 „		79	94
			4 „ 5 „		77	68

The facts concerning the ages of infants dying under one year of age are recorded with greater minuteness in Victoria than in any other part of the world. The first year of age is divided into four periods, namely, under 1 month; 1 to 3 months; 3 to 6 months; 6 to 12 months.

This careful registration of the deaths of infants gives the opportunity to make an investigation into the mortality at several stages of the first year of life; and when the importance of this period is remembered, and how large a number of infants die within it, any enquiry which will add to our knowledge of the subject, cannot fail to be of considerable interest. From the figures in Table H, I have calculated the rate of mortality for each of the four periods of the first year; the results are given in Table I (p. 319).

With the object of throwing more light on this point, it was desired to find some table showing the probability of surviving

different periods of the first year of life, in order to make a comparison with the results deduced from the Victorian returns. The only tables which occurred to me as applicable to the purpose were those compiled in 1874 by Mr. Charles Ansell, Jun., Actuary to the National Life Assurance Society, from statistics collected for the purpose by that office from families in the upper and professional classes. By the kindness of the National Office, I am enabled to make use of a table entitled "Intensity of Mortality under one year of age—Upper Class Experience Adjusted", which shows, out of 100,000 births, the numbers which survive each day from birth to the twenty-first day, and each week from the third week after birth to the end of the first year. I have rearranged the facts in this table to correspond with the Victoria returns, and the two are compared in Table I (p. 319).*

It need hardly be remarked that the "Upper Class" experience was derived from most favourable sources, and must be held to represent selected babyhood, and that the comparison with the returns of the general population of Victoria shows a very moderate rate of infantile mortality in the colony. The product of the probabilities of surviving each of the periods in the first year should amount to the total probability of surviving that year (p_0), and they will be found to do so.

We have now to deal with the mortality from ages 1 to 4.

On the assumption that the births are equally distributed throughout the year, the births of the two years 1870 and 1871 take place on 1 January 1871, and to find q_0 we have the equation

$$\frac{\text{Deaths 1871 (under 1 year)}}{\frac{1}{2}\text{ (Births in 1870 and 1871)}} = q_0.$$

By similar equations, the remaining four values of q are found, and these values appear without adjustment in the mortality table.

Table J (p. 319) is a comparison of the mortality under 5 years of age by various tables; the rate for Victoria is again seen to be unusually favourable, the only tables which surpass it being the Healthy English and the Peerage.

* Since this was written, Mr. Geo. King, of the Alliance Life Office, has kindly drawn my attention to an investigation by Dr. Farr, in the 3rd English Life Table, into the mortality for each month of the first year of age. I have arranged the results to compare with the Victoria returns, and added them to Table I.

TABLE I.—INFANTILE MORTALITY.—*Showing out of 10,000 Births the Numbers who survive, the Decrements, and the Probability of surviving at each Period of the first Year of Life.*

Ages.	Victoria.						Upper Class Experience.						English Table No. 3.					
	Males.			Females.			Males.			Females.			Males.			Females.		
	Numbers living.	Decrement.	Probability of surviving Period.	Numbers living.	Decrement.	Probability of surviving Period.	Numbers living.	Decrement.	Probability of surviving Period.	Numbers living.	Decrement.	Probability of surviving Period.	Numbers living.	Decrement.	Probability of surviving Period.	Numbers living.	Decrement.	Probability of surviving Period.
0– 1 Month	10,000	441	·9559	10,000	337	·9663	10,000	344	·9656	10,000	235	·9765	10,000	523	·9477	10,000	406	·9594
1– 3 Months	9,559	185	·9807	9,663	179	·9814	9,656	142	·9854	9,765	107	·9891	9,477	319	·9663	9,594	264	·9725
3– 6 ,,	9,374	233	·9753	9,484	212	·9777	9,514	161	·9831	9,658	149	·9846	9,158	314	·9658	9,330	265	·9718
6–12 ,,	9,141	362	·9602	9,272	330	·9647	9,353	241	·9743	9,509	208	·9782	8,814	...	·9479	9,065	...	·9541

TABLE J.—INFANTILE MORTALITY.—*Showing the Probability of surviving a Year from 0 to 4 by various Mortality Tables.*

Ages.	Victoria.		New South Wales.		English No. 1.		Healthy English.		Peerage.	
	Males.	Females.	Males.	Females.	Males.	Females.	Males.	Females.	Males.	Females.
0	·8779	·8942	·8892	·9039	·8406	·8674	·9480	·9578	·9218	·9405
1	·9613	·9644	·9548	·9558	·9370	·9396	·9845	·9859	·9838	·9836
2	·9882	·9880	·9814	·9813	·9661	·9663	·9904	·9905	·9914	·9930
3	·9930	·9915	·9882	·9881	·9756	·9766	·9932	·9933	·9961	·9944
4	·9943	·9935	·9913	·9920	·9821	·9824	·9945	·9945	·9956	·9944

Mortality over Five Years of Age.

It has been already stated that the method employed to deduce the rate of mortality at each age from the recorded data, was the graphical method applied by Milne to the Carlisle Table. A description of the process and a diagram illustrating it is given in his treatise, p. 101.

Taking the population in quinquennial groups of ages, or for any other interval that may be preferred, we draw, on a sheet of cross-ruled paper, parallelograms, of which the area represents the population in each quinquennial group, and the base the years of life, the parallelograms being ranged side by side and the bases in one straight line. Next, let a line (as little curved as the other conditions will admit of) be described through these parallelograms, so that the point describing it may continually approach from the youngest age to the oldest, "and let the line "thus described so cut each parallelogram that the area com- "prehended by the base, the two sides of the parallelogram "perpendicular thereto, and the portion of that line which is "intercepted between those sides may be equal to the area of the "same parallelogram."

Thus, for each corner of a parallelogram cut off by the curve, a corner of exactly similar area is added.

"So shall the number of the living in any assigned year of "age, be to the given number in the interval including that year, "in the ratio of the area insisting upon the portion of the base "corresponding with the year assigned, to the area of the "parallelogram in which it is found."

Similar parallelograms are set out for the deaths in the same groups of ages, and a curve drawn through them on the same conditions.

We now possess two curves, representing respectively the population and the deaths at each age; and dividing the deaths by the population we obtain the ratio m_x for all values of x*; and from

* The values of m_x were found by direct division by means of Fuller's Spiral Slide Rule, and this instrument was employed to perform all the multiplications and divisions necessary in the preparation of this paper. This useful form of the Slide Rule effects a considerable saving of labour. The Rule consists of a cylinder upon which is wound, in a spiral, a single logarithmic scale of 500 inches—or 41 feet 8 inches—long.

The setting and reading is worked by means of two indices, and the results can be read correctly to 4 and often to 5 places.

The Rule is manufactured by W. F. Stanley, Great Turnstile, London, and a full description of it is given in a pamphlet published by him. The price of the rule is £3.

this we derive p_x, by the formula $p_x = \frac{1 - \frac{1}{2}m_x}{1 + \frac{1}{2}m_x}$. On examining this column of p_x it was found that, although the general results were satisfactory, and the special features of the mortality under consideration were apparent, yet at many points it required adjustment. This was applied by a second graphical graduation. The values of p_x were set out on a sheet of cross-ruled paper, and a curve was then drawn through the points, so as to adhere as closely as possible to the original facts while removing obvious irregularities. In drawing this curve, the object is to pass among as many of the points as possible, and, by the judgment of the eye, to give due weight wherever the original observations are most strongly marked. This must not be mistaken to admit a freedom which would lead no two persons to draw the same curve from identical data, no more than liberty must be mistaken for licence. So long as the test is applied of making the expected deaths to coincide with the actual deaths, it is apprehended that no considerable difference will be found in curves deduced from the same data at various times or by different computers.

The results of the graduation are given in Table M (pp. 324-5). These show out of 10,000 births in Victoria the numbers which survive, the decrements, and the probability of living a year, at each age, for males and females. In the returns of deaths above 80, the ages are not given. I have continued the table for males to the end of life, by applying the English Table, No. 1—Males, taken one year older; and for females from age 70, by the same Table—Females, taken one year younger.

In order to test the general effect of the graduation, the population at each age was multiplied by $q_{x+\frac{1}{2}}$ (the population being that alive in the middle of the year): the result gave the expected deaths at each age; and these, arranged in quinquennial groups, will be found compared with the actual deaths in Table K (p. 322).

The results of the investigation show a remarkably favourable rate of mortality throughout life. When compared with the English table, it will be seen that the Victoria rate for males, is lower throughout life, except for the ages 50 to 54, where it is a trifle higher, and at age 55, where it is identical with Dr. Farr's value. It must be admitted, however, that the year under investigation, 1871, was a favourable one for the colony, the mortality in this and 1880 being lighter than in any one of the intermediate years; nevertheless, when due allowance has been made on this

Table K.—*Comparison of Expected and Actual Deaths by Victoria Table, Males and Females, and English Table No.* 1, *Males.*

Ages.	Victoria, 1871. Males. Expected.	Males. Actual.	Females. Expected.	Females. Actual.	English Table No. 1, Males. Expected.	Actual.
5–10	206	213	186	192	9,145	9,093
10–15	122	122	91	93	4,094	4,178
15–20	101	102	93	102	6,043	5,604
20–25	144	143	128	122	6,414	6,633
25–30	203	205	165	168	5,828	6,045
30–35	260	255	187	184	5,748	5,422
35–40	385	369	225	227	5,530	5,385
40–45	428	413	192	178	5,127	5,251
45–50	338	332	138	140	4,962	5,322
50–55	313	318	124	125	5,705	5,673
55–60	206	235	87	83	4,778	5,418
60–65	196	196	99	99	7,816	7,090
65–70	145	156	89	91	6,653	6,881
70–75	158	150	...	...	8,604	7,630
75–80	83	81	...	...	6,469	6,992
	3,291	3,290	1,804	1,804	92,916	92,917

account, the rate still remains considerably below the normal European one.

On comparing the Victoria tables with those for New South Wales, calculated by the late Professor Pell, and published in his paper (*Journal,* vol. xxi, p. 257), it is found that the Victoria rates both for males and females, excepting at ages 0 to 2, are lighter throughout the whole of life than those in the more northern colony; and that, with regard to the higher ages, the somewhat increased rate noticed in New South Wales, does not seem to have been experienced in Victoria.

The same peculiarity is observed in the relative rates for the two sexes in both colonies, that the rates for females are lower than those for males throughout the whole period of life. This would at present appear to be one of the characteristics of colonial mortality; and it must remain for the future to decide whether, as the relative numbers of the sexes approximate to the ratio in England, the relative rate will not similarly change and produce a higher rate for females during the child-bearing ages.

The Graphical Method applied to the English Table No. I.

With the object of showing the effect of a graphical graduation, I have prepared a mortality table from the returns employed by Dr. Farr in forming the 1st English Table, Males. These records, printed in the 5th Annual Report of the Registrar-General, p. 15, in a table headed "Population, Deaths, and Mortality of England, 1841", give the population in quinquennial groups, "calculated for 1 July 1841", and the deaths in 1841; and are therefore in a suitable form for graduation.

An account of the construction of the table is given by Dr. Farr in the Appendix to the 5th Report. The method of effecting the transition from the original "observations" of the mortality per-cent to the "observations corrected" is not, however, explained, and this is an obstacle in reconstructing the table. The original "observations" were derived directly from the facts returned at the census of 1841, corrected only for the increase of population and unenumerated ages.

Table L.—*Original "Observations" from which was deduced the English Table No. 1, Males.*

Age.	Mortality per-cent.	Age.	Mortality per-cent.
15–20	·718	55–60	2·860
20–25	·918	60–65	3·395
25–30	·991	65–70	5·706
30–35	·961	70–75	7·341
35–40	1·239	75–80	12·588
40–45	1·207	80–85	17·242
45–50	1·700	85–90	28·047
50–55	1·849	90–95	36·091

The results of the two methods are compared in Table M (pp. 324-5), and, in order to facilitate the comparison, I have re-calculated Dr. Farr's l_x column with a radix of 10,000, instead of his radices: males, 51,274; females, 48,726; both sexes, 100,000. At the earlier ages and up to 30, the two tables run closely together. A peculiarity is to be observed, however, in the p_x column of the graphical results at ages 18 to 23, which may fairly be claimed as illustrating the power of that method in preserving the features of the original results. A sudden increase in the rate of mortality at these ages, which does not occur in Dr. Farr's table, is manifested in mine. At any other portion of the table this might have been considered accidental; but at this important period of early manhood, the point deserves attention.

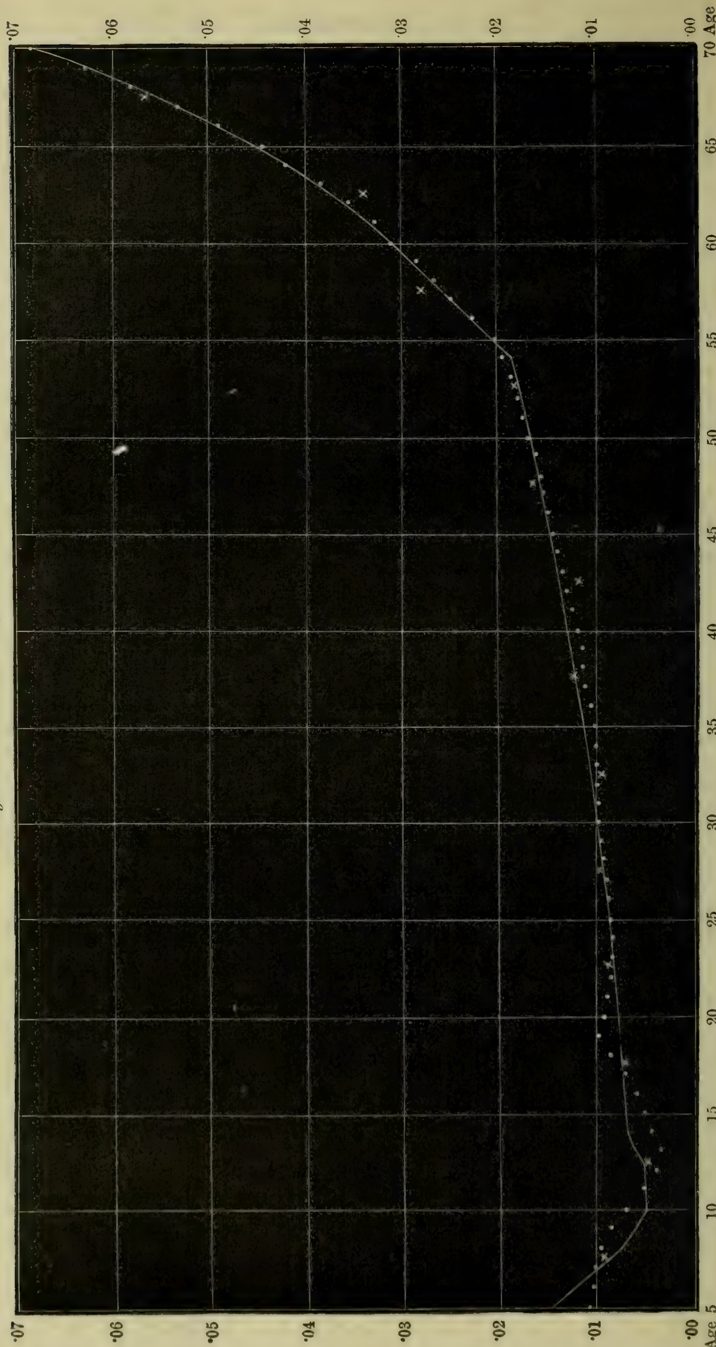
TABLE M.

Ages.	Victoria, 1871. Males. l_x	d_x	p_x	Victoria, 1871. Females. l_x	d_x	p_x	English Table No. 1, Males. Burridge. l_x	d_x	p_x	English Table No. 1, Males. Farr. l_x	d_x	p_x	Ages.
0	10,000	1,221	·8779	10,000	1,058	·8942	10,000	1,593	·8407	10,000	1,593	·8407	0
1	8,779	342	·9613	8,942	318	·9644	8,407	532	·9369	8,407	532	·9369	1
2	8,437	98	·9882	8,624	103	·9880	7,875	267	·9661	7,875	267	·9661	2
3	8,339	60	·9930	8,521	73	·9915	7,608	185	·9757	7,608	185	·9757	3
4	8,279	48	·9943	8,448	56	·9935	7,423	132	·9822	7,423	132	·9822	4
5	8,231	35	·9956	8,392	32	·9960	7,291	80	·9890	7,291	105	·9855	5
6	8,196	36	·9957	8,360	30	·9962	7,211	75	·9895	7,186	83	·9883	6
7	8,160	32	·9960	8,330	31	·9964	7,136	74	·9897	7,103	69	·9905	7
8	8,128	30	·9963	8,299	28	·9966	7,062	69	·9905	7,034	55	·9923	8
9	8,098	26	·9967	8,271	24	·9969	6,993	59	·9915	6,979	43	·9938	9
10	8,072	24	·9969	8,247	23	·9972	6,934	49	·9930	6,936	35	·9950	10
11	8,048	24	·9970	8,224	18	·9976	6,885	37	·9947	6,901	35	·9949	11
12	8,024	22	·9972	8,206	17	·9980	6,848	26	·9960	6,866	35	·9949	12
13	8,002	22	·9973	8,189	14	·9984	6,822	24	·9965	6,831	42	·9938	13
14	7,980	20	·9974	8,175	15	·9982	6,798	37	·9957	6,789	45	·9932	14
15	7,960	22	·9973	8,160	19	·9976	6,761	26	·9949	6,744	47	·9930	15
16	7,938	26	·9967	8,141	22	·9972	6,735	41	·9940	6,697	48	·9929	16
17	7,912	33	·9959	8,119	26	·9968	6,694	48	·9929	6,649	48	·9927	17
18	7,879	36	·9955	8,093	28	·9969	6,646	57	·9912	6,601	50	·9925	18
19	7,843	38	·9953	8,065	31	·9960	6,589	65	·9902	6,551	51	·9923	19
20	7,805	39	·9950	8,034	36	·9957	6,524	62	·9905	6,500	52	·9921	20
21	7,766	41	·9946	7,998	36	·9954	6,462	62	·9908	6,448	52	·9919	21
22	7,725	46	·9940	7,962	41	·9952	6,400	55	·9912	6,396	53	·9917	22
23	7,679	49	·9938	7,921	38	·9950	6,345	54	·9914	6,343	53	·9915	23
24	7,630	52	·9936	7,883	42	·9948	6,291	54	·9915	6,290	56	·9912	24
25	7,578	50	·9935	7,841	43	·9945	6,237	54	·9913	6,234	55	·9910	25
26	7,528	50	·9934	7,798	55	·9941	6,183	55	·9910	6,179	57	·9908	26
27	7,478	53	·9930	7,743	50	·9936	6,128	58	·9907	6,122	58	·9905	27
28	7,425	53	·9929	7,693	53	·9933	6,070	57	·9906	6,064	58	·9903	28
29	7,372	54	·9928	7,640	54	·9931	6,013	58	·9904	6,006	59	·9900	29
30	7,318	54	·9926	7,586	54	·9929	5,955	59	·9903	5,947	61	·9898	30
31	7,264	55	·9925	7,532	56	·9926	5,896	59	·9901	5,886	62	·9895	31
32	7,209	54	·9924	7,476	56	·9925	5,837	58	·9900	5,824	63	·9892	32
33	7,155	56	·9923	7,420	56	·9924	5,779	59	·9898	5,761	63	·9890	33
34	7,099	57	·9921	7,364	59	·9919	5,720	59	·9896	5,698	65	·9887	34
35	7,042	60	·9915	7,305	62	·9914	5,661	59	·9895	5,633	65	·9884	35
36	6,982	62	·9910	7,243	65	·9911	5,602	60	·9893	5,568	67	·9881	36
37	6,920	67	·9905	7,178	67	·9907	5,542	64	·9885	5,501	66	·9878	37
38	6,853	72	·9895	7,111	69	·9905	5,478	64	·9881	5,435	68	·9875	38
39	6,781	74	·9890	7,042	71	·9901	5,414	62	·9883	5,367	69	·9871	39
40	6,707	76	·9887	6,971	70	·9899	5,352	66	·9879	5,298	70	·9868	40
41	6,631	77	·9883	6,901	72	·9897	5,286	67	·9873	5,228	71	·9865	41
42	6,554	80	·9879	6,829	72	·9895	5,219	68	·9868	5,157	71	·9861	42
43	6,474	79	·9876	6,757	72	·9893	5,151	69	·9864	5,086	73	·9858	43
44	6,395	82	·9873	6,685	73	·9890	5,082	72	·9859	5,013	73	·9854	44
45	6,313	83	·9870	6,612	75	·9887	5,010	73	·9854	4,940	75	·9850	45
46	6,230	86	·9860	6,537	74	·9884	4,937	75	·9848	4,865	75	·9846	46
47	6,144	97	·9845	6,463	77	·9880	4,862	76	·9845	4,790	75	·9842	47
48	6,047	99	·9835	6,386	80	·9877	4,786	76	·9840	4,715	77	·9838	48
49	5,948	100	·9831	6,306	79	·9875	4,710	77	·9835	4,638	77	·9834	49
50	5,848	100	·9829	6,227	81	·9870	4,633	80	·9828	4,561	77	·9830	50
51	5,748	102	·9825	6,146	83	·9865	4,553	80	·9824	4,484	77	·9826	51

TABLE M—*(continued)*.

Ages.	Victoria, 1871.						English Table No. 1, Males.						Ages.
	Males.			Females.			Burridge.			Farr.			
	l_x	d_x	p_x	l_x	d_x	p_x	l_x	d_x	p_x	l_x	d_x	p_x	
52	5,646	102	·9817	6,063	86	·9859	4,473	82	·9817	4,407	79	·9821	52
53	5,544	107	·9807	5,977	89	·9852	4,391	82	·9812	4,328	79	·9817	53
54	5,437	111	·9797	5,888	91	·9844	4,309	84	·9805	4,249	79	·9812	54
55	5,326	113	·9789	5,797	92	·9840	4,225	86	·9796	4,170	87	·9789	55
56	5,213	115	·9780	5,705	95	·9835	4,139	96	·9770	4,083	96	·9767	56
57	5,098	120	·9767	5,610	98	·9824	4,043	100	·9750	3,987	100	·9749	57
58	4,978	121	·9757	5,512	103	·9812	3,943	106	·9732	3,887	103	·9730	58
59	4,857	125	·9740	5,409	107	·9803	3,837	110	·9715	3,784	112	·9709	59
60	4,732	130	·9725	5,302	113	·9789	3,727	116	·9686	3,672	116	·9686	60
61	4,602	133	·9713	5,189	114	·9780	3,611	120	·9670	3,556	120	·9661	61
62	4,469	138	·9693	5,075	120	·9765	3,491	124	·9643	3,436	126	·9634	62
63	4,331	143	·9670	4,955	127	·9745	3,367	129	·9617	3,310	131	·9605	63
64	4,188	149	·9645	4,828	152	·9685	3,238	139	·9575	3,179	135	·9574	64
65	4,039	155	·9615	4,676	166	·9645	3,099	139	·9550	3,044	140	·9540	65
66	3,884	163	·9583	4,510	184	·9593	2,960	147	·9505	2,904	144	·9504	66
67	3,721	170	·9545	4,326	197	·9545	2,813	150	·9465	2,760	148	·9465	67
68	3,551	177	·9500	4,129	209	·9495	2,663	156	·9415	2,612	151	·9423	68
69	3,374	186	·9450	3,920	210	·9463	2,507	159	·9370	2,461	152	·9378	69
70	3,188	194	·9392	3,710	207	·9444	2,348	161	·9315	2,309	155	·9330	70
71	2,994	197	·9340	3,503	211	·9396	2,187	161	·9265	2,154	156	·9278	71
72	2,797	205	·9270	3,292	215	·9348	2,026	160	·9210	1,998	155	·9222	72
73	2,592	225	·9130	3,077	217	·9294	1,866	160	·9144	1,843	154	·9162	73
74	2,367	237	·9000	2,860	219	·9237	1,706	156	·9085	1,689	153	·9098	74
75	2,130	232	·8910	2,641	218	·9175	1,550	151	·9025	1,536	149	·9028	75
76	1,898	224	·8823	2,423	215	·9109	1,399	146	·8958	1,387	145	·8954	76
77	1,674	205	·8770	2,208	213	·9038	1,253	140	·8880	1,242	139	·8875	77
78	1,469	192	·8690	1,995	207	·8962	1,113	134	·8795	1,103	136	·8790	78
79	1,277	179	·8605	1,788	200	·8880	979	126	·8715	967	124	·8699	79
80	1,098	165	·8500	1,588	192	·8792	853	120	·8595	843	118	·8603	80
81	933	150	·8390	1,396	182	·8698	733	114	·8450	725	109	·8500	81
83	783	135	·8273	1,214	170	·8598	619	...	...	616	...	...	82
83	648	120	·8151	1,044	158	·8489	...	...	...	...	...	...	83
84	528	104	·8021	886	143	·8377	...	...	...	...	...	...	84
85	424	90	·7883	743	130	·8254	...	...	...	...	...	...	85
86	334	76	·7738	613	115	·8127	...	...	...	...	...	...	86
87	258	62	·7586	498	100	·7991	...	...	...	...	...	...	87
88	196	50	·7427	398	86	·7850	...	...	...	...	...	...	88
89	146	40	·7259	312	72	·7696	...	...	...	...	...	...	89
90	106	31	·7086	240	59	·7539	...	...	...	...	...	...	90
91	75	23	·6905	181	47	·7372	...	...	...	...	...	...	91
92	52	17	·6717	134	38	·7197	...	...	...	...	...	...	92
93	35	12	·6521	96	29	·7018	...	...	...	...	...	...	93
94	23	9	·6323	67	21	·6829	...	...	...	...	...	...	94
95	14	5	·6115	46	15	·6634	...	...	...	...	...	...	95
96	9	4	·5903	31	11	·6433	...	...	...	...	...	...	96
97	5	2	·5685	20	8	·6224	...	...	...	...	...	...	97
98	3	1	·5466	12	5	·6010	...	...	...	...	...	...	98
99	2	1	·5241	7	3	·5792	...	...	...	...	...	...	99
100	1	1	...	4	2	·5568	...	...	...	...	...	...	100
101	0	...	...	2	1	·5340	...	...	...	...	...	...	101
102	...	...	...	1	1	...	...	...	...	...	...	...	102
103	...	...	...	0	...	...	...	...	...	...	...	...	103

English Table No. 1, Males.



The abscissa represents the age, and the ordinate the probability of dying in a year.

Dr. Farr's graduation, ———; Mr. Burridge's graduation, ... ; Dr. Farr's Original Observations

In a letter "On the Rate of Mortality at the period of Early Manhood" (*Journal*, vol. xiv, p. 247), Mr. Bailey says, "there is "abundant evidence to prove that the rate of mortality during the "quinquennial period from 20 to 24 years of age is *greater* than in "the next succeeding period from 25 to 29 . . . and yet, in the "graduated tables that have been formed from these observations, "the peculiarity has been made to disappear."

On comparing the p_x columns in quinquennial groups for the ages in question, we find that, although taking age for age, the values vary considerably, yet the average probability of the two tables approximates closely.

Ages.	Average Probability.	
	Burridge.	Farr.
15–20	·9926	·9927
20–25	·9911	·9917
25–30	·9908	·9904

In Dr. Farr's table the rate of mortality increases with regularity at each age from age 10. In the graphical method early manhood is marked by an increase in the rate, which is greater for the period 18–22 than for the period 23–27.

At the ages 30 to 50, the graphical results show a somewhat more favourable rate than Dr. Farr's; between 50 and 80 the tables cross at several points, and finally produce almost identical conclusions, namely, that out of 10,000 births, 616 persons attain age 82 by Dr. Farr's method, and 619 persons by the graphical method.

The object of this investigation has been to bring more prominently into notice an instrument by which to effect, by a simple and natural operation, the transition from the ordinary population returns to the columns of a mortality table. Until recently, this method has been almost entirely neglected in actuarial calculations. Within the past two years, its practical value has been illustrated by Mr. Sprague, who employed it in a case of peculiar difficulty, with marked success.

It is applied with great facility; it will produce results in cases where, from scantiness of data, other methods would fail; and, while bringing out orderly and progressive rates, it may be depended on for preserving in their integrity the special characteristics of the mortality under investigation.

Discussion.

The President (Mr. A. H. Bailey) thought the paper practically useful, for a knowledge of the rate of mortality in our colonies is certainly a desideratum. With regard to the Australian and other temperate colonies, we have hitherto been obliged to assume that the mortality coincides with that of this country, but Mr. Burridge has now thrown some additional light on the question. He has shown that in the colony of Victoria, at the younger ages at all events, and notably so at the period of infancy and early childhood, the mortality is more favourable than in the mother country. In consequence, however, of the comparatively small number of old men and women in this colony, we have hardly sufficient material at present to determine the average duration of life in Victoria.

Mr. George King thought the paper a very important contribution. For some time past his professional duties had made him pay some little attention to the Australian colonies, and one great difficulty he had felt was the absence of trustworthy statistics of the rate of mortality there. He should not like to say that the figures given in the paper are sufficient, or in any way final. For instance, he would not like to base, without further enquiries, the valuations of life offices upon them. But, at the same time, they throw a great deal of light on a very important branch of statistical enquiry. Such investigations have two uses; one is to give us some idea of what has been the past history of the population, and what is its present condition, and the figures in the paper are very useful in that direction. Another application is to forecast the future, but he doubted very much, looking at the somewhat peculiar condition of the Australian colonies, whether we should be able prudently to prophesy what will be the future mortality from such figures as these. The comparatively low rate of mortality is, to a certain extent, although probably not entirely, due to the fact that the lives that go out there are select. There has been, in the period under review, a very large emigration to the colony from the mother country, and, taking the average, the emigrants are of stronger health than those left behind. Therefore we may expect the rate of mortality to be somewhat lower on that account. Then for life assurance calculation we require to carry on our mortality curve much later than Mr. Burridge has been able to do. The ages above eighty very considerably affect our premiums and our reserves. Therefore he would not like to have a table of premiums or valuations based on such a table as this, when the higher ages have been dealt with arbitrarily. We may, however, by looking at this table, be able to select from some other well-known tables those which may be considered best for such purposes. Mr. Burridge has pointed out that even amongst the very early ages the rate of mortality is low, but this may be partly explained by another statement he makes. He informs us that the colony "exports infants". Now, he has based the calculation for infant mortality on the births of the preceding years, but the population at those young ages would be reduced by the children who have left the colony, so that by his method of merely looking at the births he would get his l_x somewhat too large.

With regard to the methods of construction of the tables, he thought that Mr. Burridge deserved their thanks for again introducing Mr. Milne's process. He had studied various tables constructed otherwise from census returns; the only three of any importance of which the construction has been fully explained are the first English table, the Healthy English, and Professor Pell's Australian table. The conclusion he had arrived at is that the mathematical methods of construction which have been adopted, are not satisfactory. He would not like to say the tables are wrong on that ground; but we have no means of proving their correctness. We get a kind of average ratio, and from the average ratios at different ages we interpolate other ratios, and he was not at all satisfied that the average ratio is a safe ground to work from. The method of Mr. Milne seems a very admirable one. It may produce sometimes rather a rough result, because we graduate not one curve, but two, and get our ratio from dividing the number representing one of the curves by that representing the other; but still, with the precautions Mr. Burridge has adopted, and his subsequent corrections, that is no objection. It is a very suitable method indeed for such tables, although it would not apply so well to the more accurate data obtained from life offices. For these, he would prefer a more mathematical formula; but for such tables as those derived from censuses, the graphical method appears decidedly the best. He was not quite sure that the test, as applied by Mr. Burridge, of making the expected deaths agree with the actual deaths, is quite accurate; but still it is substantially so, and although that point may deserve a little further investigation, he did not think it would in any way vitiate Mr. Burridge's figures.

Mr. Cornelius Walford quite endorsed the view expressed by Mr. Burridge, that in the Victorian Year Book, as produced by Mr. Heylyn Hayter, we have a very valuable source of statistics, which will become more and more valuable, as years roll on and the points of observation become extended. There are many interesting points arising out of the paper, but by far the most interesting is that which relates to the cycles of mortality. Those who have investigated certain branches of mortality, accidental deaths, and so on, have found cycles; but it seems somewhat new to discover cycles with regard to the general mortality, recurring with the regularity, or rather progressing with the regularity, which is remarked upon in the paper. Another point arising out of the paper is the large proportion of deaths from violence. We have had occasion to notice that fact here on other occasions, and it is quite clear that in all newly-settled countries there must be a very large proportion of deaths from violence. Of course the greater proportion of these deaths will occur to males; and when we use the expression "deaths from violence", it does not mean violence in the ordinary sense, but only in the sense in which the deaths are classed. It means, for instance, deaths from boat accidents in crossing rivers because there are no bridges, and accidents of that sort; and accidents from the stumbling of horses on newly-made roads, or in point of fact, where there are no roads at all. I have had occasion lately to investigate the deaths in the islands of Scotland, and I find that the proportion of deaths resulting there from

boat accidents is so prodigious that it almost passes belief. If you take the population of some of those islands, and see the effect in given years of those islands, and see the effect in given years of boat accidents on the population, it is quite surprising. With regard to the question of the male mortality being greater at all ages than the female, that seems to be rational too. It seems certain that in newly-settled countries the males are exposed at nearly all periods of their lives to accidents which do not surround the females, and that may account for the fact mentioned in the paper. With reference to the zymotic class of diseases, there has been a belief that the Australian colonies have been tolerably free from these diseases, but I think there is a cause which influences the zymotic diseases, and that is in the turning up of new soil for cultivation. In all new agricultural colonies there is a great deal of that necessarily going on. There is the same influence on a large scale in the western States of America, and I understand that in certain parts of those western states malarious diseases seem to arise at the moment the soil is brought into cultivation. With regard to the deaths from consumption, it certainly has been the notion that the Australian colonies present features favourable to the cure of that disease, and I think it will probably require further investigation to see whether the persons who have gone out there seeking to be cured from phthisis, have had any visible influence on the mortality. We know that at Bath, Cheltenham, Clifton, and some other most healthy spots in this country, deaths from certain kinds of disease occur in a very large proportion; and we know that in those cases at least they are deaths from imported diseases—that is, persons come to be cured, and remain to die. How far such a thing may exist in Victoria is one point to be enquired into. There is one very curious fact with regard to race, which has been observed in the United States, and still more in Australia, which may throw some light on phthisis deaths. It has been observed in the United States of America that the Irish who go over with strongly marked features, particularly the potato-headed people from the north-west parts of Ireland—Donegal for instance—do not perpetuate those distinctive features. The children of these people, born in the United States, present features of an entirely classical character; they lose altogether the appearance of their parents; and I am told that in a still greater degree is that the case in the Australian colonies—there is no following of the parental type in any way whatever. It may be that this altered type is an indication of change of constitution acquired by the change of climate from that in which their parents and the earlier members of the family had been reared.

Mr. T. Jaques Martin said that his long residence in Victoria hardly enables him to confirm Mr. Burridge's statement as to the equableness of its climate. He proceeded:—Snow is by no means an uncommon visitant at Ballarat (the second city in size and importance), and in other highly-elevated places; while on the hottest summer day I can, by a railway ride of thirty, and horse of some four or five, miles, find a deliciously cool climate. I was startled by the exposition of consumption in Melbourne and suburbs, and I think its excess there (as compared with the colony as a whole) may, to a large extent

be accounted for, first, in the ordinary remark that in Melbourne one may often, with advantage, vary the thickness of his clothing three or four times during a day. This shows the want of that equability which I have been informed by eminent medical men here is recognized now in consumption, as the necessity, rather than specially heat or cold. Mr. Burridge will, I am sure, be gratified to hear that already his paper has probably tended to lengthen the life of a young fellow, whose enquiring friends will probably now send him, not to Melbourne, but rather to Echuca, in Victoria, or to Queensland. Much unintentional cruelty is practised by sending consumptives to Australia, when too late or otherwise, regardless of district; and it is notorious that very many, if they live to land—land to live little longer, perhaps, than to qualify to swell the returns of the Registrar-General. I should like to know the number of native-born exposed to observation, with the result of consumption, according to the table (page 316) which our valuable statist, Mr. Hayter, has, I presume, provided in view of the circumstances I have last referred to. I could have wished to see this table of selection against the colony further worked out, to exhibit the different periods of residence in excess of five years. I do not think that consumption among the native-born is heavy; and what there is, is probably traceable to family history, and modified by distribution over the whole colony, as our own young people are more likely to spread, than new arrivals who have developed consumption, and who are unhappily very frequently of a class for which large towns are expected to provide employment. I noted a suggestion of Mr. Burridge, which may be regarded as prophetic, although I do not think it has yet been realized—namely, that persons may arrive at Melbourne and "pass at once to the other colonies". I hope soon to see the day when passengers may land at Adelaide, and, if they so please, be conveyed by rail, without break, as far as Northern Queensland. But the occupations of the people in the country, and the facilities of communication both by rail and horseback, enable a very large and necessarily unnumbered passage to and fro between the colonies, equally as from England to Scotland or Wales, or even from one county to another; so that nothing short of a census can in any reliable degree determine the number of the population; and even subject to the census the habits and occupations of a large number of our country population are such (shearers, sundowners, &c.) that only mechanical accuracy can be obtained as to their places of residence. The great bulk of the population of Australia occupy territory which, in proportion to the size of the island, is only a "fringe" on the sea coast; and although occupying colonies with separate names, we are in heart one people, and the dearest wish of many an Australian colonist is to see federation of the whole (like Canada). But for life assurance purposes it is absolutely necessary that there shall be practically a federation of figures, if I may use the term; and I cannot express a better wish to the interests of Australia and of science, than that Mr. Burridge may be induced to extend his researches to the other colonies, and then to blend the whole. The present paper is undoubtedly valuable; but to use the mortality of Victoria as a gauge of Australian life business, would be equal to trusting to that of, say Wales alone, as

an exponent of the mortality of the British islands. The exodus of babies referred to is, in all probability, owing in a large degree to the existence immediately north of the Murray River of a fine area of agricultural land, which is politically part of New South Wales, but geographically ought to belong to Victoria. I am not exactly aware of the relative merits of the Government statistics of the other colonies, compared with the valued records introduced by Mr. Archer, and so admirably continued and extended in scope by Mr. Hayter; but on leaving home I collected some books in relation to the colonies, in the hope that they might be useful to this Institute or to the Statistical Society. I propose to submit these to Mr. Burridge, if he will allow me, and he can take for the library any he deems useful. If I can be of any service in keeping up a regular supply of any works of the kind, I shall be glad to do so. I would submit that, valuable and interesting as is this paper, it is to the aggregate statistics of the colonies only, that we can trust for practical assurance purposes; and to such I look forward with much interest as soon as data of sufficient age and detail are available. The reference to Professor Pell's papers substantiates the feeling which, I think I may say, has prevailed generally with us, that the further you go north the higher will be the rate of mortality. But I may mention that the directors of the society of which I have the honor to be manager, caused a special investigation to be made of the physical condition of northern New South Wales and Queensland as a whole, the result of which was an extension of our geographical limit fixed originally, I think, by the Standard (late Colonial of Edinburgh) at 30° south latitude; and we consider that we can, with propriety, accept at ordinary rates far beyond the tropic of Capricorn, among residents on the elevated plateaux or "downs", where the climate is superb, while there are parts of these northern latitudes further south which may be well avoided. Since I left, however, I hear that the whole Continent has been made free of "extra", but I feel certain that the office which has led off in this thorough announcement, and which has the largest experience, has no doubt also investigated and will, like ourselves and others, notwithstanding any general announcement, make careful selection by the light of their own experience, and the supervision of resident managers of local knowledge.

Dr. Symes Thompson said that, when a colony is new, you have a natural selection in favour of good lives, because only sturdy, vigorous people go out to new colonies; but on the other hand, when a colony gets a reputation, as Victoria has done, of being a healthy resort, then you have a natural selection against it, and this natural selection against Victoria has told to a very considerable extent. He proceeded:—Of course I see more of those who go out for the sake of their health than of those who go out for the sake of their pockets; and I am sorry to say I see a good many of the class referred to by the last speaker, who ought not to go out. I very frequently have to counsel a sea voyage to persons who consider themselves so well that it is not necessary; and I very frequently have to discourage those who consider that it is the only thing which will save their lives, and this is a very difficult office. The result of a number of people in

advanced consumption going out is, that Melbourne, where they mostly land, is overstocked with imported consumptive persons, who remain there and die in the hospitals. But I confess when I come to look at these statistics which Mr. Burridge has put before us, I am rather surprised to find how small the number of these persons is. If you take all those who die before they have been two years in the colony, you find they form only 4 per-cent of the whole consumptive mortality. Of those who go out but are really too ill to undertake the voyage, I think we may assume that a large proportion die within two years. If they live two years, one may naturally assume that it was right for them to go; but these figures show that the constant complaint made that medical men are perpetually sending out consumptive people, to die soon after their arrival in the colony, is not borne out by the facts. Now I should like to say a word on the question of infant mortality. I think that Victoria deserves the thanks of all other communities for having separated infant mortality into four subdivisions. Accurate knowledge is the first step towards improved sanitation. If we knew more precisely than we do at present, when that infant mortality mainly occurs—if during the first month, or first three months, or first six months—I believe that we should be in a position to take more efficient steps for its diminution. I believe it will be found that this mortality has a very close reference to poverty, underfeeding, and especially to overcrowding. Children die off whenever they have not a sufficiency of air; so that, in proportion as a city becomes densely populated, you may expect that the mortality will increase. In the early days of Victoria, the place was comparatively small, and the towns were not excessively crowded; but now that they are becoming more closely peopled, I think that we may assume that the mortality from infantile disorders will increase. With reference to table "F", the idea which exists in the minds of English medical men with reference to the Australian climate, as compared to the English, is that it is something like the Italian—in this respect, that, whereas in this country a large majority of people die from chronic diseases, in Italy and Australia a large proportion die from acute diseases. Now, when I come to look at this table, I do not find that idea very strongly supported. If you look at "zymotic diseases", they are pretty much the same in each country. I may say in passing, with regard to that remarkable mortality referred to in another table, which shows that between 1873 and 1876 an undue wave of mortality swept over the colonies, I believe that this was due to the excessive development of scarlet fever. I have today seen two members of a family in which three died in Victoria from scarlet fever during the epidemic of 1875 and 1876, so that, although these statistics seem so very startling, I believe they are accounted for by the unusual wave of epidemic disease, a wave which may perhaps never return, but which we may, if we believe in the doctrine of cycles, expect to recur in about ten years. But, not to speak further on this point, you see a striking difference is observable in the column of "constitutional diseases"; and one is perfectly prepared to find that it should be so. The conditions of colonial life are such as to lessen the tendency to the development of those chronic diseases which are so excessively fatal

with us, including scrofula, gout, and consumption, and those maladies which occur mainly as the result of overcrowding and deficient air, and of labour in confined workshops, and insufficient food. Then when we come to look at the "local diseases", I confess myself unprepared to see the difference there shown. The mortality in England from "local diseases" is almost double what it is in the colony. This no doubt is due to the fact that we have in England a larger number of cases of brain diseases, liver disease, kidney disease, heart disease, and the like, which are dependents upon, and might be more properly classed under the head of "constitutional diseases", and I believe that these in reality magnify unduly the class of "local diseases". We have in the colony a large proportion of acute inflammatory affections, such as pneumonia, hepatitis, sunstroke, and so on, whereas in England "local diseases" are mainly degenerative affections. As regards "developmental diseases", this comprises such a curious congeries of maladies of all sorts, that one hardly knows how to deal with it. It seems rather curious that "old age" should be a "developmental disease", but so it is, according to the grouping. As regards "violent deaths", I think we might expect that mortality from that cause would be very much greater, as has been well explained by Mr. Walford. Just one word as regards climate. I think that, when you have such an enormous extent of country, varying in elevation, in vicinity to the sea, &c., so much as Victoria does, it is not easy to group the whole, and speak of the whole as one. We, as medical men, of course choose our climates, and send our patients to selected places; and the fact that there is a considerable mortality in Melbourne, which is marshy in some places, would not diminish our tendency to send those who have an inclination to early disease out to the colony, our endeavour being to send them to well-selected places. All these statistics, although they show that in Melbourne there is a considerable mortality from phthisis, do not show that it is undesirable to send well-selected cases to special parts of the colony.

Mr. R. P. HARDY said, that in the last edition of the Australian Year Book, which has only been published a few days, Mr. Hayter draws attention to a fallacy, which is not uncommon in this country, namely, in showing a comparative death-rate, not to take into sufficient consideration the fact that the average age of the population is, and must be, very different. The difference is particularly marked in the case of marriages. The impression left on his mind after studying not merely Mr. Hayter's book, but also Mr Burridge's paper, is that we must not draw our inferences too closely from the figures now before us, as there must be a considerable difficulty in arriving at the number exposed to risk.

Mr. BURRIDGE, in reply, first spoke of the method of graphical graduation which he had adopted. This had been brought before the Institute on two occasions by Mr. Sprague; who, on the second of those occasions, stated that he was gratified to observe that the same objections were not raised to the method as on the first occasion. If he had been here tonight he would have been still more gratified, as the method had met with approval. As Mr. King stated, this investigation is not a final one in any sense, but must be regarded

as a first step. He thought that, with regard to the infant mortality, which was so very favourable, it is possible that the infants going away have perhaps had an undue influence on the mortality. With regard to phthisis, an eminent physician in Melbourne, Dr. William Thompson, makes some exceedingly strong deductions from the figures, and states, in fact, that consumption is not only very heavy, but that it is on the increase. He falls, however, into a mistake by too strongly contrasting the consumption in the country districts with the consumption in Melbourne. He forgets that almost all the cases occur in Melbourne, probably not only from the reason that they originally lived there, but because sick people naturally go where hospital and medical aid can be most readily gained. But for that reason it is rather hard to say that the consumption of the whole of Victoria shows such a large increase. As it has been very happily put, you might as reasonably say the mortality in a bedroom is much larger than in a sitting-room, where it is not the difference of climate of the two rooms, but because most people go to their bedrooms when attacked by sickness.

On the Adjustment of Mortality Tables. *By* JOHN ADAMS HIGHAM, *Member of the Actuaries' Club.*

[Read before the Institute, 24 April 1882.]

MR. JOHN FINLAISON'S method of graduation (*Journal*, vol. xxi, page 50) deals with an irregularity, which, for convenience, may be assumed to be unity, in the following manner:—It diminishes the irregularity to a small fraction on its first appearance at one end of the formula; conducts it along the formula by easy stages, as over a double inclined plane; and dismisses it gently at the other end. The method has the advantage of simplicity in working and check, but it is correct to first differences only.

Mr. Woolhouse's method is correct to third differences, and the irregularity glides over a curve; but the application of the formula requires sustained attention.

The object of this paper is to combine the facility of the one method with the smoothness and correctness of the other.

Let S be the sum of nine numbers, u_0 to u_8, increasing by third differences,

$$S = 9_0u + 36\Delta_1 + 84\Delta_2 + 126\Delta_3\,,$$

$$\frac{S}{9} = u_0 + 4\Delta_1 + \frac{28}{3}\Delta_2 + 14\Delta_3\,,$$

the central term, $u_4 = u_0 + 4\Delta_1 + 6\Delta_2 + 4\Delta_3$;

whence $$u_4 = \frac{S}{9} - \left(\frac{10}{3}\Delta_2 + 10\Delta_3\right).$$

Again,—let T be the sum of

$$u_0+2u_1+3u_2+4u_3+5u_4+4u_5+3u_6+2u_7+u_8$$

$$\text{T}=25u_0+100\Delta_1+200\Delta_2+250\Delta_3$$

$$\frac{\text{T}}{25}=u_0+4\Delta_1+8\Delta_2+10\Delta_3\,.$$

Compare $$\frac{\text{S}}{9}=u_0+4\Delta_1+\frac{28}{3}\Delta_2+14\Delta_3$$

$$\frac{\text{S}}{9}-\frac{\text{T}}{25}=\frac{4}{3}\Delta_2+4\Delta_3$$

$$\frac{10}{4}\left(\frac{\text{S}}{9}-\frac{\text{T}}{25}\right)=\frac{10}{3}\Delta_2+10\Delta_3\,;$$

whence $$u_4=\frac{\text{S}}{9}-\frac{10}{4}\left(\frac{\text{S}}{9}-\frac{\text{T}}{25}\right)$$

$$=\frac{10}{4}\cdot\frac{\text{T}}{25}-\frac{6}{4}\cdot\frac{\text{S}}{9}=\frac{\text{T}}{10}-\frac{\text{S}}{6}=\frac{3\text{T}-5\text{S}}{30}\,.$$

Assume, now, that an irregularity amounting to $+1$ enters the series. It adds 1 to T when it affects u_0, and it adds 2, 3, 4, 5, 4, 3, 2, 1, when it affects in succession u_1, u_2, ... u_8.

It adds 1 to S in respect of each term of u.

Consequently, the numerator of the formula is raised in the successive terms by $(3-5)$, $(6-5)$, $(9-5)$, $(12-5)$, $(15-5)$, $(12-5)$, $(9-5)$, $(6-5)$, and $(3-5)$. And the increments of the successive terms are -2, $+1$, $+4$, $+7$, $+10$, $+7$, $+4$, $+1$, -2; all divided by 30.

If an irregularity be thus distributed, so are the normal numbers; and, applying these deductions to the graduation of the decrement in a table of mortality, the adjusted value of d_x is

$$\frac{-2d_{-4}+d_{-3}+4d_{-2}+7d_{-1}+10d+7d_{+1}+4d_{+2}+d_{+3}-2d_{+4}}{30}\,.$$

This formula has but a moderate graduating power, for two reasons; first, because it embraces a small number of terms; second, because the terms vary in straight lines instead of in a curve. The first defect may be amended by dealing, in a similar manner, with a larger number of terms. Not so the second. To correct both defects, the formula must be applied twice.

Suppose that u_0 represents d_{21}.

After one adjustment, the corrected value

$$\begin{aligned}
&\text{of } d_{25} \text{ is } \{-2d_{21}+d_{22}+4d_{23}+7d_{24}+\&c.\}\div 30,\\
&\;,,\; d_{26} \;,, \qquad \{-2d_{22}+d_{23}+4d_{24}+\&c.\}\div 30,\\
&\;,,\; d_{27} \;,, \qquad\qquad \{-2d_{23}+d_{24}+\&c.\}\div 30.
\end{aligned}$$

and so on to the corrected value of d_{33}.

And these, in the second adjustment, are multiplied respectively by

$$\{-2,\ +1,\ +4,\ +7,\ +\&c.\}\div 30.$$

Multiplying and collecting, we have, as the adjusted value of d_{29},

$$\{4d_{21}-4d_{22}-15d_{23}-20d_{24}-10d_{25}+48d_{26}+127d_{27}+200d_{28}$$
$$+240d_{29}+200d_{30}+127d_{31}+\ldots\ldots-4d_{36}+4d_{37}\}\div 900,$$

being approximately

$$\cdot004d_{21}-\cdot004d_{22}-\cdot017d_{23}-\cdot022d_{24}-\cdot011d_{25}+\cdot053d_{26}+\cdot141d_{27}$$
$$+\cdot222d_{28}+\cdot267d_{29}+\cdot222d_{30}+\&c.$$

Mr. Woolhouse's formula is

$$-\cdot024d_{22}-\cdot016d_{23}\ldots\ldots+\cdot024d_{25}+\cdot056d_{26}+\cdot168d_{27}+\cdot192d_{28}$$
$$+\cdot200d_{29}+\cdot192d_{30}+\&c.$$

T, as employed above, represents the result of summing the terms of the decrement five at a time, and again five at a time. By varying this part of the procedure, any number of formulæ may be obtained, all giving correctly the central term to third differences; but hardly any of them (so far as I have observed) available for graduation, because they have considerable prime numbers in the denominator; or because they do not maintain a proper relation between the central and outlying terms; Mr. Woolhouse's formula being taken as the standard of what that relation should be. For example:—With a view to obtain the full adjustment by one operation, let d_x be summed in fives, four times in succession. The multipliers for the successive terms of u are

1, 4, 10, 20, 35, 52, 68, 80, 85, 80, . . . 4, 1.

S includes seventeen terms, and the central term is $\left\{\dfrac{3T}{1250}-\dfrac{S}{34}\right\}$, which is useless for columnar calculation, because I cannot divide by 17 at sight. Or, with the same object, let d_x be summed in

fives, twice; and the results, seven at a time. The multipliers of u are

$$1, 3, 6, 10, 15, 19, 22, 23, 22, \ldots . 3, 1.$$

S includes fifteen terms, and the central term is $\frac{T-5S}{100}$; beautiful in simplicity, but unavailable, because the formula in detail is (quoting coefficients only),

$$-{\cdot}04-{\cdot}02+{\cdot}01+{\cdot}05+{\cdot}10+{\cdot}14+{\cdot}17+{\cdot}18+{\cdot}17+\ldots-{\cdot}02-{\cdot}04.$$

Comparing this with Mr. Woolhouse's formula, it will be seen that the central terms are too small in proportion to the others. (The central term is the largest term, the formula not being written at full length for economy of space.) In effect, it disposes too vigorously of an irregularity at the original point of incidence, reproducing it with insufficient mitigation at a distance of several terms.

Rejecting these, and returning to $\frac{3T-5S}{30}$, twice applied, we observe that the formula, as written in juxtaposition with Mr. Woolhouse's, differs therefrom in the opposite direction to that which has just now been remarked upon; and that the variation is in the right direction, if variation there must be. If a first adjustment is sufficiently even for the purpose in view, there is advantage in the greater prominence given to the central terms, because the graduated d_x is by so much nearer to the unadjusted value. And if, on the other hand, the adjustments have to be repeated, there is margin for improved smoothness without too great departure from the data in detail.

Again,—the less the outlying terms have to do with the matter, the less is the error arising from a change in the order of differences in the span of years included in the formula.

Two columns will be saved in working if, instead of

$$d_x = \frac{3T-5S}{30},$$

we write

$$10d_x = T - \frac{10S}{6}.$$

This last formula it is proposed to apply to the H^M Table.

The first three terms in the decrement are 79, 0, 40; in which the obvious correction is made, 40, 39, 40, at ages 10, 11, 12.

The large number of deaths at age 96 is assumed to include some belonging to the preceding and succeeding years. Instead

of 5, 10, 0, at ages 95, 96, and 97, I write 9, 5, 1; it appearing from one or two trials that this ending is more in harmony than another with the expectation at old ages.

The sum of $d_x + d_{x+1} + \&c.$, is $l_x = u_0$; $l_x(1+e_x)$ is the sum of the series $l_x + l_{x+1}, + \&c.$; d_x, adjusted, is $\Delta_1 l_x$; $u_n = d_{98} = 0$.

From these data Δ_2 and Δ_3 may be obtained, and the series worked out; but trial by error takes less time, and is sufficient for the purpose.

With these two exceptions, nothing will be done arbitrarily or by inspection.

The formula does not reach the first and last four years of the table. When the adjustment is suspended at age 93, we observe that the number of entries which d_{97}, d_{96}, &c., have made into T, is

1, 3, 6, 10, 15, 19, 22, 24, and 25;

the last applying to d_{89}, and being the full complement of entries. These numbers are obtained by continuous summation of 1, 2, 3, 4, 5, 4, 3, 2, 1, the factors of the terms of u in T.

The number of entries into T, which the concluding terms of d have *failed to make*, is

$$d_{90} + 3d_{91} + 6d_{92} + 10d_{93} + 15d_{94} + 19d_{95} + 22d_{96} + 24d_{97} = 953.$$

Conversely, the number of entries into T, which the commencing terms of d have failed to make, is

$$d_{17} + 3d_{16} + 6d_{15} + 10d_{14} + 15d_{13} + 19d_{12} + 22d_{11} + 24d_{10} = 3{,}676.$$

The casting of column T is 245,371, to which add 953 and 3,676.

Total, 250,000; testing the correctness of that column.

The number of entries into S, which the concluding terms of d have failed to make, is

$$d_{90} + 2d_{91} + 3d_{92} + 4d_{93} + 5d_{94} + 6d_{95} + 7d_{96} + 8d_{97} = 427.$$

The number of entries into S, which the commencing terms of d have failed to make, is

$$d_{17} + 2d_{16} + 3d_{15} + 4d_{14} + 5d_{13} + 6d_{12} + 7d_{11} + 8d_{10} = 1{,}275.$$

The casting of column S is 88,298, to which add 427 and 1,275.

Total, 90,000; testing the correctness of that column.

These figures furnish the means of completing the beginning and end of the adjustment by $\left(\mathrm{T} - \frac{10\mathrm{S}}{6}\right) = 10d$,

$$953-\frac{4270}{6}=953-712=241,$$

which is the number of deaths assignable to $10(d_{94} \ldots d_{97})$;

$$3{,}676-\frac{12750}{6}=3{,}676-2{,}125=1{,}551,$$

which is the number of deaths assignable to $10(d_{10} \, . \, . \, d_{14})$.

I have thought it sufficiently accurate to interpolate these numbers by second differences between the adjusted d_{93} and the original d_{97}; and between the original d_{10} and the adjusted d_{14}.

After the first adjustment, the one or two isolated points are plainly seen, at which Mr. Woolhouse would sanction amendment by inspection; but this I have not attempted, preferring, in a mere exercise like the present, to exhibit the untouched results of the methods employed. Partly in consequence of this, the second adjustment does not entirely satisfy me; and a third is made, before summing for numbers living and expectations.

The following table shows, in perhaps the most convenient form, the extent of departure from the unadjusted probabilities involved in these successive proceedings.

Probability of Living Five Years.

Age.	Unadjusted Probability.	Once adjusted.	Twice adjusted.	Thrice adjusted.	Woolhouse.
10	·98060	·98169	·98201	·98216	·98224
15	·98063	·98042	·98011	·97981	·97963
20	·96683	·96665	·96680	·96707	·96714
25	·96666	·96590	·96593	·96587	·96566
30	·95972	·96015	·96004	·96001	·96012
35	·95339	·95343	·95353	·95360	·95367
40	·94771	·94726	·94708	·94702	·94695
45	·93340	·93319	·93319	·93333	·93336
50	·91587	·91504	·91474	·91449	·91457
55	·88532	·88505	·88503	·88497	·88503
60	·83605	·83716	·83735	·83747	·83744
65	·76922	·77249	·77319	·77351	·77335
70	·67150	·67356	·67373	·67402	·67388
75	·54652	·54438	·54338	·54231	·54221
80	·38865	·38554	·38670	·38800	·38923
85	·27726	·27675	·27502	·27330	·26927
90	·10000	·07047	·08558	·09269	·09247

ADDENDUM.

Mr. Woolhouse's formula may be deduced from the terms of u as follows:—

For the first curve u_0, u_5, and u_{10}, are given to find u_7.
" second " u_1, u_6, and u_{11}, " "
" third " u_7, is given, and u_2 and u_{12} are not used.
" fourth " u_3, u_8, and u_{13}, are given to find u_7.
" fifth " u_4, u_9, and u_{14}, " "

First Curve.

$$u_5 = u_0 + 5\Delta_1 + 10\Delta_2 + 10\Delta_3$$
$$u_{10} = u_0 + 10\Delta_1 + 45\Delta_2 + 120\Delta_3;$$

whence
$$\Delta_1 = \frac{9u_5 - 7u_0 - 2u_{10} + 150\Delta_3}{25},$$

and
$$\Delta_2 = \frac{u_0 - 2u_5 + u_{10} - 100\Delta_3}{25}.$$

Insert these values in $u_7 = u_5 + 2\Delta_1 + 11\Delta_2 + 25\Delta_3$.

Second Curve.

$$u_1 = u_0 + \Delta_1$$
$$u_6 = u_0 + 6\Delta_1 + 15\Delta_2 + 20\Delta_3$$
$$u_{11} = u_0 + 11\Delta_1 + 55\Delta_2 + 165\Delta_3;$$

whence
$$\Delta_1 = \frac{11u_6 - 8u_1 - 3u_{11} + 275\Delta_3}{25},$$

and
$$\Delta_2 = \frac{u_1 - 2u_6 + u_{11} - 125\Delta_3}{25}.$$

Insert these values in $u_7 = u_6 + \Delta_1 + 6\Delta_2 + 15\Delta_3$.

Fourth Curve.

$$u_3 = u_0 + 3\Delta_1 + 3\Delta_2 + \Delta_3$$
$$u_8 = u_0 + 8\Delta_1 + 28\Delta_2 + 56\Delta_3$$
$$u_{13} = u_0 + 13\Delta_1 + 78\Delta_2 + 286\Delta_3;$$

(*Continued on p.* 350.)

H^M.—*First Adjustment.*

Age. x	d_x	Fives.	Fives again $=T$	$(d_{x-4} \ldots d_{x+4}) = S$	$\frac{10S}{6}$	$T - \frac{10S}{6} = 10d_x$	Δ
10	(40)	...	...	...	...	(400)	+ 9
11	(39)	...	...	...	...	(409)	− 16
12	40	194	...	...	...	(393)	− 44
13	35	176	...	...	...	(349)	− 69
14	40	137	807	316	527	280	− 50
15	22	138	803	344	573	230	+ 19
16	0	162	851	361	602	249	+109
17	41	190	1,005	388	647	358	+132
18	59	224	1,176	412	686	490	+105
19	68	291	1,337	445	742	595	+ 59
20	56	309	1,466	487	812	654	+ 7
21	67	323	1,553	535	892	661	− 21
22	59	319	1,570	558	930	640	− 2
23	73	311	1,570	559	932	638	− 21
24	64	308	1,554	562	937	617	− 27
25	48	309	1,545	573	955	590	+ 13
26	64	307	1,570	580	967	603	+ 20
27	60	310	1,599	586	976	623	+ 40
28	71	336	1,641	587	978	663	+ 31
29	67	337	1,687	596	993	694	+ 5
30	74	351	1,739	624	1,040	699	+ 11
31	65	353	1,762	631	1,052	710	+ 7
32	74	362	1,794	646	1,077	717	+ 9
33	73	359	1,819	656	1,093	726	+ 3
34	76	369	1,856	676	1,127	729	+ 17
35	71	376	1,896	690	1,150	746	+ 26
36	75	390	1,949	706	1,177	772	+ 35
37	81	402	2,002	717	1,195	807	+ 29
38	87	412	2,054	731	1,218	836	+ 21
39	88	422	2,089	739	1,232	857	− 8
40	81	428	2,117	761	1,268	849	− 3
41	85	425	2,151	783	1,305	846	+ 10
42	87	430	2,186	798	1,330	856	+ 16
43	84	446	2,235	818	1,363	872	+ 44
44	93	457	2,309	836	1,393	916	+ 35
45	97	477	2,398	868	1,447	951	+ 37
46	96	499	2,494	903	1,506	988	+ 52
47	107	519	2,607	940	1,567	1,040	+ 47
48	106	542	2,713	976	1,626	1,087	+ 53
49	113	570	2,810	1,002	1,670	1,140	+ 40
50	120	583	2,903	1,034	1,723	1,180	+ 8
51	124	596	3,006	1,091	1,818	1,188	+ 29
52	120	612	3,107	1,134	1,890	1,217	+ 43
53	119	645	3,227	1,180	1,967	1,260	+ 73
54	129	671	3,371	1,223	2,038	1,333	+ 97
Carried forward	3,338	16,377	80,328	29,052	48,421	33,458	...

H^M.—*First Adjustment*—(continued).

Age. x	d_x	Fives.	Fives again $=T$	$(d_{x-4} \ldots d_{x+4}) = S$	$\frac{10S}{6}$	$T - \frac{10S}{6} = 10d_x$	Δ
Brought forward	3,338	16,377	80,328	29,052	48,421	33,458	...
55	153	703	3,523	1,256	2,093	1,430	+ 50
56	150	740	3,673	1,316	2,193	1,480	+ 50
57	152	764	3,833	1,382	2,303	1,530	+ 47
58	156	795	4,000	1,454	2,423	1,577	+ 55
59	153	831	4,174	1,525	2,542	1,632	+115
60	184	870	4,377	1,578	2,630	1,747	+ 95
61	186	914	4,580	1,643	2,738	1,842	+ 87
62	191	967	4,781	1,711	2,852	1,929	+ 85
63	200	998	4,972	1,775	2,958	2,014	+ 44
64	206	1,032	5,156	1,859	3,098	2,058	+ 67
65	215	1,061	5,327	1,921	3,202	2,125	+ 93
66	220	1,098	5,465	1,948	3,247	2,218	+ 55
67	220	1,138	5,571	1,979	3,298	2,273	+ 11
68	237	1,136	5,696	2,047	3,412	2,284	+ 33
69	246	1,138	5,790	2,084	3,473	2,317	− 34
70	213	1,186	5,898	2,169	3,615	2,283	+103
71	222	1,192	6,036	2,190	3,650	2,386	+117
72	268	1,246	6,195	2,215	3,692	2,503	+ 89
73	243	1,274	6,262	2,202	3,670	2,592	+ 77
74	300	1,297	6,306	2,182	3,637	2,669	−100
75	241	1,253	6,215	2,188	2,646	2,569	−121
76	245	1,236	6,051	2,162	3,603	2,448	−113
77	224	1,155	5,810	2,085	3,475	2,335	−131
78	226	1,110	5,562	2,015	3,358	2,204	− 72
79	219	1,056	5,277	1,887	3,145	2,132	−101
80	196	1,005	4,973	1,765	2,942	2,031	−124
81	191	951	4,635	1,637	2,728	1,907	−163
82	173	851	4,252	1,505	2,508	1,744	−177
83	172	772	3,819	1,351	2,252	1,567	−235
84	119	673	3,342	1,206	2,010	1,332	−193
85	117	572	2,882	1,046	1,743	1,139	−203
86	92	474	2,418	889	1,482	936	−192
87	72	391	1,997	752	1,253	744	−130
88	74	308	1,641	616	1,027	614	−153
89	36	252	1,338	526	877	461	− 61
90	34	216	1,082	409	682	400	− 59
91	36	171	884	326	543	341	− 62
92	36	135	711	259	432	279	− 50
93	29	110	539	186	310	229	− 93
94	0	79	...	...	...	(136)	− 68
95	(9)	44	...	...	...	(68)	− 41
96	(5)	...	...	...	...	(27)	− 17
97	(1)	...	...	...	...	(10)	...
Total	10,000	49,571	245,371	88,298	147,163	100,000	...

H^M.—*Second Adjustment.*

Age. x	d_x	Fives.	Fives again $=T$	$(d_{x-4}\,..\,d_{x+4})$ $=S$	$\frac{10S}{6}$	$T-\frac{10S}{6}$ $=10d_x$	Δ
10	40	...	...	...	...	(400)	0
11	41	...	...	...	...	(400)	− 21
12	39	183	...	...	...	(379)	− 39
13	35	166	...	...	...	(340)	− 60
14	28	150	807	316	527	280	− 22
15	23	147	816	335	558	258	+ 28
16	25	161	884	359	598	286	+ 80
17	36	192	1,009	386	643	366	+107
18	49	234	1,165	415	692	473	+ 97
19	59	275	1,322	451	752	570	+ 64
20	65	303	1,451	490	817	634	+ 25
21	66	318	1,532	524	873	659	− 6
22	64	321	1,566	548	913	653	− 18
23	64	315	1,570	561	935	635	− 21
24	62	309	1,561	568	947	614	− 11
25	59	307	1,556	572	953	603	+ 5
26	60	309	1,568	576	960	608	+ 17
27	62	316	1,597	583	972	625	+ 28
28	66	327	1,638	591	985	653	+ 28
29	69	338	1,684	602	1,003	681	+ 19
30	70	348	1,727	616	1,027	700	+ 12
31	71	355	1,764	631	1,052	712	+ 7
32	72	359	1,796	646	1,077	719	+ 6
33	73	364	1,827	661	1,102	725	+ 11
34	73	370	1,861	675	1,125	736	+ 16
35	75	379	1,904	691	1,152	752	+ 25
36	77	389	1,952	705	1,175	777	+ 28
37	81	402	2,002	718	1,197	805	+ 25
38	83	412	2,048	731	1,218	830	+ 16
39	86	420	2,088	745	1,242	846	+ 6
40	85	425	2,120	761	1,268	852	+ 2
41	85	429	2,152	779	1,298	854	+ 8
42	86	434	2,190	797	1,328	862	+ 16
43	87	444	2,241	818	1,363	878	+ 30
44	91	458	2,310	841	1,402	908	+ 39
45	95	476	2,397	870	1,450	947	+ 45
46	99	498	2,497	903	1,505	992	+ 51
47	104	521	2,603	936	1,560	1,043	+ 48
48	109	544	2,709	971	1,618	1,091	+ 42
49	114	564	2,810	1,006	1,677	1,133	+ 34
50	118	582	2,907	1,044	1,740	1,167	+ 26
51	119	599	3,006	1,088	1,813	1,193	+ 34
52	122	618	3,114	1,132	1,887	1,227	+ 48
53	126	643	3,235	1,176	1,960	1,275	+ 63
54	133	672	3,371	1,220	2,033	1,338	+ 72
Carried forward	3,346	16,376	80,357	29,038	48,397	33,479	...

H[M].—*Second Adjustment*—(continued).

Age. x	d_x	Fives.	Fives again $=T$	$(d_{x-4}\,.\,.\,d_{x+4})$ $=S$	$\frac{10S}{6}$	$T-\frac{10S}{6}$ $=10d_x$	Δ
Brought forward	3,346	16,376	80,357	29,038	48,397	33,479	...
55	143	703	3,518	1,265	2,108	1,410	+ 60
56	148	735	3,672	1,321	2,202	1,470	+ 58
57	153	765	3,833	1,383	2,305	1,528	+ 58
58	158	797	4,003	1,450	2,417	1,586	+ 68
59	163	833	4,184	1,518	2,530	1,654	+ 89
60	175	873	4,378	1,581	2,635	1,743	+ 92
61	184	916	4,578	1,646	2,743	1,835	+ 87
62	193	959	4,780	1,715	2,858	1,922	+ 80
63	201	997	4,976	1,784	2,974	2,002	+ 72
64	206	1,035	5,156	1,849	3,082	2,074	+ 68
65	213	1,069	5,319	1,906	3,177	2,142	+ 67
66	222	1,096	5,459	1,950	3,250	2,209	+ 42
67	227	1,122	5,578	1,996	3,327	2,251	+ 27
68	228	1,137	5,686	2,045	3,408	2,278	+ 23
69	232	1,154	5,798	2,098	3,497	2,301	+ 31
70	228	1,177	5,919	2,152	3,587	2,332	+ 77
71	239	1,208	6,054	2,187	3,645	2,409	+ 94
72	250	1,243	6,178	2,205	3,675	2,503	+ 76
73	259	1,272	6,262	2,210	3,683	2,579	+ 34
74	267	1,278	6,276	2,198	3,663	2,613	− 50
75	257	1,261	6,201	2,183	3,638	2,563	− 98
76	245	1,222	6,043	2,147	3,578	2,465	−120
77	233	1,168	5,825	2,088	3,480	2,345	−118
78	220	1,114	5,565	2,003	3,338	2,227	−101
79	213	1,060	5,281	1,893	3,155	2,126	−103
80	203	1,001	4,971	1,769	2,948	2,023	−127
81	191	938	4,626	1,638	2,730	1,896	−156
82	174	858	4,238	1,499	2,498	1,740	−186
83	157	769	3,809	1,353	2,255	1,554	−209
84	133	672	3,347	1,201	2,002	1,345	−207
85	114	572	2,878	1,044	1,740	1,138	−202
86	94	476	2,424	893	1,488	936	−184
87	74	389	2,007	753	1,255	752	−148
88	61	315	1,644	624	1,040	604	−122
89	46	255	1,339	514	857	482	− 83
90	40	209	1,089	414	690	399	− 64
91	34	171	880	327	545	335	− 62
92	28	139	700	256	427	273	− 61
93	23	106	539	196	327	212	− 74
94	14	75	...	...	...	(138)	− 58
95	7	48	...	...	...	(80)	− 43
96	3	...	...	...	...	(37)	− 27
97	1	...	...	...	...	(10)	...
Total	10,000	49,563	245,370	88,292	147,154	100,000	...

H^M.—*Third Adjustment.*

Age. x	d_x	Fives.	Fives again $=T$	$(d_{x-4} \ldots d_{x+4})$ $=S$	$\frac{10S}{6}$	$T-\frac{10S}{6}$ $=10d_x$	Δ_1	Δ_2
10	40	...	...	...	...	(400)	− 10	−11
11	40	...	...	...	...	(390)	− 21	−13
12	38	180	...	...	...	(369)	− 34	−11
13	34	166	...	...	...	(335)	− 45	+33
14	28	155	822	319	532	290	− 12	+41
15	26	154	838	336	560	278	+ 29	+39
16	29	167	905	359	598	307	+ 68	+24
17	37	196	1,020	387	645	375	+ 92	− 3
18	47	233	1,164	418	697	467	+ 89	−25
19	57	270	1,311	453	755	556	+ 64	−34
20	63	298	1,433	488	813	620	+ 30	−30
21	66	314	1,515	519	865	650	0	−17
22	65	318	1,555	543	905	650	− 17	0
23	63	315	1,565	559	932	633	− 17	+ 8
24	61	310	1,561	567	945	616	− 9	+14
25	60	308	1,560	572	953	607	+ 5	+12
26	61	310	1,572	576	960	612	+ 17	+ 6
27	63	317	1,599	582	970	629	+ 23	+ 1
28	65	327	1,637	591	985	652	+ 24	− 4
29	68	337	1,681	603	1,005	676	+ 20	− 6
30	70	346	1,724	617	1,028	696	+ 14	− 4
31	71	354	1,762	631	1,052	710	+ 10	− 1
32	72	360	1,797	646	1,077	720	+ 9	+ 2
33	73	365	1,831	661	1,102	729	+ 11	+ 5
34	74	372	1,867	676	1,127	740	+ 16	+ 7
35	75	380	1,908	691	1,152	756	+ 23	+ 1
36	78	390	1,954	705	1,175	779	+ 24	− 2
37	80	401	2,000	718	1,197	803	+ 22	− 6
38	83	411	2,044	731	1,219	825	+ 16	− 8
39	85	418	2,083	745	1,242	841	+ 8	− 3
40	85	424	2,117	761	1,268	849	+ 5	+ 5
41	85	429	2,151	778	1,297	854	+ 10	+ 8
42	86	435	2,192	797	1,328	864	+ 18	+11
43	88	445	2,245	818	1,363	882	+ 29	+ 9
44	91	459	2,314	842	1,403	911	+ 38	+ 6
45	95	477	2,399	870	1,450	949	+ 44	+ 4
46	99	498	2,496	902	1,503	993	+ 48	− 3
47	104	520	2,599	935	1,558	1,041	+ 45	− 4
48	109	542	2,703	970	1,617	1,086	+ 41	− 5
49	113	562	2,805	1,007	1,678	1,127	+ 36	− 3
50	117	581	2,906	1,046	1,743	1,163	+ 33	+ 6
51	119	600	3,009	1,088	1,813	1,196	+ 39	+10
52	123	621	3,120	1,131	1,885	1,235	+ 49	+ 8
53	128	645	3,242	1,175	1,958	1,284	+ 57	+ 7
54	134	673	3,376	1,221	2,035	1,341	+ 64	− 3
Carried forward	3,348	16,383	80,382	29,034	48,390	33,486	...	...

H^M.—*Third Adjustment*—(continued).

Age. x	d_x	Fives.	Fives again $=T$	$(d_{x-4}\,.\,.\,d_{x+4})$ $=S$	$\frac{10S}{6}$	$T-\frac{10S}{6}$ $=10d_x$	Δ_1	Δ_2
Brought forward	3,348	16,383	80,382	29,034	48,390	33,486	...	0
55	141	703	3,520	1,169	2,115	1,405	+ 61	+ 0
56	147	734	3,673	1,324	2,207	1,466	+ 61	+ 3
57	153	765	3,835	1,385	2,308	1,527	+ 64	+ 7
58	159	798	4,006	1,449	2,415	1,591	+ 71	+11
59	165	835	4,187	1,515	2,525	1,662	+ 82	+ 5
60	174	874	4,379	1,581	2,635	1,744	+ 87	− 1
61	184	915	4,578	1,648	2,747	1,831	+ 86	− 3
62	192	957	4,777	1,716	2,860	1,917	+ 83	− 8
63	200	997	4,970	1,782	2,970	2,000	+ 75	− 8
64	207	1,034	5,150	1,845	3,075	2,075	+ 67	− 8
65	214	1,067	5,311	1,901	3,169	2,142	+ 59	−18
66	221	1,095	5,451	1,950	3,250	2,201	+ 41	− 9
67	225	1,118	5,574	1,999	3,332	2,242	+ 32	0
68	228	1,137	5,689	2,049	3,415	2,274	+ 32	+15
69	230	1,157	5,806	2,100	3,500	2,306	+ 47	+23
70	233	1,182	5,931	2,147	3,578	2,353	+ 70	+ 9
71	241	1,212	6,060	2,182	3,637	2,423	+ 79	−16
72	250	1,243	6,175	2,204	3,673	2,502	+ 63	−41
73	258	1,266	6,250	2,211	3,685	2,565	+ 22	−58
74	261	1,272	6,260	2,204	3,673	2,587	− 36	−45
75	256	1,257	6,191	2,184	3,640	2,551	− 81	−28
76	247	1,222	6,045	2,145	3,575	2,470	−109	− 6
77	235	1,174	5,836	2,085	3,475	2,361	−115	+ 4
78	223	1,120	5,581	2,001	3,335	2,246	−111	− 5
79	213	1,063	5,293	1,895	3,158	2,135	−116	−15
80	202	1,002	4,974	1,773	2,955	2,019	−131	−26
81	190	934	4,621	1,640	2,733	1,888	−157	−27
82	174	855	4,229	1,499	2,498	1,731	−184	−18
83	155	767	3,799	1,351	2,252	1,547	−202	− 5
84	134	671	3,342	1,198	1,997	1,345	−207	+ 7
85	114	572	2,878	1,044	1,740	1,138	−200	+19
86	94	477	2,428	894	1,490	938	−181	+32
87	75	391	2,014	754	1,257	757	−149	+30
88	60	317	1,651	626	1,043	608	−115	+31
89	48	257	1,344	513	855	489	− 88	+18
90	40	209	1,089	413	688	401	− 70	+ 5
91	34	170	876	327	545	331	− 65	+ 3
92	27	136	693	256	427	266	− 62	− 3
93	21	104	532	197	328	204	− 65	+11
94	14	74	...	...	...	(139)	− 54	+11
95	8	48	...	...	...	(85)	− 43	+11
96	4	...	...	...	...	(42)	− 32	...
97	1	...	...	...	...	(10)	...	...
TOTAL	10,000	49,564	245,380	88,290	147,150	100,000	...	...

H^{M}.—*Adjusted.*

Age. x	d_x	l_x	N_x	Average duration of Life, adjusted.	Excess.	Defect.	Average duration of Life, unadjusted.
10	400	100,000	4,979,762	50·30	·01	...	50·29
11	390	99,600	4,880,162	49·50	...	·19	49·69
12	369	99,210	4,780,952	48·69	...	...	48·69
13	335	98,841	4,682,111	47·87	...	·01	47·88
14	290	98,506	4,583,605	47·03	...	·02	47·05
15	278	98,216	4,485,389	46·17	...	·07	46·24
16	307	97,938	4,387,451	45·30	...	·04	45·34
17	375	97,631	4,289,820	44·44	·10	...	44·34
18	467	97,256	4,192,564	43·61	·08	...	43·53
19	556	96,789	4,095,775	42·82	·03	...	42·79
20	620	96,233	3,999,542	42·06	...	·03	42·09
21	650	95,613	3,903,929	41·33	...	...	41·33
22	650	94,963	3,808,966	40·61	...	·01	40·62
23	633	94,313	3,714,653	39·89	·02	...	39·87
24	616	93,680	3,620.973	39·15	...	·03	39·18
25	607	93,064	3,527,909	38·41	...	·03	38·44
26	612	92,457	3,435,452	37·66	·02	...	37·64
27	629	91,845	3,343,607	36·91	·01	...	36·90
28	652	91,216	3,252,391	36·16	·02	...	36·14
29	676	90,564	3,161,827	35·41	...	·01	35·42
30	696	89,888	3,071,939	34·68	...	...	34·68
31	710	89,192	2,982,747	33·94	...	·02	33·96
32	720	88,482	2,894,265	33·21	...	...	33·21
33	729	87,762	2,806,503	32·48	...	...	32·48
34	740	87,033	2,719,470	31·75	...	...	31·75
35	756	86,293	2,633,177	31·01	...	·02	31·03
36	779	85,537	2,547,640	30·28	...	...	30·28
37	803	84,758	2,462,882	29·56	·01	...	29·55
38	825	83,955	2,378,927	28·84	·01	...	28·83
39	841	83,130	2,295,797	28·12	...	...	28·12
40	849	82,289	2,213,508	27·40	...	·02	27·42
41	854	81,440	2,132,068	26·68	...	·01	26·69
42	864	80,586	2,051,482	25·96	...	...	25·96
43	882	79,722	1,971,760	25·23	...	·01	25·24
44	911	78,840	1,892,920	24·51	·01	...	24·50
45	949	77,929	1,814,991	23·79	...	...	23·79
46	993	76,980	1,738,011	23·08	...	...	23·08
47	1,041	75,987	1,662,024	22·37	...	...	22·37
48	1,086	74,946	1,587,078	21·68	...	...	21·68
49	1,127	73,860	1,513,218	20·99	·01	...	20·98
50	1,163	72,733	1,440,485	20·31	·01	...	20·30
51	1,196	71,570	1,368,915	19·63	...	...	19·63
52	1,235	70,374	1,298,541	18·95	...	·02	18·97
53	1,284	69,139	1,229,402	18·28	...	·01	18·29
54	1,341	67,855	1,161,547	17·62	·02	...	17·60
Carried forward	...	...	...	...	·36	·55	...

H^M.—*Adjusted*—(continued).

Age. x	d_x	l_x	N_x	Average duration of Life, adjusted.	Excess.	Defect.	Average duration of Life, unadjusted.
Brought forward	...	...	...	...	·36	·55	...
55	1,405	66,514	1,095,033	16·96	·03	...	16·93
56	1,466	65,109	1,029,924	16·32	...	...	16·32
57	1,527	63,643	966,281	15·68	...	·01	15·69
58	1,591	62,116	904,165	15·06	...	·01	15·07
59	1,662	60,525	843,640	14·44	...	...	14·44
60	1,744	58,863	784,777	13·83	·03	...	13·80
61	1,831	57,119	727,658	13·24	·01	...	13·23
62	1,917	55,288	672,370	12·66	...	...	12·66
63	2,000	53,371	618,999	12·10	·01	...	12·09
64	2,075	51,371	567,628	11·55	·01	...	11·54
65	2,142	49,296	518,332	11·01	...	...	11·01
66	2,201	47,154	471,178	10·49	...	...	10·49
67	2,242	44,953	426,225	9·98	...	...	9·98
68	2,274	42,711	383,514	9·48	·02	...	9·46
69	2,306	40,437	343,077	8·98	...	·01	8·99
70	2,353	38,131	304,946	8·50	...	·04	8·54
71	2,423	35,778	269,168	8·02	...	...	8·02
72	2,502	33,355	235,813	7·57	·05	...	7·52
73	2,565	30,853	204,960	7·14	·02	...	7·12
74	2,587	28,288	176,672	6·75	·06	...	6·69
75	2,551	25,701	150,971	6·37	...	·05	6·42
76	2,470	23,150	127,821	6·02	...	·01	6·03
77	2,361	20,680	107,141	5·68	...	·01	5·69
78	2,246	18,319	88.822	5·35	·02	...	5·33
79	2,135	16,073	72,749	5·03	·03	...	5·00
80	2,019	13,938	58,811	4·72	·01	...	4·71
81	1,888	11,919	46,892	4·43	·03	...	4·40
82	1,731	10,031	36,861	4·17	·03	...	4·14
83	1,547	8,300	28,561	3·94	·04	...	3·90
84	1,345	6,753	21,808	3·73	...	·06	3·79
85	1,138	5,408	16,400	3·53	·02	...	3·51
86	938	4,270	12,130	3·34	...	...	3·34
87	757	3,332	8,798	3·14	·01	...	3·13
88	608	2,575	6,223	2·92	·05	...	2·87
89	489	1,967	4,256	2·66	...	·15	2·81
90	401	1,478	2,778	2·38	·03	...	2·35
91	331	1,077	1,701	2·08	·17	...	1·91
92	266	746	955	1·78	·23	...	1·55
93	204	480	475	1·49	·09	...	1·40
94	139	276	199	1·22	...	·99	2·21
95	85	137	62	·95	...	·26	1·21
96	42	52	10	·69	·19	...	·50
97	10	10	...	·50	·50	...	...
Total	...	...	...	...	2·05	2·15	...

From age 10 to 90, there are 22 cases of coincidence.
" " 33 " excess; aggregate ·87, average +·026.
" " 26 " defect; " ·90, average −·035.

whence $$\Delta_1 = \frac{15u_8 - 10u_3 - 5u_{13} + 600\Delta_3}{25},$$

and $$\Delta_2 = \frac{u_3 - 2u_8 + u_{13} - 175\Delta_3}{25}.$$

Insert these values in $u_7 = u_3 + 4\Delta_1 + 18\Delta_2 + 34\Delta_3$.

FIFTH CURVE.

$$u_4 = u_0 + 4\Delta_1 + 6\Delta_2 + 4\Delta_3$$
$$u_9 = u_0 + 9\Delta_1 + 36\Delta_2 + 84\Delta_3$$
$$u_{14} = u_0 + 14\Delta_1 + 91\Delta_2 + 364\Delta_3;$$

whence $$\Delta_1 = \frac{17u_9 - 11u_4 - 6u_{14} + 800\Delta_3}{25},$$

and $$\Delta_2 = \frac{u_4 - 2u_9 + u_{14} - 200\Delta_3}{25}.$$

Insert these values in $u_7 = u_4 + 3\Delta_1 + 15\Delta_2 + 31\Delta_3$.

RESULTS.

First	curve,	$u_7 = -{\cdot}12u_0 + {\cdot}84u_5 + {\cdot}28u_{10} - 7\Delta_3$
Second	,,	$u_7 = -{\cdot}08u_1 + {\cdot}96u_6 + {\cdot}12u_{11} - 4\Delta_3$
Third	,,	$u_7 = \ldots\ldots\ldots u_7 \ldots\ldots\ldots$
Fourth	,,	$u_7 = +{\cdot}12u_3 + {\cdot}96u_8 - {\cdot}08u_{13} + 4\Delta_3$
Fifth	,,	$u_7 = +{\cdot}28u_4 + {\cdot}84u_9 - {\cdot}12u_{14} + 7\Delta_3$

When these are summed for a mean curve, Δ_3 disappears, and the formula is correct to third differences.

DISCUSSION.

The PRESIDENT (Mr. A. H. Bailey) welcomed this contribution from a former Fellow of the Institute, who, in its early years, made several contributions to the *Journal*. His paper on the selection of lives had thrown much light upon a subject which was not nearly so well understood then as it is now, and it was to be hoped he would make still other communications.

Mr. G. W. BERRIDGE thought they were very much indebted to Mr. Higham for a very easy method of graduation. Many methods are very complicated and take a great deal of time. As to Mr. Higham's method, as you go on you get something very near the mark; and the result of the first application of the method may be sufficient for what you have in hand. Mr. Woolhouse's method, which is one of the best ever published, is very laborious to work, especially

when the numbers run irregularly. He (Mr. Berridge) had, in one or two cases, tried its effect on tables, and had been considerably troubled—not how to use it, but so to use it as to get practically useful results. We all know that the mortality curve is a curve, but we none of us know what sort of curve it is. Any system which gives us easily adjusted results, and which does not tend to straighten the curve in any way—that is to say, has no inherent false bias—is useful. The comparison with the original table is a very useful way of showing how a table, when graduated, conforms to the original figures, and how far not; but that of course is no necessary part of the method, and he himself thought that the method which Mr. Sprague had pointed out, of comparing the actual deaths produced by the ungraduated table, is more perfect in that respect.

Mr. Adler could speak from practical experience of working the adjustment of mortality tables. Finlaison's method was simple, introducing merely an equation of the first degree. Mr. Woolhouse's method was more complex. There is an infinite number of curves of the third degree, which can be chosen in the vicinity of each age; and he supposed that Mr. Higham's method is a mode of selecting the particular curve which he considers most suitable of the infinite number which Mr. Woolhouse offers.

Mr. R. P. Hardy desired to express his pleasure at seeing Mr. Higham again taking part in the proceedings of the Institute, and again throwing so much light upon subjects in which they were all interested. Mr. Higham had produced a graduation by which no violence whatever had been done to the original facts, since the expectation of the premium income would be the same, practically, whether the adjusted or unadjusted results be used; and that is as close an agreement as can be expected, and as close as is necessary.

Mr. J. A. Higham, in reply to Mr. Adler's question, said the curve is one of many. He would not say that it is the best, but it is simply that which is most easily got at. The formula can be worked with great ease and simplicity. There are no plus or minus signs and no decimal points, and the figures can easily be written down at sight and checked by addition.

Mr. G. F. Hardy remarked that Mr. Higham had taken Mr. Woolhouse's formula as the standard by which to test his own, which is intended to obviate the complicated processes involved in the usual method of applying Mr. Woolhouse's. The latter, however, can itself, by a new way of applying it, be worked in almost as little time as the single application of Mr. Higham's formula. Suppose that u_0, u_1, u_2, &c., represent the functions which we have to graduate. The first operation is to form a column of u_0-u_5, &c. This is done simply by differencing the first column for intervals of 5 values. Call this first column α; we then repeat the process, forming a column of $\alpha_0-\alpha_5$; call this β, and repeat the process again, calling the final column γ. Now this last row will contain the values of $u_0-3u_5+3u_{10}-u_{15}$ corresponding to the ordinary expression for the third difference. This last column is summed continuously from the bottom to the top of the table, and the process is repeated with the resulting column. We shall now have our d_x column, with the exception that no correction has been made for the second difference.

This correction is at once applied, by simply taking three times the value of γ_{-1} (our third column) and deducting it from the last column; thus the result of summing our column γ will be the expression

$$u_0 + \ldots + u_4 - 2u_5 - \ldots - 2u_9 + u_{10} + \ldots + u_{14}.$$

Summing this again, we get

$$u_0 + 2u_1 + 3u_2 + 4u_3 + 5u_4 + 3u_5 + u_6 - u_7 - 3u_8 - 5u_9 - 4u_{10} - 3u_{11} - 2u_{12} - u_{13};$$

subtracting from this $3\gamma_{-1}$ (that is $3u_{-1} - 9u_4 + 9u_9 - 3u_{14}$), we have

$$-3u_{-1} + u_0 + 2u_1 + 3u_2 + 4u_3 + 14u_4 + 3u_5 + u_6 - u_7 - 3u_8 - 14u_7 - 4u_{10} - 3u_{11} - 2u_{12} - u_{13} + 3u_{14},$$

which is our value of d_{x+6}, which summed gives the value of

$$l_{x+6} = -3u_{-1} - 2u_0 + 3u_2 + 7u_3 + 21u_4 + 24u_5 + 25u_6 + 24u_7 + 21u_8 + 7u_9 + 3u_{10} - 2u_{11} - 3u_{12};$$

this value being divisible by 125 if we wish to keep the original radix. By this method, all that we have to do in order to get our final table, is to take the l_x column, to difference it three times, sum those differences twice, deduct as a correction three times the third difference at the age one year younger, and we have got the d_x column. Sum that again and we get the l_x. The whole of this process can be gone through in something under an hour for a mortality table. Mr. Woolhouse's method, as ordinarily applied, certainly requires continuous attention; but it will be seen that the method here described is one of great simplicity, and one that could be put at once into the hands of an assistant with simple directions to perform the successive steps. It has this advantage, too, that the whole of it can be checked at each stage by the summation of each of these columns, the sum of each column of differences being equal to the sum of the five top values of the preceding column.

On the Graduation of Mortality Tables. By THOMAS G. ACKLAND, F.I.A., *of the Gresham Life Assurance Society.*

THE problem investigated by Mr. Higham is the discovery of some easy means of graduating a mortality table, so as to obtain the smoothness and scientific accuracy of Mr. Woolhouse's formula with the facility of working of Mr. Finlaison's method. By an ingenious combination of two different systems of summation, Mr. Higham arrives, after three separate graduations, at a result closely approximating to that of Mr. Woolhouse. I propose to show how the formula of Mr. Woolhouse may be *accurately* obtained by columnar summation upon the principles of Mr. Finlaison's method.

Mr. Finlaison's mode of graduation is to sum the original facts

in groups of five, and take the mean value as the graduated result at the central age. The summation in fives may be repeated any number of times. If it be performed thrice in succession, the formula representing the result of the third summation for the central value u_0, is

$$\Sigma_5^3 = 125u'_0 = u_{-6} + 3u_{-5} + 6u_{-4} + 10u_{-3} + 15u_{-2} + 18u_{-1} + 19u_0 + 18u_{+1} + 15u_{+2} + 10u_{+3} + 6u_{+4} + 3u_{+5} + u_{+6} \quad . \quad (\alpha)$$

Mr. Woolhouse's formula may be thus expressed :—

$$125u'_0 = -3u_{-7} - 2u_{-6} + 3u_{-4} + 7u_{-3} + 21u_{-2} + 24u_{-1} + 25u_0 + 24u_{+1} + 21u_{+2} + 7u_{+3} + 3u_{+4} - 2u_{+6} - 3u_{+7} \quad . \quad (\beta)$$

If these two expressions be compared, it will be found that the difference is equal to

$$9(u_{-2} + u_{-1} + u_0 + u_{+1} + u_{+2}) - 3(u_{-7} + u_{-6} + u_{-5} + u_{-4} + u_{-3} + u_{-2} + u_{-1} + u_0 + u_{+1} + u_{+2} + u_{+3} + u_{+4} + u_{+5} + u_{+6} + u_{+7}) \quad . \quad . \quad . \quad (\gamma)$$

The exact results of Mr. Woolhouse's system of graduation can therefore be obtained, from Mr. Finlaison's thrice performed, by any operation which will introduce the value of the expression (γ). This expression is equal to nine times the sum of the five values having u_0 as their central value, less three times the sum of the fifteen values having u_0 as their central value. Mr. Woolhouse's formula may therefore be thus symbolically stated in terms of Mr. Finlaison's method :—

$$\Sigma_5^3 + 9\Sigma_5^1 - 3\Sigma_{15}.$$

In practical application, this formula is very simple. I append a table (A) (pp. 354-5) showing the full operation for the graduation of the decrements of the H^M table. The second column contains the ungraduated values of d_x at all ages; and in columns (3), (4), and (5), the successive summations in groups of five are shown. The results of column (5), if divided by 125, would give the graduated decrement according to Mr. Finlaison's formula (α). In the following columns, (6) to (10), the operation of obtaining Mr. Woolhouse's results is shown, the values in column (5) being increased by nine times the first summation in fives, and diminished by three times the summation of the original values in fifteens. Finally, in column (11) the results are divided by 12·5 (or multiplied by ·08) in order to obtain the graduated values, the radix being increased to 100,000 for comparison with Mr. Woolhouse's results. The summation in fifteens, column (8), is most readily obtained by the addition of the three appropriate values in column

TABLE (A).

Age.	Ungraduated Decrement (Radix=10,000).	First Summation in Fives.	Second Summation in Fives.	Third Summation in Fives.			Summation in Fifteens.			Graduated Decrement (Radix=100,000).
x	d_x	Σ_5^1	Σ_5^2	Σ_5^3	$9\Sigma_5^1$	$\Sigma_5^3+9\Sigma_5^1$ =(5)+(6)	Σ_{15}	$3\Sigma_{15}$	$\Sigma_5^3+9\Sigma_5^1-3\Sigma_{15}$ =(7)−(9)	d'_x =(10)÷12·5
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
(3)	...	...	...	...	...	...	79	237	− 237	(− 19·0)
(4)	...	...	...	79	...	79	79	237	− 158	(− 12·6)
(5)	...	...	...	237	...	237	119	357	− 120	(− 9·6)
(6)	...	...	79	514	...	514	154	462	52	(4·2)
(7)	...	...	158	945	...	945	194	582	363	(29·0)
(8)	...	79	277	1,570	711	2,281	216	648	1,633	(130·6)
(9)	...	79	431	2,174	711	2,885	216	648	2,237	(179·0)
10	79	119	625	2,757	1,071	3,828	257	771	3,057	(244·6)
11	0	154	683	3,240	1,386	4,626	316	948	3,678	(294·2)
12	40	194	741	3,577	1,746	5,323	384	1,152	4,171	(333·7)
13	35	137	760	3,716	1,233	4,949	440	1,320	3,629	(290·3)
14	40	137	768	3,884	1,233	5,117	507	1,521	3,596	(287·7)
15	22	138	764	4,148	1,242	5,390	566	1,698	3,692	(295·4)
16	0	162	851	4,564	1,458	6,022	639	1,917	4,105	(328·4)
17	41	190	1,005	5,133	1,710	6,843	703	2,109	4,734	378·7
18	59	224	1,176	5,835	2,016	7,851	672	2,016	5,835	466·8
19	68	291	1,337	6,537	2,619	9,156	736	2,208	6,948	555·8
20	56	309	1,466	7,102	2,781	9,883	756	2,268	7,615	609·2
21	67	323	1,553	7,496	2,907	10,403	792	2,376	8,027	642·2
22	59	319	1,570	7,713	2,871	10,584	819	2,457	8,127	650·2
23	73	311	1,570	7,792	2,799	10,591	871	2,613	7,978	638·2
24	64	308	1,554	7,809	2,772	10,581	936	2,808	7,773	621·8
25	48	309	1,545	7,838	2,781	10,619	969	2,907	7,712	617·0
26	64	307	1,570	7,909	2,763	10,672	983	2,949	7,723	617·8
27	60	310	1,599	8,042	2,790	10,832	991	2,973	7,859	628·7
28	71	336	1,641	8,236	3,024	11,260	1,006	3,018	8,242	659·4
29	67	337	1,687	8,428	3,033	11,461	1,014	3,042	8,419	673·5
30	74	351	1,739	8,623	3,159	11,782	1,036	3,108	8,674	693·9
31	65	353	1,762	8,801	3,177	11,978	1,050	3,150	8,828	706·2
32	74	362	1,794	8,970	3,258	12,228	1,074	3,222	9,006	720·5
33	73	359	1,819	9,127	3,231	12,358	1,107	3,321	9,037	723·0
34	76	369	1,856	9,314	3,321	12,635	1,128	3,384	9,251	740·0
35	71	376	1,896	9,522	3,384	12,906	1,155	3,465	9,441	755·3
36	75	390	1,949	9,757	3,510	13,267	1,168	3,504	9,763	781·0
37	81	402	2,002	9,990	3,618	13,608	1,194	3,582	10,026	802·1
38	87	412	2,054	10,211	3,708	13,919	1,217	3,651	10,268	821·4
39	88	422	2,089	10,413	3,798	14,211	1,248	3,744	10,467	837·4
40	81	428	2,117	10,597	3,852	14,449	1,281	3,843	10,606	848·5
41	85	425	2,151	10,778	3,825	14,603	1,314	3,942	10,661	852·9
42	87	430	2,186	10,998	3,870	14,868	1,351	4,053	10,815	865·2
43	84	446	2,235	11,279	4,014	15,293	1,400	4,200	11,093	887·4
44	93	457	2,309	11,622	4,113	15,735	1,449	4,347	11,388	911·0
45	97	477	2,398	12,043	4,293	16,336	1,488	4,464	11,872	949·8
46	96	499	2,494	12,521	4,491	17,012	1,520	4,560	12,452	996·2
47	107	519	2,607	13,022	4,671	17,693	1,561	4,683	13,010	1,040·8
48	106	542	2,713	13,527	4,878	18,405	1,633	4,899	13,506	1,080·5
49	113	570	2,810	14,039	5,130	19,169	1,698	5,094	14,075	1,126·0
50	120	583	2,903	14,539	5,247	19,786	1,763	5,289	14,497	1,159·8
51	124	596	3,006	15,053	5,364	20,417	1,835	5,505	14,912	1,193·0
52	120	612	3,107	15,614	5,508	21,122	1,895	5,685	15,437	1,235·0
53	119	645	3,227	16,234	5,805	22,039	1,982	5,946	16,093	1,287·4
54	129	671	3,371	16,901	6,039	22,940	2,072	6,216	16,724	1,337·9
Forward	3,338	16,769	84,004	420,770	150,921	571,691	51,033	153,099	419,107	33,487·4

TABLE (A)—(*continued*).

Age.	Ungraduated Decrement (Radix= 10,000).	First Summation in Fives.	Second Summation in Fives.	Third Summation in Fives.			Summation in Fifteens.			Graduated Decrement (Radix= 100,000).
x.	d_x	Σ_5^1	Σ_5^2	Σ_5^3	$9\Sigma_5^1$	$\Sigma_5^3+9\Sigma_5^1$ =(5)+(6)	Σ_{15}	$3\Sigma_{15}$	$\Sigma_5^3+9\Sigma_5^1-3\Sigma_{15}$ =(7)−(9)	d'_x =(10)÷12·5
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Forward	3,338	16,769	84,004	420,770	150,921	571,691	51,033	153,099	419,107	33,487·4
55	153	703	3,523	17,627	6,327	23,954	2,156	6,468	17,486	1,398·9
56	150	740	3,673	18,400	6,660	25,060	2,250	6,750	18,310	1,464·8
57	152	764	3,833	19,203	6,876	26,079	2,343	7,029	19,050	1,524·0
58	156	795	4,000	20,057	7,155	27,212	2,438	7,314	19,898	1,591·8
59	153	831	4,174	20,964	7,479	28,443	2,534	7,602	20,841	1,667·3
60	184	870	4,377	21,912	7,830	29,742	2,634	7,902	21,840	1,747·2
61	186	914	4,580	22,884	8,226	31,110	2,752	8,256	22,854	1,828·3
62	191	967	4,781	23,866	8,703	32,569	2,869	8,607	23,962	1,917·0
63	200	998	4,972	24,816	8,982	33,798	2,929	8,787	25,011	2,000·9
64	206	1,032	5,156	25,701	9,288	34,989	3,001	9,003	25,986	2,078·9
65	215	1,061	5,327	26,491	9,549	36,040	3,117	9,351	26,689	2,135·1
66	220	1,098	5,465	27,215	9,882	37,097	3,204	9,612	27,485	2,198·8
67	220	1,138	5,571	27,849	10,242	38,091	3,351	10,053	28,038	2,243·0
68	237	1,136	5,696	28,420	10,224	38,644	3,408	10,224	28,420	2,273·6
69	246	1,138	5,790	28,991	10,242	39,233	3,467	10,401	28,832	2,306·6
70	213	1,186	5,898	29,615	10,674	40,289	3,500	10,500	29,789	2,383·1
71	222	1,192	6,036	30,181	10,728	40,909	3,526	10,578	30,331	2,426·5
72	268	1,246	6,195	30,597	11,214	41,811	3,539	10,617	31,194	2,495·5
73	243	1,274	6,262	31,014	11,466	42,480	3,520	10,560	31,920	2,553·6
74	300	1,297	6,306	31,029	11,673	42,702	3,491	10,473	32,229	2,578·3
75	241	1,253	6,215	30,644	11,277	41,921	3,444	10,332	31,589	2,527·1
76	245	1,236	6,051	29,944	11,124	41,068	3,379	10,137	30,931	2,474·5
77	224	1,155	5,810	29,015	10,395	39,410	3,252	9,756	29,654	2,372·3
78	226	1,110	5,562	27,673	9,990	37,663	3,156	9,468	28,195	2,255·6
79	219	1,056	5,277	26,257	9,504	35,761	3,026	9,078	26,683	2,134·6
80	196	1,005	4,973	24,699	9,045	33,744	2,830	8,490	25,254	2,020·3
81	191	951	4,635	22,956	8,559	31,515	2,661	7,983	23,532	1,882·6
82	173	851	4,252	21,021	7,659	28,680	2,397	7,191	21,489	1,719·1
83	172	772	3,819	18,930	6,948	25,878	2,190	6,570	19,308	1,544·6
84	119	673	3,342	16,713	6,057	22,770	1,981	5,943	16,827	1,346·2
85	117	572	2,882	14,458	5,148	19,606	1,793	5,379	14,227	1,138·2
86	92	474	2,418	12,280	4,266	16,546	1,596	4,788	11,758	940·6
87	72	391	1,997	10,276	3,519	13,795	1,377	4,131	9,664	773·1
88	74	308	1,641	8,476	2,772	11,248	1,186	3,558	7,690	615·2
89	36	252	1,338	6,938	2,268	9,206	1,005	3,015	6,191	495·3
90	34	216	1,082	5,649	1,944	7,593	832	2,496	5,097	407·8
91	36	171	880	4,544	1,539	6,083	660	1,980	4,103	328·2
92	36	135	708	3,586	1,215	4,801	541	1,623	3,178	254·2
93	29	106	536	2,764	954	3,718	424	1,272	2,446	195·7
94	0	80	380	2,048	720	2,768	332	996	1,772	141·8
95	5	44	260	1,424	396	1,820	260	780	1,040	83·2
96	10	15	164	928	135	1,063	186	558	505	40·4
97	...	15	84	573	135	708	150	450	258	20·6
98	...	10	40	323	90	413	116	348	65	5·2
99	...	...	25	159	...	159	80	240	−81	−6·5
100	...	...	10	75	...	75	44	132	−57	−4·5
101	...	...	...	35	...	35	15	45	−10	− ·8
102	...	...	...	10	...	10	15	45	−35	−2·8
103	...	...	...	...	...	...	10	30	−30	−2·4
	10,000	50,000	250,000	1,250,000	450,000	1,700,000	150,000	450,000	1,250,000	100,000·

(3); thus the value of Σ_{15} for central age 18 would be equal to the sum of the values of Σ_5^1 at central ages 13, 18, 23; and this operation can be further facilitated by the use of a card so perforated as to show only the three values required. The total of each column is a complete check upon the accuracy of the work step by step.*

It will be found that the values from age 17 inclusive to the end of the table, agree closely with Mr. Woolhouse's results as printed in the Institute tables. Deviations at certain ages, between these limits, arise from further adjustment of individual values performed by Mr. Woolhouse for the purpose of eliminating accidental irregularities; but the final column of the table here given expresses in every case (after age 16) the exact numerical result of Mr. Woolhouse's formula.

The values in column (11) at ages 3 to 16 inclusive, are worked out on the assumption that the ungraduated values of d_x at ages earlier than 10 are equal to 0. This assumption, which holds good at the end of the table, is not applicable at its commencement; and the values in column (11) at ages before 17 do not therefore correctly express the graduated decrement, but are inserted only to render complete the operations of summation and verification. I find, however, that these imperfect values at the early ages may be employed to obtain graduated results which appear to be practically good. The operation, which is perfectly simple, and is shown in the following table (B), consists in folding back, so to speak, the

Table (B).

Age.	Values of Column (11), Table (A).		Sum of Columns (2) and (3).	Mr. Woolhouse's Graduated Decrements.
	Ages 10 to 16.	Ages (3) to (9), in reverse order.		
(1)	(2)	(3)	(4)	(5)
10	244·6	179·0	423·6	490
11	294·2	130·6	424·8	397
12	333·7	29·0	362·7	329
13	290·3	4·2	294·5	288
14	287·7	− 9·6	278·1	272
15	295·4	−12·6	282·8	282
16	328·4	−19·0	309·4	318
			2,375·9	2,376

* The relation between Mr. Finlaison's and Mr. Woolhouse's methods may be readily seen by a comparison of columns (5) and (10) of Table (A). It will be found that there is a close agreement between ages 20 and 70, and that from age 70 to 84 Mr. Finlaison's graduated decrement is much lower, while after age 84 it is much higher, than Mr. Woolhouse's.

overlapping values in column (11), at ages earlier than 10, upon the values from age 10 to 16, and the sum of the values thus brought together gives the graduated result. The adjusted decrements obtained by Mr. Woolhouse at the same ages are given in column (5) for purposes of comparison. The sum of the adjusted values at ages 10 to 16 agrees by the two methods;—an essential condition, in order that the sum of the adjusted decrements at all ages may make up the value of the radix (100,000).

This process of obtaining the adjusted values at ages 10 to 16 is not entirely empirical; and a short explanation of the *rationale* of the method may here be given. The analysis of each of the graduated values of Table (A), from $d'_{(3)}$ to d'_{16} inclusive, is shown in the following table (C), the graduated decrements (d') being

TABLE (C).

Graduated Decrement as in Table (A), col. (10).	Ungraduated Decrements.														Total of Coefficients.
	d_{10}	d_{11}	d_{12}	d_{13}	d_{14}	d_{15}	d_{16}	d_{17}	d_{18}	d_{19}	d_{20}	d_{21}	d_{22}	d_{23}	
$d'_{(3)}$	−3	...	...	...	...	...	...	...	...	...	...	...	...	...	− 3
$d'_{(4)}$	−2	−3	...	...	...	...	...	...	...	...	...	...	...	...	− 5
$d'_{(5)}$	0	−2	−3	...	...	...	...	...	...	...	...	...	...	...	− 5
$d'_{(6)}$	3	0	−2	−3	...	...	...	...	...	...	...	...	..	...	− 2
$d'_{(7)}$	7	3	0	−2	−3	...	...	...	...	...	...	...	...	...	5
$d'_{(8)}$	21	7	3	0	−2	−3	...	...	...	...	...	...	...	...	26
$d'_{(9)}$	24	21	7	3	0	−2	−3	...	...	...	...	...	...	...	50
d'_{10}	25	24	21	7	3	0	−2	−3	...	...	...	...	...	...	75
d'_{11}	24	25	24	21	7	3	0	−2	−3	...	...	...	...	...	99
d'_{12}	21	24	25	24	21	7	3	0	−2	−3	...	...	...	...	120
d'_{13}	7	21	24	25	24	21	7	3	0	−2	−3	...	...	...	127
d'_{14}	3	7	21	24	25	24	21	7	3	0	−2	−3	...	...	130
d'_{15}	0	3	7	21	24	25	24	21	7	3	0	−2	−3	...	130
d'_{16}	−2	0	3	7	21	24	25	24	21	7	3	0	−2	−3	128

expressed on the horizontal lines of the table in terms of the ungraduated values (d) set forth along the top of the table. Referring now to the numerical coefficients contained in the full formula (β), it will be seen from this table that the graduated values at the ages therein included do not in any case contain the complete expression of the formula (β), the most extended value (d'_{16}) having only 14 of the 15 terms contained in the full expression. But it will be further seen that, *as regards the numerical coefficients only*, the values of d' may be so taken in pairs as to include the complete expression in every case. Thus the several values

$$d'_{10},\ d'_{11},\ d'_{12},\ d'_{13},\ d'_{14},\ d'_{15},\ d'_{16},$$

may be respectively made up to the complete expression (β), as regards the coefficients, by the addition of

$$d'_{(9)},\ d'_{(8)},\ d'_{(7)},\ d'_{(6)},\ d'_{(5)},\ d'_{(4)},\ d'_{(3)}.$$

Thus, for example, we have

$$125d'_{10} = 25d_{10} + 24d_{11} + 21d_{12} + 7d_{13} + 3d_{14} + 0d_{15} - 2d_{16} - 3d_{17},$$
$$125d'_{(9)} = 24d_{10} + 21d_{11} + \ 7d_{12} + 3d_{13} + 0d_{14} - 2d_{15} - 3d_{16};$$

and in the sum of these two expressions is included the whole of the coefficients of the full formula (β). Summing, then, in pairs according to the following scheme, we obtain in every case a complete expression, including 125 ungraduated values (the sum of the coefficients being 125):—

$$d''_{10} = d'_{10} + d'_{(9)}$$
$$d''_{11} = d'_{11} + d'_{(8)}$$
$$d''_{12} = d'_{12} + d'_{(7)}$$
$$d''_{13} = d'_{13} + d'_{(6)}$$
$$d''_{14} = d'_{14} + d'_{(5)}$$
$$d''_{15} = d'_{15} + d'_{(4)}$$
$$d''_{16} = d'_{16} + d'_{(3)}$$

The following table (D), obtained by summing the values of Table (C) in pairs as indicated above, shows the exact construction of the graduated values finally obtained:—

Table (D).

Graduated Decrement (Final Value).	Ungraduated Decrements.														Total of Coefficients.
	d_{10}	d_{11}	d_{12}	d_{13}	d_{14}	d_{15}	d_{16}	d_{17}	d_{18}	d_{19}	d_{20}	d_{21}	d_{22}	d_{23}	
d''_{10}	49	45	28	10	3	−2	−5	−3	...	...	...	...	...	...	125
d''_{11}	45	32	27	21	5	0	0	−2	−3	...	...	...	...	...	125
d''_{12}	28	27	25	22	18	7	3	0	−2	−3	...	...	...	...	125
d''_{13}	10	21	22	22	24	21	7	3	0	−2	−3	...	...	...	125
d''_{14}	3	5	18	24	25	24	21	7	3	0	−2	−3	...	...	125
d''_{15}	−2	0	7	21	24	25	24	21	7	3	0	−2	−3	...	125
d''_{16}	−5	0	3	7	21	24	25	24	21	7	3	0	−2	−3	125

The distribution of the ungraduated values at these early ages is not, in all cases, perfectly satisfactory; but the spirit of Mr. Woolhouse's formula is, on the whole, well maintained, and the simple operation here suggested may perhaps be found to give results sufficiently near for all practical purposes. It will be remembered that Mr. Woolhouse obtains the initial values of his table

by constant third differences derived from the values at older ages, but entirely independent of the ungraduated values at ages 10 to 16; and Mr. Higham has adopted a similar plan to obtain his initial and terminal values.

A Method of Solving approximately Questions in Compound Interest, without the aid of Tables. By MARCUS N. ADLER, M.A., *Fellow of University College, London, &c.*

[Read before the Institute, 24 April 1882.]

$(1+i)^x$ being the amount of 1 put out at compound interest for x years, it follows, solving for x, that the number of years in which 1 will amount to m is $\frac{\log_e m}{\log_e(1+i)}$; which is $\frac{\log_e m}{i}$ multiplied by (1 + a very convergent series, the sum of which is nearly half the rate of interest). Moreover, 1 becomes m^n in $n\frac{\log_e m}{\log_e(1+i)}$ years. Let m be 2: $\log_e 2$ being ·6931, we deduce at once that money doubles itself at 2 per-cent in $\frac{70}{2}$ or 35 years, and becomes 2^2 or 4 times its original value in twice 35 years, 2^3 or 8 in thrice 35 years, and so on. For every additional 3 per-cent, it will be necessary to add 1 to 70 and to divide by the interest, in order to find the time in which money will double. Thus, at 8 per-cent, £1 will become £2 in $72\div 8$ or 9 years, £4 in twice 9 or 18 years, and so on.

Similarly, 1 becomes $1\frac{1}{4}$ in $\cdot 23:i$ years nearly,
1 " $1\frac{1}{2}$ in $\cdot 4:i$ "
1 " $1\frac{3}{4}$ in $\cdot 56:i$ "

The figures given above, ·23, ·4, ·56, and ·7, form a sequence which is easily remembered. If this and the ordinary formulas of compound interest be remembered, any question can be solved with fair exactness after a little practice.

Example 1.—What is the amount of £15 in 89 years at 3 per-cent? 1 becomes 2 in $\frac{70}{3}$ or $23\frac{1}{3}$ years,
1 " 2^2 or 4 in $46\frac{2}{3}$ "
1 " 2^3 or 8 in 70 "

As 1 also becomes $1\frac{3}{4}$ in $\frac{56}{3}$, say 19 years, we have at once that £1 in $70+19$ or 89 years becomes $£8\times 1\frac{3}{4}=£14$. Therefore £15 becomes £210, the true result being £208. 5*s*.

Example 2.—What is the present value of £1,000 payable 80 years hence at 5 per-cent? 1 becomes 2^5 or 32 in $5 \times \frac{71}{5}$ years, and also $1\frac{1}{2}$ as much in $\frac{40}{5}$ or 8 years more. 1 becomes therefore 48 in 79 years, and 50 in 80 years. The present value of £1,000 in 80 years is found at once by dividing £1,000 by 50, which is £20—the correct answer being £20. 3*s*.

Example 3.—What will an annuity of £10 accumulate to in 78 years at 3 per-cent? Following the rule, we have to deduct from the accumulations of £1 in 78 years 1, and to divide by the rate of interest. Now, 1 becomes 2^3 or 8 in 70 years, and 10 in $70 + \frac{23}{3}$ or 78 years, $\frac{10-1}{\cdot 03} = 300$, so that the accumulations of £10 per annum amount to £3,000—the true answer being £3,010.

Example 4.—What sinking fund will be required to pay off £10,000 at 8 per-cent in 50 years? 1 becomes 2^5 or 32 in $5 \times \frac{72}{8}$ years, that is, in 45 years, and in $\frac{40}{8}$ or 5 years more, that is, in 50 years, 32 will become half as much again, namely, 48. The reciprocal of the accumulations of the annuity gives the sinking fund, $\frac{\cdot 08 \times 10000}{48-1}$, or £17, the correct result being £17. 8*s*. 7*d*. If we wish to find the annuity for 50 years which £10,000 will purchase at 8 per-cent, we have simply to add one year's interest, £800, to the sinking fund, £17, making £817. This gives the annuity for 50 years which £10,000 will purchase at 8 per-cent.

In this manner all ordinary questions in compound interest may be solved with ease.

The Cost of Life Assurance.

[Abridged and adapted from the *Review*.]

In 1874, two officials of the Board of Trade issued a report containing a table of the proportion of expenses to premiums, in the case of each life assurance company transacting business in this country; but this report was not continued in subsequent years. A similar table of expense-rates was published annually for a time in the *Insurance Register*. This latter table was also discontinued, the editor stating that unfair and unjust conclusions had been drawn from it in regard to companies of the highest respectability, and that it was evident no comparative statement of the kind could fairly be published without supplying, at the same

time, copious explanatory notes. Some persons have gone so far as to say that the publication of mere ratios of expenses to income is, by itself, an invidious and absolutely unreliable test. Others, however, have expressed great regret at the discontinuance by the Board of Trade of the publication of an official analysis of insurance companies' accounts; alleging that the only objection to such an analysis was that it told the truth about some companies in so plain a manner as to be unpleasant to them.

Those who have carefully studied the accounts of the various companies have probably noticed that an expenditure much below the average rate is frequently to be accounted for by one or more of the following causes:—

1. The premium income being larger than the average amounts, and the fixed charges of management, apart from commission, representing less than the average rate.

2. The company being Mutual, and material help in extending the business being obtained gratuitously from the policyholders. In a small class of offices, remarkable results have been attained without the aid of agents or payment of commission.

3. A separate branch, such as Fire business, being worked under the same management, thus lightening the expenses falling on the Life business.

4. The company being one of the London Legal offices and its policies being for large amounts, and, in many cases, being in connection with loans, thus not requiring expenditure in advertizing or canvassing.

5. No new business being transacted.

6. The new business being much below the average amount for a company of the income shown.

The opposite causes to those given above have of course the contrary effect, and tend to increase the expenditure beyond the average rate.

In the following tables the companies have been divided, as far as possible, into classes, so as to ascertain how much weight ought on the average to be attached to the several main causes of increase or diminution in the expense-rates. Other minor causes, not capable of being so easily classified, may affect the expenses of individual companies; but none of these minor causes can materially affect the average expense-rates brought out for any of the classes in the tables.

In Table I the figures relating to 95 life offices are summarized in 9 groups, which are arranged according to the magnitude of the

TABLE I.

Life Premium Incomes of the following Amounts.	Number of Companies.	Total Life Premium Incomes.	Total Expenses (including Commission).	Rate per-cent of Expenses.	Names of Companies.
		£	£		
Under £10,000 . .	14	50,913	17,656	35·1	City, *Colonial*, *Farmers'*, *Imperial Union*, Law Property, *London, East India, and Colonial*, *London and Southwark*, *Masonic*, *Midland Counties*, *National Guardian*, *Patriotic*, Preserver, Scottish Metropolitan, *Security*.
£10,000 to £25,000	8	145,941	40,053	27·4	Argus, Briton, Emperor, *Marine and General*, *National of Ireland*, Scottish Commercial, *United Kent*, Western Counties.
£25,000 ,, £50,000	7	258,894	51,813	20·0	Positive, *Provincial*, Sceptre, Scottish Imperial, Whittington, Westminster, *Yorkshire*.
£50,000 ,, £75,000	9	568,803	95,537	16·8	Caledonian, Great Britain, Lancashire, Law Union, London and Lancashire, National, Queen, Sovereign, University.
£75,000 ,, £100,000	11	953,159	130,827	13·7	*Atlas*, *Church of England*, Commercial Union, Friends' Provident, *Imperial*, London and Provincial Law, Mutual, Pelican, Reliance, Union, West of England.
£100,000 ,, £150,000	20	2,507,247	325,070	13·0	Alliance, British Empire, British Equitable, Briton Medical, City of Glasgow, Crown, English and Scottish Law, Equitable, Equity and Law, General, Guardian, Hand-in-Hand, Legal and General, Provident Clerks', Prudential, Rock, *Royal Exchange*, Scottish Provincial, *Sun*, Universal.
£150,000 ,, £200,000	8	1,340,929	152,907	11·4	Clerical, Edinburgh, London Assurance, Metropolitan, Northern, Norwich Union, Provident, Scottish Amicable.
£200,000 ,, £250,000	9	2,060,062	214,439	10·4	Clergy Mutual, Eagle, Economic, Liverpool and London, Royal, Scottish Equitable, Star, United Kingdom Temperance, Law.
Above £250,000 . .	9	3,531,429	470,018	13·3	Gresham, Life Association of Scotland, London Life, National Provident, North British, Scottish Provident, Scottish Union and National, Scottish Widows' Fund, Standard.
	95	11,417,377	1,498,320	13·1	

premium incomes of the companies. As some of these companies do not publish the amount of their new premiums, the second table, which is based on the amount of the new premiums, relates only to 75 of the companies; and in Table I the names of those offices which are not included in Table II are printed in italics.

Table II gives two separate estimates of the expense-rates, allocating the same between renewal premiums and new premiums, in accordance with the suggestions contained in Mr. Deuchar's paper of Oct. 1874 (see *J.I.A.*, xviii, 323). Method A consists in assuming that 7½ per-cent (which nearly represents the average cost of management of companies transacting no new business) is the actual cost of managing the old business, and that any excess beyond that rate is applicable to the new premiums. This method is applicable to the circumstances of many companies, and is very useful for comparison in all cases; but in the case of very small companies it is open to the objection that the 7½ per-cent allotted for managing the old business is insufficient. There is also the further objection, that in a good many instances the rate of expense brought out as applicable to the new premiums, is equal to more than 100 per-cent, while it might fairly be contended that, if the expenses and risk of first year (including risk of deterioration as well as risk of death) be in excess of the total amount of new premiums, such excess ought to be charged on renewals. Method B, which is not open to the objections above-named, consists in assuming that the limit of total expenditure in respect of new business shall be two-thirds of the new premiums; that the remaining one-third, equal to £1 per-cent on the sum assured, is required to meet the risk of death or deterioration of the life during the first year (*see* Mr. Sprague's table of first year's risk, *J.I.A.*, xxii, 419, January 1881, taking 39 as the average age at entry); and that any expenditure beyond two-thirds of the new premiums shall be held as applicable to the renewal premiums.

In this table the companies are arranged according to their class, and (as far as practicable) to the amount of income. In some classes the numbers were too small to admit of subdivision according to the amount of each company's income. In the case of companies transacting also fire, marine, or industrial business, such a classification would have served no purpose, as in such cases it is generally the relative amount of the life income to the total income, rather than the absolute amount of the former, which affects the rate.

TABLE II.

Class of Companies.	Number of Company.	Total Premium Income.	Expenses (including Commission).	New Premiums (estimated).	Rates per-cent of Expenses. Ordinary Method. On Total Premiums.	Method A. On New Premiums, in addition to 7½ per-cent of the Renewal Premiums.	Method B. On Renewal Premiums, in addition to two-thirds of the New Premiums.
		£	£	£			
I. Mutual Companies employing no Agents and paying no Commission	8	206,274	10,748	9,351	5·2	0·0	2·3
	17	147,638	8,307	5,550	5·6	0·0	3·2
	32	310,930	13,052	9,737	4·2	0·0	2·2
	35	151,794	8,169	8,124	5·4	0·0	1·9
Total, Class I (Four Offices)	...	816,636	40,276	32,762	4·9	0·0	2·4
II. Mutual Companies employing Agents:— Premium Incomes under £100,000	19	87,282	9,122	6,734	10·5	45·8	5·7
	21	61,108	15,467	2,910	25·3	381·5	23·2
	36	81,170	9,854	3,302	12·1	121·6	9·8
	37	69,125	8,504	4,293	12·3	84·8	8·7
	48	81,404	18,285	10,224	22·5	126·6	16·1
	68	15,159	6,612	3,006	43·6	189·6	37·9
Total, Subdivision 1 (Six Off.)	...	395,248	67,844	30,469	17·2	132·9	13·0
Premium Incomes between £100,000 and £200,000	2	107,870	22,073	8,288	20·5	176·2	16·6
	24	139,505	11,872	5,805	8·5	31·8	6·0
	41	154,688	19,980	7,915	12·9	113·4	10·0
	45	100,551	13,738	7,611	13·7	88·9	9·3
	52	182,744	20,227	11,416	11·1	64·6	7·4
Total, Subdivision 2 (Five Off.)	...	685,358	87,890	41,035	12·8	96·4	9·4
Premium Incomes above £200,000	13	227,491	22,959	8,200	10·1	79·4	8·0
	38	324,994	26,989	11,582	8·3	30·1	6·1
	54	217,118	26,030	14,542	12·0	74·5	8·1
	57	359,793	36,957	31,411	10·3	39·2	4·9
	60	600,776	66,864	34,548	11·1	70·6	7·7
	65	235,988	28,667	14,504	12·2	83·1	8·6
Total, Subdivision 3 (Six Off.)	...	1,966,160	208,466	114,787	10·6	60·6	7·1
Total, Class II (Seventeen Off.)	...	3,046,766	364,200	186,291	12·0	80·3	8·4
III. London Legal Life Offices (Proprietary)	18	129,083	12,630	13,710	9·8	29·0	3·0
	26	247,786	19,491	11,216	7·9	15·6	5·1
	28	137,348	16,212	8,780	11·8	74·8	8·1
	34	76,513	8,227	4,752	10·8	59·9	7·0
Total, Class III (Four Offices)	...	590,730	56,560	38,458	9·6	39·4	5·6
IV. Proprietary Companies having a separate Income from Fire, Marine, or Industrial Premiums, *more than twice the amount* of their Life Premium Income	1	100,295	12,016	9,268	12·0	56·0	6·4
	10	99,469	13,975	9,790	14·1	74·0	8·3
	23	114,041	15,240	7,499	13·4	96·7	9·6
	25	62,633	6,936	7,541	11·1	37·2	3·5
	30	229,043	22,904	10,189	10·0	63·7	7·4
	39	319,646	38,291	27,661	12·0	59·3	6·8
	40	156,662	15,666	9,872	10·0	47·2	6·2
	46	107,535	12,305	15,527	11·4	28·3	2·1
	47	57,199	8,182	7,064	14·3	62·6	6·9
	50	247,194	24,897	12,976	10·1	56·5	6·9
	53	15,307	1,429	2,382	9·3	19·3	0·0
	55	29,338	3.749	5,211	12·8	37·2	1·1
Total, Class IV (Twelve Offices)	...	1,538,362	175,590	124,980	11·4	55·7	6·5

TABLE II—(*continued*).

Class of Companies.	Number of Company.	Total Premium Income.	Expenses (including Commission).	New Premiums (estimated).	Rates per-cent of Expenses. Ordinary Method. On Total Premiums.	Method A. On New Premiums, in addition to 7½ per-cent of the Renewal Premiums.	Method B. On Renewal Premiums, in addition to two-thirds of the New Premiums.
		£	£	£			
V. PROPRIETARY COMPANIES HAVING A SEPARATE INCOME FROM FIRE, MARINE, OR INDUSTRIAL PREMIUMS, *less than twice the amount* OF THEIR LIFE PREMIUM INCOME	5	67,167	11,349	7,107	16·9	96·3	11·1
	20	104,952	22,699	14,471	21·6	110·0	14·4
	27	68,760	9,095	6,385	13·2	69·2	7·8
	31	156,198	16,226	6,395	10·4	78·0	8·0
	58	129,470	19,106	8,550	14·8	117·4	11·1
	59	266,205	41,856	18,429	15·7	126·2	11·9
	64	98,566	13,375	7,240	13·6	90·1	9·4
	70	98,044	12,960	5,492	13·2	109·6	10·0
Total, Class V (Eight Offices) .	...	989,362	146,666	74,069	14·8	105·3	10·6
VI. ORDINARY LIFE ASSURANCE COMPANIES (PROPRIETARY) NOT BELONGING TO ANY OF THE FOREGOING CLASSES:— Prem. Incomes under £40,000	4	17,743	11,395	5,289	64·2	197·8	63·2
	6	2,228	2,776	1,788	124·6	153·4	360·0
	15	13,865	4,815	1,449	34·7	268·0	31·0
	43	36,885	9,515	1,974	25·8	349·4	23·5
	51	31,409	8,041	4,709	25·6	128·2	18·4
	56	8,231	3,449	3,243	41·9	94·8	25·8
Total, Subdivision 1 (Six Off.)	...	110,361	39,991	18,452	36·2	179·4	30·1
Premium Incomes £40,000 to £100,000 . . .	33	59,388	19,156	10,144	32·3	152·4	25·2
	42	88,376	10,732	6,949	12·1	66·6	7·5
	61	71,409	10,833	5,070	15·2	115·5	11·2
	67	52,014	6,015	3,186	11·6	73·9	8·0
	69	46,245	8,169	3,404	17·7	145·6	13·8
	71	42,501	11,096	4,689	26·1	176·2	21·1
Total, Subdivision 2 (Six Off.)	...	359,933	66,001	33,442	18·3	124·1	13·4
Premium Incomes £100,000 to £200,000 . . .	3	132,955	33,584	12,109	25·3	202·5	21·1
	7	138,866	19,314	9,351	13·9	102·7	10·1
	9	182,226	20,195	7,826	11·1	90·9	8·6
	11	137,226	18,556	8,236	13·5	107·8	10·1
	14	171,013	23,909	14,194	14·0	85·6	9·2
	16	129,072	20,571	12,988	15·9	91·4	10·3
	44	185,604	28,535	14,118	15·4	111·0	11·2
	49	134,879	14,156	6,541	10·5	69·3	7·6
	66	117,725	12,957	4,058	11·1	109·2	9·0
Total, Subdivision 3 (Nine Off.)	...	1,329,566	191,777	89,421	14·4	110·5	10·7
Premium Incomes above £200,000 . . .	12	241,536	23,634	5,400	9·8	109·7	8·5
	22	446,993	115,019	52,810	25·7	161·8	20·2
	29	329,683	48,430	20,851	14·7	121·2	11·2
	62	572,409	82,560	34,033	14·4	123·7	11·1
	63	207,632	35,109	20,281	16·9	103·3	11·5
Total, Subdivision 4 (Five Off.)	...	1,798,253	304,752	133,375	16·9	134·9	12·9
Total, Class VI (26 Offices) .	...	3,598,113	602,521	274,690	16·7	128·6	12·6
VII. TRANSACT NO NEW BUSINESS	*a*	23,186	2,797	*Nil.*	12·0	...	12·0
	b	135,621	8,560	,,	6·3	...	6·3
	c	2,189	271	,,	12·4	...	12·4
	d	33	100	,,	303·0	...	303·0
Total, Class VII (Four Offices)	...	161,029	11,728	*Nil.*	7·3	...	7·3
GRAND TOTAL (75 Offices) .	...	10,740,998	1,397,561	731,250	13·0	91·2	9·1

No information on the subject of new business being given in the accounts printed by the Board of Trade, the amounts of new assurances were obtained by collating the new business given in the *Review Almanac* for 1881, with that given in the *Post Magazine Almanac* and in White's *Insurance Register* for the same year. The Net new business, after deducting reinsurances, being rarely obtainable, it was found necessary, for the sake of uniformity, to use the Gross new assurances in all cases; and for a similar reason it was found advisable to treat the new premiums as at the uniform rate of three per-cent on the amount of new assurances. When the new premiums alone are given, the new assurances were approximated therefrom.

Owing to these circumstances the percentages brought out must be regarded merely as approximations.

HOME AND FOREIGN INTELLIGENCE.

THE COLONIAL MUTUAL LIFE ASSURANCE SOCIETY, LIMITED.

First Quinquennial Investigation, as at 31 *March* 1879.

EXTRACT FROM THE REPORT OF THE DIRECTORS.

The active prosecution of the business of the Society was commenced in June 1874.

The constitution of the Society is based upon the principle of federation of the colonies. The Articles of Association provide that the local affairs of each colony shall be administered by a Local Board of Directors, the whole business being reported to the principal Office, while the members of each Board of Directors form collectively the General Committee of Management. The business of the Society is now conducted at six offices, in the principal colonies of Australia, being in effect six separate life assurance offices blended into one by a common interest and general management, with a branch in the colony of Fiji, which is an adjunct to the Melbourne office, and has been successful. In connection with these offices there are no less than 546 agencies.

During the period under review, there have been issued 9,085 policies. The business offered to the Directors was considerably larger, over 14,000 proposals having been made. A large part of these were absolutely declined on account of the lives being below the high standard of vitality fixed by the Society, others were surcharged for various reasons and objected to paying the extra premium, and therefore did not complete. Others were allowed to withdraw their proposals or themselves did not proceed further; many being in progress towards completion at the date under review. The results, so far, fully justify the vigilance exercised in the selection

of lives by the various Boards, and promise a favourable effect upon the mortality experience of the future.

The progress of the business is shown in the following table:—

Year ending 31 March.	New Policies. Number.	New Policies. Assuring.	Premium Revenue.	Invested Funds.
		£	£	£
1875	457	119,654	3,406	2,864
1876	1,203	313,167	10,692	7,892
1877	2,132	564,912	19,775	16,989
1878	2,388	633,033	22,833	34,823
1879	2,905	770,300 and £204 per ann.	28,250	63,628

The average age at entrance of the lives assured was about $34\frac{1}{2}$ years, and the average amount of each assurance policy was about £264; the largest amount retained at risk of the Society in any one instance being £1000.

The policies discontinued have been as follows:—

	Policies.	Assuring.
By Maturity at Death	69	£20,028
By Surrender	81	23,990
By Default in paying Premiums	1,733	508,532
Total	1,883	£552,550

The expected mortality was about 89 lives, insured for about £28,994.

The policies in force on 31 March 1879, numbered 7,202, and are as follows:—

	Policies.	Assuring.
Life and Endowment Assurance	6,972	£1,812,666
Endowments	226	35,850
Annuities	4	For £204 per ann.
Total	7,202	£1,848,516

Included in the above were 139 policies in abeyance at the end of the fifth year, the validity of which was maintained by the existence of a surrender value at the date of default of payment of premium, and which value on 31 March 1879, was not absorbed by effluxion of time, thus securing in favour of the representatives of policyholders the payment of the amount assured, in event of death occurring before such surrender value had been completely absorbed. But for this provision in the Articles of Association of this Society, any claim on its funds, in respect of a policy not renewed within one month after the premium became due, would have been terminated by such default.

This principle is now adopted by several other offices in the colonies with various differences in details. The benefits of the system have been practically exemplified in this Society by several

death payments for claims arising within the period covered by the surrender value. The practical adoption in Australia of this liberal concession to assurants may be regarded as an epoch in life assurance history.

The following is the special Report on the Valuation, made to the Directors by Mr. T. B. Sprague and Mr. G. King:—

1. We beg leave to report as follows on the results of the first quinquennial investigation into the affairs of your Society made as at 31 March 1879:—

2. We observe that the valuation has been made by your actuary, Mr. Robert Thomson, strictly upon the principles laid down in our report of 11 March 1879;* also, that you specially retained the services of Mr. Robert M'Caskie, the actuary of another office in Melbourne, to check the calculations, and that he reports that the formulas given in our report have been properly applied and acted upon; also, that he has examined the valuations from day to day as the work proceeded, and satisfied himself from working numerous valuations taken indiscriminately, that the valuation is correct.

3. We have, however, had the opportunity given us of satisfying ourselves independently on these points; for we have received sheets containing classified lists of all the policies issued, given in sufficient detail to enable us to verify the calculations; and we have accordingly made such an examination of these as was sufficient to satisfy us of the substantial accuracy of the arithmetical work.

4. We take this opportunity of expressing our opinion that the manner in which the valuation has been conducted, and its results laid before us, is very creditable to your actuarial staff.

5. We will proceed now to consider the various points brought specially under our notice in Mr. Martin's letter of the 23rd of January last. One of the most important of these is thus stated:—

> "The question is, whether we are entitled to add to the "assets, or otherwise bring into the valuation as a credit, the "present value of that which may be considered an annuity, "namely, the extra charge made upon half-yearly or quarterly "premiums in excess of the aliquot part of an annual premium."

* Mr. Thomson says, in his report:—The leading features of the canon laid down by these gentlemen were (1) the adoption of *four* per-cent per annum interest in *all* the valuations; (2) the use of the tables published under the auspices of the Institute of Actuaries, known in the profession as the H^M (or *healthy males*) tables (now regarded as a standard by the profession), throughout:—except (3) in case of endowments, mostly to children on arriving at majority, and present annuities, chiefly on the lives of persons far advanced in years. In respect of these the use of the Carlisle Table was recommended, and it has accordingly been applied; (4) that no risk of the Society *should be taken to account as an asset.* This cannot arise when a valuation is made on sound principles. Although methods have occasionally been adopted by some offices bringing out such results, they are quite indefensible, inasmuch as they introduce *negative values,* the effect of which is to exhibit results less or more illusory as to the present position of an office, and therefore tending to future disappointment, and, to some degree, to injustice between member and member; (5) that a small portion of the future loading may be attached for the equalization of the heavy initial expenses as compared with the lighter ones of future years.

And this, it appears from the documents submitted to us, is a question upon which some difference of opinion exists. After careful consideration, we feel no hesitation in stating that it would not be right to take credit for the present value of this extra charge. There is no question that, when premiums are payable half-yearly or quarterly, the pure premium (as well as the office premium) is increased, the pure half-yearly premium being greater than half the yearly premium, and the pure quarterly premium greater than half of the half-yearly; and under one aspect it is no doubt correct to use these increased pure premiums in the valuation. When this is done, however, the pure premiums must be multiplied by a somewhat smaller annuity than when the yearly premium is valued, and the effect of thus increasing the premium and reducing the annuity is to leave the value of the pure premiums almost unaltered. This view of the subject is confirmed by the fact that the members paying half-yearly and quarterly premiums are entitled at any time to have them changed to the yearly rate of premium corresponding to their age at entry.

6. Interest.—We are pleased to observe that the funds of your Society are now rapidly going into investment, and that, therefore, a higher average rate of interest may confidently be reckoned upon in the future than has hitherto been realized. We are nevertheless of opinion that it would not be wise to use a higher rate of interest than 4 per-cent in the general valuation of the society's liabilities. If, as is probable, a higher rate of interest is hereafter realized for many years to come, the excess beyond the 4 per-cent on which the valuation is based, will fall into the divisible profit, as realized from time to time, and will, in future years, become a most important source of profit, from which your members may expect to derive considerable bonuses.

7. We observe that you request our opinion as to whether a higher rate of interest might not properly be used in valuing the immediate annuities, the foundation policies, and the endowments. The last of these classes has been specially treated, the whole of the premiums received having been reserved to meet the liability under the policies; and, from the nature of the contract, we do not think a smaller reserve than this should be made. The liability of the society would, therefore, not be altered if the endowments were valued (say) at 5 per-cent instead of 4. The immediate annuities and the foundation policies stand upon a different footing. We understand that they were granted at the outset at exceptionally low rates, in order to provide the capital necessary to start the Society. Under these circumstances, it would be more logically consistent to value them at the same rate of interest at which the premiums for them were calculated; and, under the circumstances, we should be quite prepared to follow this course if it were necessary, in order to prevent the estimated liability of the Society under its contracts exceeding its assets. This, however, is very far from being the case. Furthermore, an examination of the valuation schedules has satisfied us that the reduction of the estimated liability consequent upon valuing these classes of policies at high rates of interest, would be comparatively small; and we are of opinion that the Society will

stand better before the public if it uses throughout the whole of the calculations the same rate of interest which we believe is generally adopted by its rivals in the colonies.

8. The valuation balance-sheet shows a surplus of £6,844, and from our examination of the schedules, we are satisfied that this is a genuine surplus, which might, with perfect safety, be divided among the members. Bearing in mind the fact that the principles of valuation acted upon are such as to effectually prevent the introduction of any negative policy-values, so that in no instance has a policy been reckoned as an asset in the valuation, we consider the above result to be one on which the members of the Society may fairly be congratulated. Having regard to the large expense which has been incurred in building up a large and extensive business, and bearing in mind that the special circumstances of the Society have hitherto prevented it from investing the greater part of its funds at remunerative rates of interest, it would not at all have surprized us if the first valuation had shown no surplus at all.

9. The position of the Society in these respects is now greatly altered for the better. If, as appears to be the case, the greater part of the policies obtained at so much expense are permanently kept in force, the pressure of the expenditure upon the resources will rapidly become less, while the income of the Society, from interest upon its invested funds, will correspondingly increase.

10. Under these circumstances, the members may hopefully look forward to the result of the next investigation into the Society's affairs. For the present, however, we recommend that no division of surplus be made. It will, in our opinion, be more advantageous for the Society that the present surplus be retained in hand as an additional provision for unforeseen contingencies. If not required for that purpose, it will, at the next valuation, be available for division among the members; and it should then, we consider, be exclusively appropriated to those of the existing members whose policies shall then still remain in force.

The following resolution was passed at the general meeting of the Society, held on 31 May 1880:—

> "That the surplus of £6,844. 9*s*. 11*d*. shall be dealt with in "the manner suggested in the report of Messrs. Sprague "and King, with the addition that the executors or "administrators of the persons who were members of the "Society on 31st March 1879, and who shall die before "the 31st day of March 1884, shall then be entitled to "participate in such division."

From the returns made under "The Victorian Life Assurance Companies' Act 1873", we extract the following particulars:—

Whole-Life and Endowment-Assurance policies secured by premium payable until maturity have, as far as practicable, been valued in groups. All others have been individually valued.

The principles upon which Valuations and Distributions of Profits among the members are made, are laid down and determined in the Articles of Association of the Society.

The Tables of Mortality known as the H^M, have been used throughout, except in the case of Endowments and Present Annuities, which have been valued by the Carlisle Table of Mortality.

The rate of interest used in the calculations has been 4 per-cent per annum throughout the whole valuation.

The portion of the premium revenue of the Society reserved for future expenses and profits varies according to the special features of each of the several classes into which the business is subdivided; but it is 24·23 per-cent on the *net* or *pure* premiums. The total value of such reserve on the present occasion is £188,881. 8*s*. 7*d*.

No policy which has not entered on its second year of existence is entitled to share in profits.

Consolidated Revenue Account *from the beginning of the Society to* 31 *March* 1879.

	£	s.	d.
Amount of Funds at the beginning of the Society	Nil.		
Premiums (Reassurance Premiums being deducted)	147,710	7	11
Consideration for Annuities granted	1,626	8	0
Interest	4,336	8	0
Other Receipts	1,531	18	9
	£155,205	2	8
Claims under Policies (after deduction of Sums Reassured)	20,028	0	10
Surrenders (Reassured Policies deducted)	513	3	7
Annuities	700	6	3
Commission and Brokerage	26,649	6	6
Expenses of Management	41,693	16	1
Other Payments	1,992	7	11
Amount of Funds on 31 March 1879	63,628	1	6
	£155,205	2	8

Balance Sheet *as on* 31 *March* 1879.

LIABILITIES.

	£	s.	d.
Assurance Fund	63,628	1	6
Claims admitted but not matured	1,500	0	0
Other sums owing by the Society	229	0	0
	£65,357	1	6

ASSETS.

	£	s.	d.
Cash on Deposit	13,650	0	0
Cash on Current Account and in hand	6,028	19	7
Loans on Mortgage	23,735	1	8
Leasehold Property	4,118	2	6
Loans on Policies within Surrender Value	614	16	3
Loans on Personal and other Security	8,733	3	7
Agents' Balances	393	10	1
Outstanding Premiums	4,579	10	5
Interest accrued but not yet payable	842	15	10
Interest outstanding	125	18	7
Office Property	2,535	3	0
	£65,357	1	6

SUMMARY and VALUATION of Policies extant on 31 March 1879.

Description of Transactions.	Particulars of Policies.				Valuation.			
	Number of Policies.	Sums Assured.	Office Premiums per Annum.	Net Yearly Premiums.	Of Sums Assured.	Of Office Yearly Premiums.	Of Net Yearly Premiums.	Net Liabilities.
ASSURANCES.		£	£	£	£	£	£	£
1.—*With Participation in Profits.*								
For Whole Term of Life, secured by periodical Premiums payable throughout Life	4,023	1,212,831·000	37,129·685	29,067·584	468,929·951	574,985·007	445,592·819	23,337·132
Other Classes.								
Endowment Assurances	2,575	569,528·000	25,527·538	21,322·374	284,533·261	315,747·965	261,588·565	22,944·696
Partnership Policies	120	29,283·333	1,375·454	1,073·675	14,460·059	17,513·877	13,645·484	814·575
Single Premium Policies	73	8,973·972	2,374·071	...	3,758·976	...	...	3,758·976
Endowments	226	35,850·000	1,871·555	1,663·652	19,397·273	18,323·057	16,160·255	3,237·018
Absolute Policies	38	14,750·000	798·916	569·548	5,624·492	6,370·311	4,543·131	1,081·361
Special Premiums	...	...	43·134	...	...	...	...	...
Total Assurances with Profits	7,055	1,871,216·305	66,746·282	53,696·833	796,704·012	932,940·217	741,530·254	55,173·758
2.—*Without Participation in Profits.*								
For Whole Term of Life	2	800·000	24·400	22·766	339·887	337·307	314·625	25·262
Others	2	700·000	9·917	5·510	17·212	28·190	14·277	2·935
Total Assurances without Profits	4	1,500·000	34·317	28·276	357·099	365·497	328·902	28·197
Total Assurances—all classes	7,059	1,872,716·305	66,780·599	53,725·109	797,061·111	933,305·714	741,859·156	55,201·955
Deduct Reassurances	...	24,200·000	865·513	731·522	10,388·532	12,373·118	9,807·988	580·544
Net Amount of Assurances	...	1,848,516·305	65,915·086	52,993·587	786,672·579	920,932·596	732,051·168	54,621·411
ANNUITIES.								
Immediate	4	204·266	1,626·400	...	1,713·550	...	...	1,713·550
Policies in Abeyance—all classes	139	...	...	...	448·619	...	...	448·619
Total of the Results	7,202	1,848,516·305	65,915·086	52,993,587	788,834·748	920,932·596	732,051·168	56,783·580

Annual Premiums for Assurance of £100, with participation in profits, payable within one calendar month after proof of death.

Age next Birthday.	Class A.	Class B.	Age next Birthday.	Class A.	Class B.
	£ s. d.	£ s. d.		£ s. d.	£ s. d.
20	1 16 4	1 15 10	40	3 0 9	2 17 6
1	1 16 11	1 16 3	1	3 2 9	2 19 5
2	1 17 7	1 16 8	2	3 4 9	3 1 3
3	1 18 3	1 17 1	3	3 6 10	3 3 0
4	1 18 11	1 17 7	4	3 8 11	3 4 10
25	1 19 7	1 18 1	45	3 11 1	3 7 1
6	2 0 5	1 18 8	6	3 13 5	3 9 6
7	2 1 4	1 19 4	7	3 16 0	3 12 1
8	2 2 4	2 0 2	8	3 18 11	3 14 11
9	2 3 6	2 1 2	9	4 2 3	3 18 1
30	2 4 8	2 2 5	50	4 6 2	4 1 7
1	2 5 11	2 3 7	1	4 10 4	4 5 3
2	2 7 3	2 4 10	2	4 14 9	4 9 1
3	2 8 8	2 6 1	3	4 19 5	4 13 2
4	2 10 2	2 7 5	4	5 4 5	4 17 6
35	2 11 9	2 8 10	55	5 9 9	5 2 0
6	2 13 4	2 10 4	6	5 15 5	5 6 8
7	2 15 6	2 11 11	7	6 1 6	5 11 5
8	2 17 1	2 13 8	8	6 8 1	5 16 4
9	2 18 8	2 15 6	9	6 15 3	6 1 6
			60	7 3 3	6 6 9

Class A—provides for immediate participation in profits at each periodical investigation, in respect of every *policy* of more than one year's duration.

Class B—does not entitle the assured or his representatives to any participation in the periodically vested surplus until after the premiums received, with interest at 4 per-cent per annum, actually amount to the sum originally assured by the policy.

The average rate of interest at which the life assurance fund of the company was invested at the close of each year during the period since the last investigation :—

31 March 1875	£2·41	per-cent per annum.
„ 1876	£3·52	„ „
„ 1877	£4·43	„ „
„ 1878	£5·31	„ „
„ 1879	£6·23	„ „

(It is stated in the report that the average upon the funds actually invested is 6·96 per-cent, and that the low rate in the earlier years has been caused by the decentralization of the Society's funds, and the difficulties of massing them together for investment, after providing a local fund at each colonial centre.)

The Society does not publish any table of Surrender Values. But Article 56 provides as follows :—

"Persons on whose policies a complete year's premium at "least has been paid may surrender the same to the Society, "receiving therefor such consideration as the actuary may "determine, according to principles laid down by the Board."

INSTITUTE OF ACTUARIES.

PRELIMINARY EXAMINATION, 1882.

Examiners.—H. W. MANLY, Esq.; A. F. BURRIDGE, Esq.; W. T. GRAY, Esq.; and W. VAUGHAN, Esq.

I.

1. How should the following accounts be closed?

   1. Merchandize.
   2. Cash.
   3. Profit and Loss.
   4. Petty Expenses.

2. A policy assuring the sum of £10,000 at the death of the life assured, on payment of an annual premium for the whole of life of £300, is bought for £1,000, the next annual premium being just due. If the purchaser reckons to obtain 5 per-cent interest on his investment, and the life assured dies at the end of the second year, what Journal Entries would he have to make in respect of the transaction, (1) at the time of purchase, (2) at the close of the first year, and (3) at the close of the second year?

3. State and prove the rule for converting a recurring decimal into a vulgar fraction; and for example use $\cdot 13\dot{7}$.

4. Reduce 2 weeks, 3 days, 4 hours, 5 minutes, and 6 seconds, to the decimal of a month of 28 days.

5. Solve the equations—

$$(\alpha) \quad \sqrt{x}-\sqrt{a+x}=\sqrt{\frac{a}{x}}.$$

$$(\beta) \quad \frac{x+2}{x-1}-\frac{4-x}{2x}=2\tfrac{1}{3}.$$

6. Two Agents, A and B, introduce, at an average premium of 1*s*. 6*d*. per-cent, Fire insurances, on which the commission amounts to £12. 16*s*. B introduces £10 more premium than A; but, if he had introduced the amount A introduced, he would have received £7. 4*s*., while A would have received £4. 12*s*. for what B introduced. Find the Sum Assured introduced by each, and the rate of commission which they each received.

7. The page of a book has a margin of $\frac{1}{8}$th of its breadth down each side, and of $\frac{1}{9}$th of its length along the top and bottom. The total area of these 4 margins is $22\frac{1}{2}$ square inches, while the outer perimeter is 7 inches more than the inner perimeter. Find the dimensions of the page.

8. Prove that if any number of Arithmetical Progressions be written one under another, and the vertical columns of first terms, second terms, third terms, &c., so formed, be added, the totals will also form an Arithmetical Progression.

9. Define a geometric series; and show how to find the sum of any number of terms of such a series.

10. Sum to n terms $\frac{1}{1.2.3}+\frac{1}{2.3.4}+\frac{1}{3.4.5}+$ &c.

11. In a Geometrical Progression, the product of the sum of the 13th, 14th, and 15th terms, by the sum of the 10th, 11th, and 12th, is 392, and the quotient is 8: find the first term.

12. Write down the expression for the number of combinations that can be formed out of m things by always taking r together; and find how many things must be taken together that the number of combinations may be the greatest possible.

II.

13. A system of signalling at sea is arranged by the use of rockets of seven different colours, each signal being produced by firing 4 different coloured rockets in succession. How many signals can be given?

14. At a Board Meeting of 12 Directors, the Chairman having taken his place, in how many different ways can the other Directors be arranged at the table?

15. Write down the $(r+1)$th term in the expansion of

$$\frac{1}{\sqrt[3]{1+x}}.$$

16. Write down the expansion of $(a+b)^n$, where n is any number, whole or fractional, and determine for what values of a, b, and n, the expansion is $1.2-2.3+3.4-4.5+5.6-$&c.

17. Define the logarithm of a number, and state why Briggs's (or the common) system is more convenient for numerical computations than any other.

18. Write down the series for $\log_e(1+x)$, and calculate by means of it the logarithm of 11 to the ordinary decimal base, 10. (The *modulus* of the common system is ·43429448.)

19. Having given $\log 2 = \cdot 301030$, solve the equations—

$$\left.\begin{aligned}\frac{5^{x+y}}{4^x}&=32\\ 2^x&=128^y\end{aligned}\right\}$$

20. Express $\Delta^n u_x$ in terms of u_x and succeeding values.

21. The series whose general term is represented by the formula $u_x=2^{x+1}$ gives $u_0=\Delta^1 u_0=\Delta^2 u_0=\Delta^3 u_0= \ldots\ldots =2$. If to simplify calculation the third differences are assumed constant, what corrected formula represents the general term? Compare this corrected formula with the original one, and show that, on the assumption of any order of differences being constant in an exponential series, the method of finite differences is inadmissible as a means of obtaining any but the earlier terms.

22. Given $\log 101 = \cdot 0043214$,
„ $103 = \cdot 0128372$,
„ $105 = \cdot 0211893$,
„ $109 = \cdot 0374265$.
Find $\log 107$.

23. Two boys, the first of whom has one penny and the second two, arrange to toss for pennies till one of them has nothing left. Calculate the probability for each boy of his winning the other one's money, and the value of each one's prospects.

24. A speaks truth 2 out of 3 times, and B 3 out of 4. What is the probability of the truth of a fact which (1) they both assert; (2) A asserts and B denies; (3) A reports having heard from B.

25. The proprietor of a gambling booth at a fair throws a pair of dice, after first betting level bets against the number thrown being below 7, level bets against the number thrown being above 7, and 4 : 1 against exactly 7; what percentages of profit does he charge his customers?

INTERMEDIATE EXAMINATION, 1882.

Examiners.—G. W. BERRIDGE, Esq.; T. G. ACKLAND, Esq.; and J. H. DUNCAN, Esq.

I.

1. Explain the symbols $a_{\overline{xyz}}{}^2$, $a_{\overline{xy}.\overline{wz}}$, $a_{\overline{yz}|x}$.

2. Find the values of ${}_nq_{\overline{xy}}$ and ${}_{n-1|}q_{\overline{xy}}$.

3. Given the present value, at rate of interest i, of an annuity of 1 deferred d years. Investigate a formula to find n, the number of years it has to run.

4. Express the rate of mortality at age x in terms of the force of mortality at age $(x+\frac{1}{2})$.

5. A community propose to establish pensions for such of their number as attain a certain age, to be provided by equal annual subscriptions payable up to that age. How would you proceed to find the amount of such subscriptions?

6. Describe the construction of the M column by a continuous method.

7. Give a verbal explanation of the formulas

$$|_nA_x=v(1+|_{n-1}a_x)-|_na_x$$
$$A_{x\overline{n}|}=v-d|_{n-1}a_x\,.$$

8. Find the probability that of two lives, (x) and (y), (x) shall die in the nth year, and the other, (y), be alive at the moment of (x)'s death.

9. Show how to find the single premium for an assurance payable on the death of B, aged y, provided he die after A, aged x, and before C, aged z. If the premium is to be paid annually until the risk determines, by what annuity would you divide the single premium?

10. Explain how you would construct tables of the values of $\bar{a}_x$ and $\bar{A}_x$.

11. State a formula for the net annual premium for an assurance on a life (x), commencing with 1 and decreasing ·05 every year until its extinction at the end of 20 years; the premium being also reduced by $\frac{1}{20}$th of its original amount each year.

12. How would you proceed to obtain the value of a reversionary annuity to a after b, in the case where the two lives are now resident

in India, but a has the intention of living in England after the death of b?

II.

13. Prove that

$$1+e_x=q_x+p_x(1+q_{x+1})+{}_2p_x(1+q_{x+2})+\ \cdots$$

14. An annuity-certain of £729 is granted for 25 years, the rate of interest assumed being 5 per-cent. Calculate the value of such annuity. Given $\log 2=\cdot30103$, $\log 3=\cdot47712$, $\log 7=\cdot84510$, $\log 2952=3\cdot47012$, $\log 2953=3\cdot47026$.

15. Show that the probability of a life (x) dying before a life (y), may be expressed by the formula

$$\frac{(\mathrm{D}_{xy})+(\mathrm{N}_{x-1.y})-(\mathrm{N}_{x.y-1})}{2(\mathrm{D}_{xy})},$$

where (D), (N), denote the values of D, N, when the rate of interest $=0$.

16. Describe any two of the commonly used Tables of Mortality, naming the sources from which they were drawn, their advantages, weakness, and applicability.

17. Given the values of l_x for every fifth age of life, show how you would proceed to deduce an approximate value of P_x at the same ages.

18. Given only the D and N columns, show how to deduce all the others from them.

19. Show clearly into what the component parts of an annuity-certain may be divided, and give a means of determining how much of each payment goes to repay the capital originally invested. If the rate of interest at which that part of each payment which goes to repay capital is invested, is different from the rate of interest received on the original investment, how will the answer to the above question be affected?

20. Explain how a table of the values of Endowment Assurances $\mathrm{A}_{x\overline{n}|}$ may be made use of to find the values of temporary assurances $|_n\mathrm{A}_x$.

21. How would you proceed if required to value a contingent assurance (a sum payable if A die in B's lifetime, secured by an annual premium) by a table other than one like the Carlisle, for which the values of A^1_{xy} have been tabulated?

22. The common rule for finding the value of an annuity payable half-yearly or quarterly is to add $\frac{1}{4}$ or $\frac{3}{8}$ to the value when payable yearly. What is the amount of the error?

23. Investigate a formula (1) for an annuity payable until the death of the last survivor of x and y, the payments being reduced to $\frac{m}{n}$ of their former amount at the first death. (2) For an annuity until the death of the last survivor of x and y, the payments being reduced to $\frac{m}{n}$ of their former amount in the event of x dying before y.

24. In a Society consisting of n members, 1 is payable at the death of each member, by the survivors. Find an expression for the present value, approximately, of the future payments.

25. Draw up a form of endorsement, changing an ordinary policy for the whole of life into a paid-up policy for a smaller amount, and dealing with the question of bonuses.

FINAL EXAMINATION, 1882.

Examiners.—T. G. C. BROWNE, Esq.; T. E. YOUNG, Esq.; and T. H. COOKE, Esq.

I.

1. Describe briefly the methods you would adopt to determine the rate of mortality when the materials for obtaining it are derived—

   (1) From the Registers of an Assurance Company.

   (2) From the Populations enumerated at two Censuses, and the deaths registered at different ages for each intermediate year.

2. Describe the graphical method of graduation, and state its merits and defects as compared with those of Woolhouse, Makeham, and the Method of Differences. Which would you prefer, and on what grounds, for the graduation of the experience of a Life Office?

3. Discuss the merits of the various tables that might be used in the valuation of an Annuity business.

4. State how you would ascertain the addition to be made to the Reserve in respect (1) of claims being payable immediately on *proof of title;* and (2) the unequal distribution of the premium income throughout the year.

5. It has been determined, in the case of a Mutual Assurance Company which has become insolvent, to settle a scheme for the reduction of the contracts, in place of winding up the business. Upon what basis would you, as the Actuary consulted, make the necessary valuation to determine the reductions to be made, and what points should be specially dealt with in your report to the members?

6. On what data does the Act of 1870 require Policies in an insolvent Office to be valued? Have you any objection to urge to the Table of Mortality, and on what ground?

7. The surplus in a life fund is ascertained annually by valuation. Explain how you would check the surplus thus ascertained by an independent analytical method exhibiting the several sources whence it is derived.

8. Describe Mr. Sheppard Homans's method of distributing the surplus of a Life Office.

9. Some offices give a lower rate of Reversionary Bonus to Endowment Assurance Policies than to ordinary Whole-Life Assurances. In deciding the relations between the rates of Bonus, what points should be considered?

10. Describe the chief methods in use for distributing surplus, and express your opinion upon their suitability for adoption in Mixed and Mutual Offices.

11. Describe the course that should be pursued in carrying out the following transactions:—

   (1) A loan upon bills.

   (2) A loan on the Stock Exchange from account to account.

12. State the various methods proposed for the valuation of the Stock Exchange securities of a Life Office. Give the reasons for and against each method: and what appears to you to be the most correct principle of valuation.

II.

13. What was the intention, and what has been the effect, of the change introduced in 1844 in the form of accounts of the Bank of England?

14. Discuss the question of the bi-metallic standard in relation to the essential function of money.

15. Specify the chief differences between the Married Women's Policies of Assurance (Scotland) Act, 1880, and the provisions relating to Life Assurance in the Married Women's Property Act, 1870.

16. What is the legal position of the Mortgagee of a base-fee upon its falling into possession?

17. Explain the position and rights of Joint Tenants and Tenants in Common. Who are always made Joint Tenants, and why?

18. It is required to fix the single premium for an assurance to be paid on A, aged 60 (whose present wife, aged 58, is alive and in good health), having issue which shall attain the age of 21. How would you estimate the maximum limit of the premium?

19. An assurance is payable in the event of the survivor of two lives, A and B, predeceasing C. A goes to India. Explain how you would assess the premium for foreign residence.

20. A, aged 26 last birthday, is entitled to a reversionary life interest in an estate producing £10,000 a year on the death of his father, aged 52, subject to a jointure of £2,400 a year to A's mother, aged 45, for the remainder of her life after her husband's death. The estate is further charged with the payment of portions, amounting to £40,000, to the younger children, bearing interest at 4 per-cent from the death of A's father until payment. What sum should be given for A's reversionary life interest, the purchaser to make 6 per-cent while the interest is in reversion, and 5 per-cent while in possession?

21. A, aged 25, is tenant in tail of an estate after the death of his uncle (a widower aged 60), without issue. Give an expression for the annual reversionary charge per £100 of advance, providing for the policies being made world-wide.

22. A (60) and B (40) have successive life interests in a sum of £10,000 Consols, over which they have a power of appointment which they can only jointly exercise. They now determine to exercise this power and divide the fund. Give in symbols their respective shares, and state the table of mortality and rate of interest you would employ.

23. A is absolutely entitled, on the death of B aged 65, to a trust fund invested as follows:—£1,000 Consols; £1,100 Great Western Railway 5 per-cent Preference Stock; well-secured freehold ground-rents, yielding £100 a year, the leases having 80 years to run; a leasehold house, let at £75 a year, held under a lease having 50 years

to run, at a ground-rent of £10. State how you would estimate the value of the various securities in valuing A's interest for sale.

24. How, in your opinion, should the Income-Tax be assessed on the profits of Life Insurance Companies (Mutual and Proprietary)?

25. If you were called upon to audit the accounts of a Life Insurance Company, state briefly how you would proceed, and to what points you would give special attention.

PROCEEDINGS OF THE INSTITUTE.—Session 1881-2.

First Ordinary Meeting, 28 *November* 1881.

The President in the Chair.

The following gentlemen were elected members of the Institute:—

Fellows.

Elderton, F. F.	Puckle, Henry John.

Associates.

Beddall, Charles.	Nash, Oscar.
Bell, Frederick.	Oakley, Walter.
Fisher, Frederick.	Purves, Thomas Peter.
Hamilton, Andrew.	Rusher, Edward Arthur.
Hose, Frederick Edward Arthur.	Stuart, John Moody.
Molyneux, Arthur Ernest.	Tilt, Robert Ruthven.

The President delivered an Inaugural Address. Mr. D. J. McG. McKenzie then read a paper on "The Transformation of Annuities and Annuity-Values payable yearly, into the like when payable in Fractional Intervals of a Year, by means of Constant Factors; with Specimens of Tables computed for this purpose, and Examples of their application."

The following gentlemen took part in the discussion:—The President, Messrs. Ackland, Berridge, Gray, G. King, and Walford.

Second Ordinary Meeting, 2 *January* 1882.

The President in the Chair.

The following gentlemen were elected Associates of the Institute:—

Milner, John William.	Pease, Arthur Henry.

Mr. W. J. H. Whittall then read a paper on "The Rates of Fatal Accidents in various Occupations."

The following gentlemen took part in the discussion:—The President, Messrs. Adler, Humphreys, Neison, and Walford.

Third Ordinary Meeting, 30 *January* 1882.

The President in the Chair.

The following gentleman was elected an Associate of the Institute:—

Day, Christian Richard Tyler.

Mr. H. W. Manly read a paper on "A certain Method of Distributing the Surplus among the Assured, and the Construction of an Equitable Scale of Office Premiums with reference thereto"; and the following gentlemen

took part in the subsequent discussion:—The President, Messrs. Baden, Crisford, R. P. Hardy, Justican, G. King, Searle, and Sorley.

Fourth Ordinary Meeting, 27 *February* 1882.

The President in the Chair.

The following gentlemen were elected Associates of the Institute:—

Burke, David.	Goldman, Leopold.
Fowler, William Henry.	Perks, Sydney.

Mr. G. King read a paper "On the Valuation of Policies subject to Half-yearly and Quarterly Premiums", being an Extract from a Joint Report by Messrs. T. B. Sprague and G. King.

Mr. G. F. Hardy read a paper "On the Rate of Interest in Annuities-Certain", and another on "An Improved Method of Approximating to the Value of Annuities involving Three Lives."

The following gentlemen took part in the discussion on these papers:—The President, Messrs. Ackland, Bumsted, Chisholm, Gordon, R. P. Hardy, Ryan, and Sutton.

Fifth Ordinary Meeting, 27 *March* 1882.

The President in the Chair.

The following gentleman was elected an Associate of the Institute:—

Colquhoun, Robert Douglas.

Mr. A. F. Burridge read a paper on "The Rates of Mortality in Victoria, and on the Construction of Mortality Tables from Census Returns by the Graphical Method of Graduation"; and the following gentlemen took part in the discussion:—The President, Messrs. R. P. Hardy, G. King, Martin, and Walford, and Dr. Symes Thompson (a visitor).

Sixth Ordinary Meeting, 24 *April* 1882.

The President in the Chair.

Mr. C. D. Higham read a paper by Mr. J. A. Higham, on "The Adjustment of Mortality Tables", and Mr. M. N. Adler "Some Notes on Compound Interest." The following gentlemen took part in the discussion:—The President, Messrs. Berridge, R. P. Hardy, and G. F. Hardy.

The Thirty-third Annual General Meeting, 3 *June* 1882.

The President (Mr. A. H. Bailey) in the Chair.

Mr. G. Humphreys (Hon. Sec.) read the circular convening the meeting, the minutes of the ordinary meeting held in April, which were signed as correct by the President, and the following Report of the Council and Statement of Accounts:—

"In meeting the members of the Institute at the close of another session, the Council have to report that the number of members on 31 March 1882 was 352, as against 355 at the corresponding period of 1881.

"Twenty-one new members have been elected during the year, and 24 removed from the list by death or resignation.

"The accounts, which have been duly audited, show that the income of the year was £955. 18*s*., the expenditure £826. 6*s*. 10*d*., and that the funds of the Institute now amount to £3,298. 0*s*. 7*d*. It may be noticed that the

expenditure includes an item of £134. 13*s*. on account of the printing of the Text-book.

"The ordinary meetings of the Institute have been well attended. At the first meeting of the session, the President (Mr. A. H. Bailey) delivered an inaugural address. The following papers have been read and the subjects discussed during the session:—

"*First*—'On the Transformation of Annuities and Annuity-Values payable yearly, into the like when payable in Fractional Intervals of a Year, by means of Constant Factors, with Specimens of Tables computed for this purpose, and Examples of their application'—by Mr. D. J. McG. McKenzie.

"*Second*—'On the Rates of Fatal Accidents in various Occupations'—by Mr. W. J. H. Whittall.

"*Third*—'On a Certain Method of Distributing a Surplus among the Assured, and the Construction of an Equitable Scale of Office Premiums with reference thereto'—by Mr. H. W. Manly.

"*Fourth*—'On the Valuation of Policies subject to Half-yearly and Quarterly Premiums' (being an extract from a joint report of Messrs. T. B. Sprague and G. King).

"*Fifth*—'On the Rate of Interest in Annuities-Certain'—by Mr. G. F. Hardy.

"*Sixth*—'On an Improved Method of Approximating to the Values of Annuities involving Three Lives'—by Mr. G. F. Hardy.

"*Seventh*—'On the Rates of Mortality in Victoria, and on the Construction of Mortality Tables from Census Returns by the Graphical Method of Graduation'—by Mr. A. F. Burridge.

"*Eighth*—'On the Adjustment of Mortality Tables'—by Mr. J. A. Higham.

"*Ninth*—'Notes on Compound Interest'—by Mr. M. N. Adler.

"The Council have to express their regret that, owing to the increase of official duties, Mr. Wm. Sutton has been compelled to resign the editorship of Part II. of the Text-book; but at the same time they have the satisfaction to announce that Mr. George King, who succeeded Mr. W. Sutton as lecturer to the students preparing for the intermediate examinations, has consented to undertake the completion of the work.

"The Council have arranged for a continuation of the classes during the next session. The subjects treated are those for students preparing for the preliminary and intermediate examinations. The usual notice of the commencement of the classes will be given.

"The reports of the examiners for the examinations of the Institute, held on 29 and 30 April last, give as the result:—

"PRELIMINARY EXAMINATION.

"Fourteen gentlemen presented themselves, one withdrew, and seven passed in the following order of merit:—

1.—F. E. A. Hose, J. M. Stuart, } Eq.	5.—W. Oakley, E. A. Rusher, } Eq.
3.—J. W. Milner.	7.—F. M. T. Byers.
4.—W. P. Pulley.	

"INTERMEDIATE EXAMINATION.

"Nine gentlemen presented themselves, of whom five passed, in the following order of merit:—

1.—H. Ansell.	3.—B. A. Berry.
2.—A. Carter.	4.—S. Day.

5.—F. M. Ashton.

"FINAL EXAMINATION.

"Nine gentlemen presented themselves, and of these five passed, in the following order of merit:—

1.—H. J. Rothery, } Eq.
G. H. Ryan, }
3.—A. W. Sunderland.
4.—J. Graham.
5.—E. A. Colquhoun.

[*Statement of Accounts, p.* 384.]

The adoption of the Report, and of the Accounts for the year, having been proposed by the President and seconded by Mr. G. Cutcliffe, was agreed to unanimously.

The following list was declared to constitute the President, Vice-Presidents, Council, and Officers, for the ensuing year:—

President.

THOMAS BOND SPRAGUE, M.A.

Vice-Presidents.

MARCUS N. ADLER, M.A.
GEORGE WM. BERRIDGE.
CHARLES JOHN BUNYON, M.A.
RALPH PRICE HARDY.

Council.

MARCUS N. ADLER, M.A.
ARTHUR H. BAILEY.
GEORGE WILLIAM BERRIDGE.
CHARLES JOHN BUNYON, M.A.
HENRY COCKBURN.
*GEORGE STEPHEN CRISFORD.
FRANK A. CURTIS.
ARCHIBALD DAY.
DAVID DEUCHAR.
*ALEXANDER JOHN FINLAISON.
JOHN RALPH GRIMES.
MAJOR-GEN. J. C. HANNYNGTON.
RALPH PRICE HARDY.
STEWART HELDER.
AUGUSTUS HENDRIKS.
*WILLIAM HUGHES.
GEORGE HUMPHREYS, M.A.
FRANK MCGEDY.
HENRY WILLIAM MANLY.
JAMES MEIKLE.
*EDWARD ALGERNON NEWTON, M.A.
ARTHUR PEARSON.
HENRY WILLIAM PORTER, B.A.
H. AMBROSE SMITH.
*ARTHUR SMITHER.
THOMAS BOND SPRAGUE, M.A.
SPENCER CAMPBELL THOMSON, B.A.
JOHN WHITCHER.
JOHN HILL WILLIAMS.
THOMAS EMLEY YOUNG, B.A.

NOTE.—Those marked * are new members.

Treasurer.

JOHN RALPH GRIMES.

Honorary Secretaries.

GEORGE HUMPHREYS, M.A.
HENRY WILLIAM MANLY.

The following gentlemen were elected Auditors:—Mr. C. Windett, Mr. H. W. Andras, and Mr. G. B. Bellamy.

The President then moved, and Mr. G. Humphreys seconded—"That this meeting approves the recommendation of the Council in reference to a Charter of Incorporation for the Institute, and requests the Council to take the necessary steps towards obtaining such Charter", which was agreed to unanimously.

INSTITUTE OF ACTUARIES.

Income and Expenditure for the Year ending 31 March 1882.

Dr.

1881-82.	£	s.	d.	£	s.	d.
Amount of Funds, 31 March 1881—						
Messenger Legacy Fund	275	2	6			
Brown Prize Fund	275	13	11			
General Fund	2,617	13	0			
				3,168	9	5
Annual Subscriptions—						
Town Fellows	248	17	0			
Country ,,	70	7	0			
Town Associates	294	0	0			
Country ,,	97	13	0			
				710	17	0
Journal Sales				86	18	0
Mortality Experience Sales				28	7	6
Log. Card Sales				1	3	6
Examination Fees				34	13	0
Dividends				93	19	0
				£4,124	7	5

Cr.

1881-82.	£	s.	d.	£	s.	d.
Rent	76	17	0			
Salaries	179	2	6			
Journal Expenses	196	15	4			
Stationery, Printing, &c.	82	10	0			
Ordinary Meetings	11	9	0			
House Expenses	76	17	5¼			
Text-Book	134	13	0			
Postage	11	12	0½			
Tutor	31	10	0			
Examination Expenses	21	0	0			
Miscellaneous	4	0	6			
				826	6	10
Amount of Funds, 31 March 1882—						
Messenger Legacy Fund	281	6	6			
Brown Prize Fund	283	10	5			
General Fund	2,733	3	8			
				3,298	0	7
				£4,124	7	5

Balance Sheet, 31 March 1882.

Dr.

	£	s.	d.	£	s.	d.
Messenger Legacy Fund (£211. 1*s.* 10*d.* Consols), cost	203	17	8			
Unappropriated Dividends	77	8	10			
				281	6	6
Brown Prize Fund (£200 Metropolitan Railway 4 per-cent Debenture Stock)	200	0	0			
Unappropriated Dividends	83	10	5			
				283	10	5
General Fund				2,733	3	8
				£3,298	0	7

Cr.

	£	s.	d.	£	s.	d.
Consols, £1,600 Stock, cost				1,498	19	3
Metrop. Railway 4 per-cent Debenture Stock, £1,200, cost				1,225	17	3
Cash at London and Westminster Bank	167	4	5			
Less Cheque not presented	2	2	0			
				165	2	5
Cash at London and Westminster Bank on Deposit				400	0	0
Cash in hand				0	14	8
Arrears of Subscriptions:—						
Country Fellows	6	6	0			
,, Associates	1	1	0			
				7	7	0
				£3,298	0	7

Examined and found correct,

E. A. Colquhoun,
Thos. J. Searle,
Chas. Windett.
} *Auditors.*

JOURNAL

OF THE

INSTITUTE OF ACTUARIES

AND

ASSURANCE MAGAZINE.

Policy Life-lines.—The Relation of Tables of Mortality to Values of Policies. By James Meikle, *Actuary, Fellow of the Institute of Actuaries, and of the Faculty of Actuaries, Edinburgh; and Actuary to the Scottish Provident Institution.*

[Being Extracts from a paper read before the Actuarial Society, Edinburgh, and separately printed in 1871.]

The actuarial student may derive very great advantage from viewing any of his problems under various hypothetical changes, even though these be entirely imaginary, and having for their objects merely the exercise of his ingenuity and the sharpening of his analytic powers. This operation will tend to deepen the subject in his mind, and to impress him with a more extended and enlarged view of the nature of the problem. I have in this way, for my own benefit, taken up the formula for the *Value of a Policy;* and by making variations in the symbols, or variations in the data, and tracing their effects practically, I have obtained a much clearer notion of the relation subsisting between tables showing the law of mortality, and the values of policies calculated thereupon. The Law or Table of Mortality, when plotted out in the form of a curve, assumes the name of a *Life-line;* and as it

is proposed to consider the merits of such Life-lines in respect of the magnitude of the values of policies thereunder, I have given my paper the title of *Policy Life-lines.*

It will not be necessary to enter into any explanations regarding the primary formulas for the value of a policy. The one principally followed is

$$_{n}V_{x}=1-\frac{1+a_{x+n}}{1+a_{x}},$$

in which 1 represents the unit or £1 assured; $1+a_x$ the value of an annuity-in-advance on life x; and $\frac{1+a_{x+n}}{1+a_x}$ the liability for the amount assured which is uncovered by the funds on hand, or the liability which is secured by *future* premiums and *future* accumulations.

In the expression for the value of a Life Annuity—

$$a_x=vp_x+v^2{}_2p_x+v^3{}_3p_x+\ldots\ldots$$

the vitality of the life and the discount of the money may be interchanged, without affecting the result.

Let $v=w(1+k)$, so that

$$a_x=w(1+k)p_x+w^2(1+k)^2{}_2p_x+w^3(1+k)^3{}_3p_x+\ldots\ldots$$

Separating the $1+k$ from the w and attaching it to the p, the value will become that of an annuity at a higher rate of interest and a higher rate of vitality, because the probabilities of surviving each year are increased—

$$(1+k)p_x=p'_x$$
$$(1+k)^2{}_2p_x=(1+k)p_x(1+k)p_{x+1}={}_2p'_x$$
$$(1+k)^3{}_3p_x=(1+k)p_x(1+k)p_{x+1}(1+k)p_{x+2}={}_3p'_x$$
&c.

$$\therefore\quad a_x=wp'_x+w^2{}_2p'_x+w^3{}_3p'_x+\ldots\ldots$$

In $v=w(1+k)$,

$$\text{let } \frac{v}{w}=1+k=\frac{1}{1\cdot03}\div\frac{1}{1\cdot035}=\frac{1\cdot035}{1\cdot03}=1\cdot004854;$$

that is to say, increasing the probability of surviving one year at each age by $\frac{4854}{1000000}$, is equivalent to increasing the rate of interest, that at which the discount of money is computed, from 3 to $3\frac{1}{2}$ per-cent. On certain terms, rate of vitality and rate of discount may thus interchange. We may permit a larger number of survivors each year to draw the annuity, provided we receive a larger rate of interest for the capital sum invested.

We further see that, since the values of life annuities are unaffected, when calculated according to a "percentage increased" vitality and "percentage increased" rate of interest, interchanging with ordinary vitality and ordinary interest, the Values of Policies will also remain unaffected.

It is scarcely necessary to prove that the values of policies, computed at any given rate of interest, are always *greater* than when computed at a higher rate.

Assuming this, it may be shown that a "percentage increased" vitality will of itself increase the values of Policies.

Thus Carl.-3-per-cent $a_x = (1+k)$-Carl.-$3\frac{1}{2}$-per-cent a_x.

But ${}_nV_x$ Carl.-3-per-cent $> {}_nV_x$ Carl.-$3\frac{1}{2}$-per-cent.

$\therefore \quad {}_nV_x(1+k)$-Carl.-$3\frac{1}{2}$-per-cent $> {}_nV_x$ Carl.-$3\frac{1}{2}$-per-cent.

It will be better, however, to take up this point in a more practical manner.

Let V_x and V'_x = values of policies by any two tables.

Then $$V'_x >=< V_x \text{ according as}$$

$$1 - \frac{1+a'_{x+1}}{1+a'_x} >=< 1 - \frac{1+a_{x+1}}{1+a_x},$$

or as $$\frac{1+a_{x+1}}{1+a_x} >=< \frac{1+a'_{x+1}}{1+a'_x},$$

or as $$\frac{1+a_{x+1}}{1+a'_{x+1}} >=< \frac{1+a_x}{1+a'_x}.$$

So that, if values of annuities-in-advance be prepared according to any two tables, and their ratios at each age be taken, we at once ascertain whether the values of policies will be greater or less by the one table than by the other. If the ratio of $\frac{1+a_{x+1}}{1+a'_{x+1}}$ be greater than $\frac{1+a_x}{1+a'_x}$, the values of policies referring to the accented annuities in the denominator will be greater; and if the ratio is less, the values will be less.

The above reasoning is quite general, and may be applied to any tables; but for practical exemplification, I have increased by 5 per-cent the rate of mortality deduced from the experience of Scottish Life Offices—

$$q'_x = 1{\cdot}05 q_x,$$
$$\therefore \quad p'_x = 1 - q'_x,$$
$$p'_{x+1} = 1 - q'_{x+1},$$
$$\&c.,$$

and from a radix of 100,000 at age 15, the corresponding new life-line has been ascertained, and also the values of annuities at 3 per-cent interest. The values at the first six ages by both tables are the following:—

x	l'_x	$1+a'_x$	$1+a_x$	l_x
15	100000	23·7667	23·9841	100000
16	99504	23·5665	23·7859	99528
17	98968	23·3694	23·5906	99017
18	98397	23·1742	23·3969	98473
19	97799	22·9792	23·2031	97903
20	97175	22·7831	23·0088	97309

And there has been prepared the ratio of the annuities by the two tables.

$$\frac{1+a_x \text{ (ordinary mortality)}}{1+a'_x \text{ (increased mortality)}}=\mathrm{R}_x.$$

If the ratio is greater at age $x+1$ than at age x, or at age $x+n+1$ than $x+n$, or age $x+n$ than age x, the values of policies of corresponding ages and durations by the increased mortality table will be the greater.

Table *showing the Value of* $\mathrm{R}_x=\dfrac{1+a_x}{1+a'_x}$.

Age x	R_x	Age x	R_x	Age x	R_x	Age x	R_x	Age x	R_x
15	1·0091	32	1·0116	49	1·0157	66	1·0241	83	1·0345
16	1·0093	33	1·0117	50	1·0160	67	1·0248	84	1·0345
17	1·0094	34	1·0119	51	1·0164	68	1·0254	85	1·0346
18	1·0096	35	1·0121	52	1·0168	69	1·0260	86	1·0348
19	1·0097	36	1·0123	53	1·0172	70	1·0267	87	1·0351
20	1·0099	37	1·0125	54	1·0177	71	1·0274	88	1·0355
21	1·0100	38	1·0127	55	1·0181	72	1·0280	89	1·0359
22	1·0102	39	1·0129	56	1·0186	73	1·0287	90	1·0365
23	1·0103	40	1·0131	57	1·0190	74	1·0293	91	1·0368
24	1·0104	41	1·0133	58	1·0196	75	1·0300	92	1·0381
25	1·0105	42	1·0136	59	1·0201	76	1·0307	93	1·0388
26	1·0107	43	1·0138	60	1·0206	77	1·0313	94	1·0397
27	1·0108	44	1·0141	61	1·0212	78	1·0321	95	1·0441
28	1·0110	45	1·0144	62	1·0217	79	1·0328	...	...
29	1·0111	46	1·0147	63	1·0223	80	1·0336	...	...
30	1·0113	47	1·0150	64	1·0229	81	1·0341	...	...
31	1·0114	48	1·0153	65	1·0233	82	1·0344	...	...

These ratios being thus shown to be progressively increasing, the values of policies by the table derived from $1{\cdot}05q_x$ will therefore always be greater than by the table derived from q_x.

A less conclusive method of exhibiting this law, but perhaps one more easily comprehended, is showing the actual values of policies of various durations, by the two tables referred to, thus:—

Comparison of values of policies of 100, *based on ordinary mortality, with the values based on the same rate of mortality increased by* 5 *per-cent.*

n After years	Age at Entry 20. $_nV'_x$ − $_nV_x$	Age at Entry 30. $_nV'_x$ − $_nV_x$	Age at Entry 40. $_nV'_x$ − $_nV_x$
5	4·435 − 4·372 = ·063	5·929 − 5·851 = ·078	8·442 − 8·326 = ·116
10	9·278 − 9·154 = ·124	12·546 − 12·386 = ·160	17·752 − 17·516 = ·236
15	14·656 − 14·468 = ·188	19·928 − 19·680 = ·248	27·776 − 27·420 = ·356
20	20·660 − 20·406 = ·254	28·071 − 27·732 = ·339	38·174 − 37·717 = ·457
25	27·357 − 27·033 = ·324	36·837 − 36·409 = ·428	48·482 − 47·954 = ·528
30	34·744 − 34·347 = ·397	45·931 − 45·431 = ·500	58·205 − 57·645 = ·560
35	42·697 − 42·230 = ·467	54·945 − 54·400 = ·545	66·953 − 66·401 = ·552
40	50·947 − 50·426 = ·521	63·449 − 62·891 = ·558	74·565 − 74·051 = ·514
45	59·125 − 58·574 = ·551	71·099 − 70·563 = ·536	80·209 − 79·789 = ·420
50	66·840 − 66·288 = ·552	77·756 − 77·265 = ·491	86·175 − 85·855 = ·320
55	73·780 − 73·257 = ·523	82·692 − 82·292 = ·400	... ...
60	79·820 − 79·346 = ·474	87·909 − 87·607 = ·302	... ...
65	84·298 − 83·913 = ·385	... ...	... ...
70	89·031 − 88·741 = ·290	... ...	... ...

Increasing the Rate of Mortality, or, in other words, increasing the probability of dying in one year, by a uniform percentage, thus augments the values of policies. This has been shown with reference to one table, and it may be fairly assumed with reference to any other.

Similar processes might have been gone through to show that increasing the rate of *vitality*, or probability of *surviving* one year, by a uniform percentage, also increases values of policies, as has already been briefly pointed out. With the view of practically exemplifying this, I have augmented the probability of surviving one year—Scottish Offices Experience—by $\frac{4}{1000}$, or multiplied it by 1·004, and worked out the values after 5 years' duration at each quinquennial age, as shown in the following table. A comparison of the numbers-living is also given.

Age at Entry x	$100{}_5V'_x$ $100{}_5V_x$	l'_x	l_x	$l'_x - l_x$	Percentage of Increase. $100\frac{l'_x - l_x}{l_x}$
15	4·467 − 4·067 = ·400	100000	100000		
20	4·790 − 4·372 = ·418	99271 −	97309 =	1962	2·0
25	5·436 − 5·000 = ·436	97917 −	94085 =	3832	4·1
30	6·304 − 5·850 = ·454	96138 −	90550 =	5588	6·2
35	7·410 − 6·941 = ·469	93948 −	86739 =	7209	8·3
40	8·803 − 8·326 = ·477	91295 −	82624 =	8671	10·5
45	10·500 − 10·025 = ·475	88068 −	78128 =	9940	12·7
50	12·475 − 12·007 = ·468	84026 −	73071 =	10957	15·0
55	14·630 − 14·188 = ·442	78742 −	67121 =	11621	17·3
60	16·940 − 16·435 = ·505	71566 −	59798 =	11768	19·7
65	19·028 − 18·621 = ·407	61854 −	50604 =	11250	22·2
70	21·052 − 20·674 = ·378	49104 −	39379 =	9725	24·7
75	23·114 − 22·767 = ·347	34141 −	26838 =	7303	27·2
80	22·309 − 22·112 = ·197	19336 −	14903 =	4433	29·7
85	... ...	7803 −	5895 =	1908	32·4
90	... ...	2064 −	1529 =	535	35·0
95	... ...	160 −	116 =	44	37·9

It will be observed that the amount of increase on ${}_5V_x$ is nearly constant—and that the rate per-cent of increase is greatest at the younger ages. The percentage of increase to the l_x is progressively increasing.

Next suppose the number-living at each age increased by a constant quantity, so that $l'_x = l_x + c$.

In this case the constant affects the probability of surviving one year in a slightly increasing ratio, because

$$p'_x = \frac{c + l_{x+1}}{c + l_x} > \frac{l_{x+1}}{l_x} = p_x.$$

$$\therefore \text{ If } \frac{c + l_{x+1}}{c + l_x} = p_x(1+k),$$

$$\frac{c + l_{x+2}}{c + l_{x+1}} = p_{x+1}(1+k+l), \text{ and } \frac{c + l_{x+3}}{c + l_{x+2}} = p_{x+2}(1+k+l+m);$$

l, m, &c., being very small in comparison with k. Upon the whole, this is a very imperfect form of modification, but as it presents an illustration of the general relation between life-lines and values of policies, the calculated results are here given in detail. The value of a policy effected at age 20, and of 5 years' duration, is thus expressed:—

$${}_5V_{20} = 1 - \frac{\dfrac{v^{25}(c+l_{25}) + v^{26}(c+l_{26}) + v^{27}(c+l_{27}) + \ldots}{v^{25}(c+l_{25})}}{\dfrac{v^{20}(c+l_{20}) + v^{21}(c+l_{21}) + v^{22}(c+l_{22}) + \ldots}{v^{20}(c+l_{20})}}.$$

And the values by the two life-lines are compared in the following table:—

x	$c=100=\frac{\text{Radix}}{1000}$	$c=1000=\frac{\text{Radix}}{100}$	
	$100{}_5V_x \quad 100{}_5V'_x$	$100{}_5V''_x$	$100({}_5V_x-{}_5V''_x)$
15	4·067 − 4·062 = ·005	4·024	·043
20	4·372 − 4·367 = ·005	4·324	·048
25	5·000 − 4·993 = ·007	4·910	·060
30	5·850 − 5·843 = ·007	5·775	·075
35	6·941 − 6·931 = ·010	6·844	·097
40	8·326 − 8·313 = ·013	8·196	·130
45	10·025 − 10·006 = ·019	9·846	·179
50	12·007 − 11·981 = ·026	11·752	·255
55	14·188 − 14·148 = ·040	13·805	·383
60	16·435 − 16·371 = ·064	15·825	·610
65	18·621 − 18·509 = ·112	17·365	1·256
70	20·674 − 20·442 = ·232	18·573	2·101
75	22·767 − 22·210 = ·557	17·974	4·793
80	22·112 − 20·415 = 1·697	9·614	12·498
85	30·015 − 23·554 = 6·461	2·572	27·443

The differences of ${}_5V''_x$ seem about 10 times those of ${}_5V'_x$, and this is no doubt caused in some way by the one constant being 10 times the other. The deviations in the values are not great, but they point out that a constant increase to the number-living has the tendency to *lessen* values of policies.

The next variation upon values of policies and their life-lines, to which attention is drawn, is the relations of certain life-lines to each other which will give *equal* values of policies opened at the same ages and of equal durations.

It has frequently been expressed, in ordinary off-hand conversation, that values of policies, by the tables which charged high premiums for assurance, would necessarily be larger than by tables charging smaller premiums. It appears, however, that there is no such necessity; and, accordingly, I wish to point out that it is quite possible for values of policies to be always equal, although the premiums may vary immensely.

In the present illustration, I shall not take my start from the formula

$$ {}_nV_x = 1 - \frac{1+a_{x+n}}{1+a_x}, $$

but from its equivalent form, expressed in terms of the annual premiums ϖ_x—

$$ {}_n\mathrm{V}_x = \frac{\varpi_{x+n} - \varpi_x}{\varpi_{x+n} + d}. $$

Let $$ \varpi'_{x+n} = (1+k)\varpi_{x+n} + dk, $$

$$ \therefore \quad {}_n\mathrm{V}'_x = \frac{\varpi'_{x+n} - \varpi'_x}{\varpi'_{x+n} + d} = \frac{(1+k)\varpi_{x+n} + dk - \{(1+k)\varpi_x + dk\}}{(1+k)\varpi_{x+n} + dk + d} $$

$$ = \frac{\varpi_{x+n} - \varpi_x}{\varpi_{x+n} + d} = {}_n\mathrm{V}_x. $$

Thus values of policies remain the same, whether derived from ordinary net premiums, or from the same premiums increased by a percentage and a constant, as indicated above, a percentage addition to the annuity-due being equivalent to an addition to the premium composed of a percentage and a constant.

The first step, therefore, appears to be to find the values of

$$ \varpi'_x = (1+k)\varpi_x + dk, $$
$$ \varpi'_{x+1} = (1+k)\varpi_{x+1} + dk, $$
&c., &c.,

and from the connection between these at contiguous ages, to ascertain the probability of surviving one year, and therefrom the number living at each age.

Since $$ vp_x(1 + a_{x+1}) = a_x, $$

and $$ a_x = \frac{v - \varpi_x}{\varpi_x + d} \text{ and } \frac{1}{1 + a_{x+1}} = \varpi_{x+1} + d, $$

$$ \therefore \quad p_x = \frac{a_x}{v(1 + a_{x+1})} = \frac{(v - \varpi_x)}{(\varpi_x + d)} \frac{(\varpi_{x+1} + d)}{v} = \frac{\varpi_{x+1} + d}{\varpi_x + d} \cdot \frac{v - \varpi_x}{v}; $$

and accordingly,—

$$ p'_x = \frac{\varpi'_{x+1} + d}{\varpi'_x + d} \{1 - (\varpi'_x + d)\}(1+i) $$

$$ p'_{x+1} = \frac{\varpi'_{x+2} + d}{\varpi'_{x+1} + d} \{1 - (\varpi'_{x+1} + d)\}(1+i) $$
&c., &c.,

and from a radix at age 15, the ordinary tables, showing the several laws of mortality, have been computed as follows:—

$$ l'_{15} = 100000 $$
$$ l'_{16} = p'_{15} \times 100000 $$
$$ l'_{17} = p'_{15} \times p'_{16} \times 100000 $$
$$ l'_{18} = p'_{15} \cdot p'_{16} \cdot p'_{17} \times 100000 $$
&c., &c.

Table showing the Annual (3 *per-cent*) *Premium for Assurance of* 100 *at Death, derived from the formula* $(1+k)\varpi_x + kd$.

Age.	Values of k.						
	$-\frac{10}{100}$	$-\frac{5}{100}$	$-\frac{1}{100}$	0	$+\frac{1}{100}$	$+\frac{5}{100}$	$+\frac{10}{100}$
15	·8393	1·0483	1·2151	1·2568	1·2985	1·4653	1·6737
20	·9989	1·2162	1·3901	1·4336	1·4770	1·6509	1·8682
25	1·1778	1·4050	1·5868	1·6323	1·6777	1·8595	2·0868
30	1·3931	1·6323	1·8236	1·8715	1·9193	2·1107	2·3499
35	1·6606	1·9147	2·1179	2·1687	2·2196	2·4228	2·6769
40	2·0017	2·2747	2·4932	2·5478	2·6024	2·8208	3·0938
45	2·4481	2·7459	2·9841	3·0437	3·1032	3·3415	3·6393
50	3·0453	3·3763	3·6411	3·7073	3·7735	4·0383	4·3693
55	3·8583	4·2345	4·5354	4·6106	4·6859	4·9868	5·3630
60	4·9778	5·4161	5·7668	5·8545	5·9122	6·2928	6·7312
65	6·5296	7·0542	7·4739	7·5788	7·6837	8·1034	8·6279
70	8·6902	9·3348	9·8505	9·9794	10·1083	10·6240	11·2686
75	11·7050	12·5171	13·1668	13·3392	13·4916	14·1413	14·9534
80	16·0258	17·0779	17·9196	18·1301	18·3405	19·1822	20·2343
85	21·4022	22·7531	23·8337	24·1039	22·3740	25·4547	26·8055
90	31·8302	33·7604	35·3045	35·6906	36·0766	37·6207	39·5509

There is further shown a diagram of the numbers living; see p. 404.

When k is positive, that is, when the premiums are increased by a percentage and a constant, combined in the form of

$$\varpi'_x = (1+k)\,\varpi_x + dk = \varpi_x + \frac{k}{1+a_x},$$

the numbers-living, or the life-lines, range themselves on the under side of the normal line, at distances respectively indicated by the value of k.

And *when k is negative,* the numbers range on the upper side of the normal line. In short, the life-lines on the under side result from high premiums and high mortality; on the upper side from small premiums and light mortality; and yet each of these life-lines produces the same value to a whole-life annual-premium policy, calculating at 3 per-cent interest.

It is obvious, from an inspection of the diagram, that the whole plane upon which these equipolicy life-lines are drawn, based upon certain values of k positive and negative, might be filled up with an indefinite number of other equipolicy life-lines, by giving to k certain varying values accordingly; and, further, it will be seen that any other line, drawn in any other manner, may equally well be compared with any of these indefinitely numerous lines as with the normal line, or with any of those

which have been actually computed and drawn. The limits of this plane have not been investigated, except when k was assumed $=-\cdot15$, in which case the numbers living increased for a few years after the youngest age in the table.

It now becomes interesting to know the nature of the values of policies prepared from any two or more lines cutting this plane in different directions. Let first one be drawn passing through the numbers (see table of equipolicy life-lines).

15 = 10000
25 = 8991
35 = 8270
45 = 7694
55 = 6864
65 = 5933
75 = 4263
85 = 1733
95 = 0

And filling up the intermediate numbers by any process which, without much trouble, will give the entire life-line unbroken from 10000 to 0.

For this purpose I have employed Mr. Sang's equation,

$$l_{95-x}=l_t=\mathrm{B}t^2+\mathrm{C}t^3+\,.\,.\,.\,.\,.$$

giving the line as plotted in the diagram from which values of policies have been computed at 3 per-cent interest.

Comparison of values of Policies, based thereupon.

Age.	$100_5\mathrm{V}_x$ (1)	$100_5\mathrm{V}'_x$ (2)	Difference. (2)−(1)	(1)−(2)
15	4·067	2·391	...	1·676
20	4·372	·703	...	3·669
25	5·000	2·206	...	2·794
30	5·850	4·639	...	1·211
35	6·941	6·110	...	·831
40	8·326	6·654	...	1·672
45	10·025	7·332	...	2·693
50	12·007	9·018	...	2·989
55	14·188	11·762	...	2·426
60	16·435	14·914	...	1·521
65	18·621	17·870	...	·751
70	20·674	20·767	·093	...
75	22·767	24·343	1·576	...
80	22·112	31·069	8·957	...
85	30·015	44·218	14·203	...

Excepting the last four, the values by this hypothetical life-line are all smaller, so that, this exception satisfactorily explained, we seem to have fallen upon a life-line which shows *small* values of

policies. Its track cuts the orbits of the equipolicy life-lines. During the youngest ages it shows heavy mortality, gradually, however, at older ages becoming lighter. At age 50 it crosses the path of the normal line, and recovering from its unhealthy character, ultimately, about age 75, shows a much greater number-living than any of the other lines. Since, by terms of the original assumption, all the lives die before age 95, and as a larger number are alive at age 75, the mortality from 75 to 95 must be greater than by any of the other lines, and therefore the sum required to meet the claims—that is, the values of policies—must be greater than by the normal line. If, in this case, life had been assumed to terminate at age 105 or 115, the increased value would not have appeared. Upon the whole, however, it appears that a table, which shows heavier mortality in its earlier years, and lighter thereafter, will produce small values of policies.

Taking another instance of a life-line crossing the equipolicy lines from the opposite direction, and let one be drawn through the numbers (see table of equipolicy lines)—

15	10000
25	9843
35	9097
45	7933
55	6564
65	4314
75	1680
85	195
90	0

interpolating by the same formula $l_{95-x}=l_t=Bt^2+Ct^3+\&c.$, and preparing values of policies for comparison with the values according to the normal or any of the other equipolicy life-lines.

Ages.	$100_5V'_x$	100_5V_x	Difference.
15	5·54 –	4·067 =	1·473
20	7·57 –	4·372 =	3·198
25	7·33 –	5·000 =	2·330
30	6·83 –	5·850 =	·980
35	7·47 –	6·941 =	·529
40	9·46 –	8·326 =	1·134
45	12·07 –	10·025 =	2·045
50	14·48 –	12·007 =	2·473
55	16·33 –	14·188 =	2·142
60	17·81 –	16·435 =	1·175
65	19·32 –	18·621 =	·699
70	21·40 –	20·674 =	·726
75	24·96 –	22·767 =	2·193
80	31·73 –	22·112 =	9·618

At all ages the values by this hypothetical life-line are greater. It shows light mortality during the earlier years, and crossing the normal line at age 50 appears to show heavier mortality during the remainder of life. We may almost infer, therefore, that any other table of mortality which describes a course similar in character, though not necessarily to the same extent, will give greater values of policies than the table with which the comparison is made.

Let a comparison be made with the Carlisle. If its life-line be drawn in the diagram, it will be seen that it keeps to the under side of the normal line for a considerable period of life, its distance at first increasing; about age 50–55, it approaches the normal line without crossing it; it does not keep in the path of any of the equipolicy lines; it shows a greater mortality in the early years, without in the later years so far recovering as to necessitate a still greater mortality towards the close of life, and is therefore indicative of small values of policies.

If a comparison be made with Davies's Equitable mortality table, it will also be seen that it also bears the character of a small value of policy life-line.

A comparison of the values by these tables with those of the normal line is here given.

Comparison of values of Policies for 100, *based upon*—

Age	100_5V_x			Difference.		Difference.	
	Carlisle. (1)	Davies's Equitable. (2)	Normal Life-line of Scottish Offices. (3)	(1)–(3)	(3)–(1)	(2)–(3)	(3)–(2)
15	3·760	4·302	4·067	...	·307	·235	...
20	4·534	4·709	4·372	·162	...	·337	...
25	5·120	5·146	5·000	·120	...	·146	...
30	5·464	5·612	5·850	...	·386	...	·238
35	6·644	6·440	6·941	...	·297	...	·501
40	7·052	7·845	8·326	...	1·274	...	·481
45	9·255	9·375	10·025	...	·770	...	·645
50	12·374	10·178	12·007	·367	...	...	1·829
55	14·299	11·495	14·188	·111	...	...	2·693
60	13·698	13·817	16·435	...	2·737	...	2·618
65	18·091	16·207	18·621	...	·530	...	2·414
70	19·835	19·248	20·674	...	·839	...	1·426
75	17·617	21·038	22·767	...	5·150	1·271	...
80	21·17	25·709	22·112	...	·942	3·597	...
85	17·27	23·461	30·015	...	12·745	...	6·554

It would seem, therefore, that we are able to predicate the reserve powers of different life-lines, or we may draw the line which will show to any extent a greater or smaller reserve than any other table. Two offices on different life-lines may, notwithstanding, reserve equal values of policies. It will further be seen, that it is the table which shows the highest rate of mortality at the younger ages, and inferentially the office charging the highest premiums at these ages (apart from loading), which makes the smallest reserve; though we may easily conceive a table, based upon a life-line which lies always on the under side of the normal line, not keeping in the path of the equipolicy lines, but gradually increasing its distance from the normal line, which would show higher premiums at all ages, and at same time a greater value to the policies,—for instance, such a table as that produced, by increasing the rate of mortality by an uniform percentage; or we may conceive of a table, based upon a line always on the upper side of the normal line, but which is gradually nearing it throughout life,—such a table would show smaller premiums and higher values than the normal line. We may also conceive other lines pursuing different paths, and invariably producing smaller values.

Another illustration of the effect of the foregoing changes upon the life-line may be traced, even in some of the methods which have been proposed for graduation. I have observed, in several of the processes, that there is a tendency to disturb the curvature of the life-line. Such a tendency has its effect, distant perhaps, but one which should be kept in view as one of the possible tendencies of all graduation.

I give only one illustration—a simple one:—

$l_x, l_{x+1}, \ldots$ being given in the unadjusted series,

$$l'_{x+3} = \frac{l_x + 6l_{x+1} + 15l_{x+2} + 20l_{x+3} + 15l_{x+4} + 6l_{x+5} + l_{x+6}}{64}.$$

Applying this process to the experience of the Scottish offices, it is found that the values of policies resulting from the readjusted table are slightly *less*.

x	100_5V_x	$100_5V'_x$	Difference.	x	Numbers Living. l_x	l'_x	Difference. $l'_x - l_x$	$l_x - l'_x$
20	4·372 −	4·378 =	− ·006	20	97,309	97,294	...	15
25	5·000 −	5·000 =	·000	25	94,085	94,077	...	8
30	5·850 −	5·849 =	+ ·001	30	90,550	90,542	...	8
35	6·941 −	6·938 =	·003	35	86,739	86,730	...	9
40	8·326 −	8·315 =	·011	40	82,624	82,613	...	11
45	10·025 −	10·007 =	·018	45	78,128	78,112	...	16
50	12·007 −	11·967 =	·040	50	73,071	73,045	...	26
55	14·188 −	14·136 =	·052	55	67,121	67,080	...	41
60	16·435 −	16·329 =	·106	60	59,798	59,741	...	57
65	18·621 −	18·457 =	·164	65	50,604	50,541	...	63
70	20·674 −	20·419 =	·255	70	39,379	39,337	...	42
75	22·767 −	22·194 =	·573	75	26,838	26,854	16	...
80	22·112 −	21·933 =	·179	80	14,903	14,964	61	...
85	30·015 −	27·882 =	2·133	85	5,895	6,043	148	...
				90	1,529	1,616	87	...
				100	116	154	38	...

The reason of the values being less lies in the fact that the graduation makes the number at each age less up to age 73, and thereafter greater. By tracing in the diagram the effect of such alterations, the character of a small-value table will at once appear. At the earliest ages, 18–30, the difference in the numbers living is a diminishing one, and therefore produces the small negative difference in the ${}_5V_{20}$ above, of ·006. The difference in the numbers, however, although diminishing for a few years, does not change its sign, but increases, and therefore the smaller value is maintained to the oldest age.

It would appear, in conclusion, that if the ratio of the probabilities of living one year, according to any two tables of equipolicy life-lines, were known, it might be applied to any other table to obtain another set of equipolicy lines on that basis.

It was shown that

$$p'_x = \frac{\varpi'_{x+1}+d}{\varpi'_x+d} \cdot \frac{1-(\varpi'_x+d)}{v},$$

which, since $\varpi'_x = \dfrac{1+k}{1+a_x} - d$ and $\varpi'_{x+1} = \dfrac{1+k}{1+a_{x+1}} - d$,

$$= \frac{\dfrac{1+k}{1+a_{x+1}}}{\dfrac{1+k}{1+a_x}} \cdot \frac{1-\dfrac{1+k}{1+a_x}}{v} = \frac{1+a_x}{1+a_{x+1}}\left(\frac{1}{v} - \frac{1+k}{v(1+a_x)}\right).$$

But it may also be shown that

$$p_x = \frac{a_x}{v(1+a_{x+1})} = \frac{1+a_x}{1+a_{x+1}}\left(\frac{1}{v} - \frac{1}{v(1+a_x)}\right);$$

$$\therefore \quad \frac{p'_x}{p_x} = \frac{\frac{1}{v} - \frac{1+k}{v(1+a_x)}}{\frac{1}{v} - \frac{1}{v(1+a_x)}} = \frac{a_x - k}{a_x} = 1 - \frac{k}{a_x},$$

or, generally, tables based upon the formula

$$p'_x = p_x\left(1 \pm \frac{k}{a_x}\right)$$

will give equal values of policies.

Referring to the illustration on page 390, where the variation on the l_x was a uniform constant *addition*, it will at once be perceived, from the results of the foregoing investigations, that such a variation will give a *smaller* value to the policy. The new life-line will lie on the upper side of the normal line, and although at all ages at an equal distance from it—the addition c being constant--still it will, with reference to the contiguous equipolicy lines, at first lie on the under side thereof, and at increasing ages gradually appear on the upper side, and thus realize the character of the small-value life-line already drawn. When the equal constant is deducted from l_x, the new life-line lies on the *under* side, but crosses the neighbouring equipolicy lines, so as to show heavier mortality in the later years of life, and therefore exhibits the character of a larger-policy life-line. When the l_x is increased by a percentage, the increase is greatest at the *youngest* ages, gradually becoming less at older ages—thus showing a large-policy life-line—and when the number-living is diminished by a percentage, the opposite results follow, and a small-policy life-line is produced.

Values of Policies on Underaverage Lives.

In dealing with the values of policies of persons who have been charged extra for deteriorated health or unfavorable family history, it is a common principle at investigations to measure the liability of the office on the assumption that the policy is of n years' standing of a life of the age corresponding to the premiums actually charged —say, d years older than x, the actual age at entry. Such a system,

no doubt, is quite safe, assuming, of course, that the life has been properly rated. It, however, proceeds on the assumption that the extra which would be charged were the life assuring at age $x+n$, and again rated as an unhealthy life, would equal the difference between the ordinary premiums at ages $x+n+d$ and $x+n$. It is difficult to see the correctness of this assumption. Viewing unhealthy lives as one class, although they may be divided into several distinct classes, I am inclined to think this series of extra excessive. The method may also prove inconvenient, should any of the lives survive the tabular duration, and thus hypothetically be viewed as a life d years older than the extreme age of healthy lives.

If, according to the figures in the table, page 393, a life of 30, on which the ordinary premium is ·018715, were rated as a life say of $37\frac{1}{2}$ years, and charged the premium of ·023499, it would, by the foregoing principle, be held at each investigation to be reinsurable at the following extras:—

Age 30	·004784 = (say)	$\varpi_{37\frac{1}{2}} - \varpi_{30}$
40	·008038 =	$\varpi_{47\frac{1}{2}} - \varpi_{40}$
50	·014770 =	$\varpi_{57\frac{1}{2}} - \varpi_{50}$
60	·028289 =	$\varpi_{67\frac{1}{2}} - \varpi_{60}$
70	·055404 =	$\varpi_{77\frac{1}{2}} - \varpi_{70}$

Whereas, if the life be placed on a table of mortality corresponding to the premium charged at the actual age = ·023499, the value of the policy may at each investigation be computed on that table in place of the table for healthy lives; and the series of extra premiums would, in place of the foregoing, become:—

Age 30	·004784 = ϖ'_{30}	(by Table of Increased	Mortality)	$-\varpi_{30}$	(of Ordinary	Mortality)
40	·005460 = ϖ'_{40}	,,	,,	$-\varpi_{40}$	,,	,,
50	·006620 = ϖ'_{50}	,,	,,	$-\varpi_{50}$	,,	,,
60	·008767 = ϖ'_{60}	,,	,,	$-\varpi_{60}$	,,	,,
70	·012892 = ϖ'_{70}	,,	,,	$-\varpi_{70}$	,,	,,

And the value of the policy would issue from the formula

$$p'_x = p_x\left(1+\frac{k}{a_x}\right), \quad \text{in which } k = -\cdot 1.$$

To take another illustration. A life of 30 is rated at 40, the extra premium being (·025478 − ·018715) = ·006763, or 13*s*. 6*d*. per-cent.

$$\therefore \quad \varpi'_{30} = (1+k)\varpi_{30} + dk$$

$$k \text{ being } = \frac{\varpi'_{30} - \varpi_{30}}{\varpi_{30} + d} = (\varpi'_{30} - \varpi_{30})(1 + a_{30}) = \cdot 14136$$

$$= \text{nearly } A'_{30} - A_{30}.$$

Therefore the value of the policy would be derived from the formula

$$p'_{30} = p_{30}\left(1 - \frac{\cdot 14136}{a_{30}}\right)$$

$$p'_{31} = p_{31}\left(1 - \frac{\cdot 14136}{a_{31}}\right)$$

But values of policies prepared from the formula

$$p'_x = p_x\left(1 - \frac{k}{a_x}\right)$$

have been shown to be equal to ordinary values prepared from p_x; and therefore it would appear from the foregoing arguments that it is unnecessary to give to whole-life-premium policies, on which extra premiums for impaired health have been charged, any higher value than to policies on ordinary healthy lives.

There is, however, a wide difference between the theory and the practice of assuring unhealthy lives. Should the life not recover, so as to be thereafter viewed as an average life, the assurance is invariably maintained against the office; and should the life recover health so as to be assured elsewhere without extra, the extra-premium-charging assurance is generally discontinued, and the office is left with the worst of the unhealthy lives. In practice, therefore, the high extras imposed are required to serve the double purpose of protecting the offices against the extra mortality of inferior lives, and against the injurious consequences resulting from the powers exercised by the public in withdrawing the better and leaving the worse lives.

A table is annexed, showing the numbers living at each age, and the deaths during each year, according to the normal life-line and several of the equipolicy life-lines. The numbers at each tenth age of the hypothetical lines, giving large and small values, are printed in blacker type.

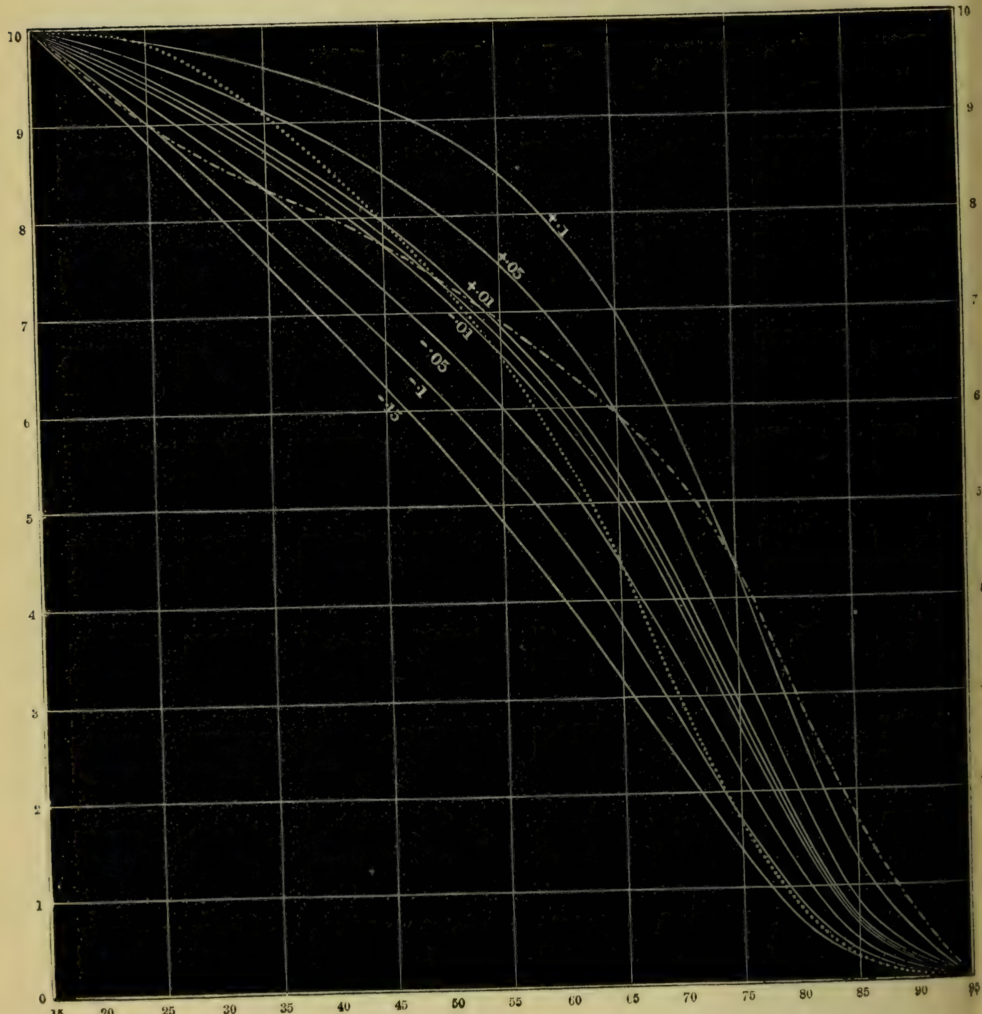
Equipolicy Life-Lines.

Tables of Mortality, which, when computed with 3 per-cent Interest, will give equal Values of Policies.

Age.	$k = +\cdot15$		$k = +\cdot1$		$k = +\cdot05$		$k = +\cdot01$		Ordinary Unaltered Law of Mortality. $k = \cdot0$		$k = -\cdot01$		$k = -\cdot05$		$k = -\cdot1$		$k = -\cdot15$		Age.
	l_x	d_x	l_x	d_x	l_x	d_x	l_x	d_x	l_x	d_x	l_x	d_x	l_x	d_x	l_x	d_x	l_x	d_x	
15	10,000	−18	10,000	4	10,000	26	10,000	43	10,000	47	10,000	51	10,000	69	10,000	91	10,000	112	15
16	10,018	−14	9,996	8	9,974	29	9,957	47	9,953	51	9,949	56	9,931	72	9,909	94	9,888	116	16
17	10,032	−11	9,988	10	9,945	33	9,910	50	9,902	55	9,893	59	9,859	76	9,815	97	9,772	118	17
18	10,043	− 9	9,978	14	9,912	35	9,860	52	9,847	57	9,834	61	9,783	79	9,718	99	9,654	120	18
19	10,052		9,964	16	9,877	38	9,808	56	9,790	59	9,773	64	9,704	80	9,619	102	9,534	122	19
20			9,948	18	9,839	40	9,752	57	9,731	61	9,709	65	9,624	83	9,517	103	9,412	123	20
21			9,930	19	9,799	41	9,695	59	9,670	64	9,644	68	9,541	84	9,414	104	9,289	124	21
22			9,911	21	9,758	44	9,636	60	9,606	64	9,576	68	9,457	85	9,310	105	9,165	125	22
23			9,890	23	9,714	44	9,576	62	9,542	66	9,508	71	9,372	87	9,205	107	9,040	125	23
24			9,867	24	9,670	46	9,514	63	9,476	67	9,437	71	9,285	87	9,098	107	8,915	126	24
25			**9,843**	25	9,624	48	9,451	64	9,409	69	9,366	73	9,198	89	**8,991**	108	8,789	126	25
26			9,818	26	9,576	48	9,387	66	9,340	70	9,293	73	9,109	90	8,883	108	8,663	127	26
27			9,792	28	9,528	50	9,321	66	9,270	70	9,220	75	9,019	90	8,775	110	8,536	127	27
28			9,764	28	9,478	51	9,255	68	9,200	72	9,145	76	8,929	92	8,665	110	8,409	127	28
29			9,736	30	9,427	52	9,187	69	9,128	73	9,069	77	8,837	92	8,555	110	8,282	127	29
30			9,706	31	9,375	53	9,118	70	9,055	74	8,992	78	8,745	93	8,445	111	8,155	128	30
31			9,675	32	9,322	54	9,048	71	8,981	75	8,914	79	8,652	94	8,334	112	8,027	128	31
32			9,643	33	9,268	56	8,977	72	8,906	76	8,835	80	8,558	95	8,222	112	7,899	128	32
33			9,610	35	9,212	57	8,905	73	8,830	78	8,755	81	8,463	96	8,110	113	7,771	128	33
34			9,575	36	9,155	58	8,832	75	8,752	78	8,674	82	8,367	97	7,997	114	7,643	129	34
35			9,539	37	**9,097**	60	8,757	76	8,674	80	8,592	84	**8,270**	98	7,883	114	7,514	129	35
36			9,502	38	9,037	60	8,681	77	8,594	81	8,508	85	8,172	99	7,769	115	7,385	129	36
37			9,464	40	8,977	63	8,604	78	8,513	82	8,423	86	8,073	100	7,654	116	7,256	129	37
38			9,424	41	8,914	63	8,526	80	8,431	84	8,337	87	7,973	101	7,538	116	7,127	130	38
39			9,383	43	8,851	66	8,446	81	8,347	85	8,250	89	7,872	103	7,422	117	6,997	130	39
40			9,340	44	8,785	66	8,365	83	8,262	86	8,161	90	7,769	103	7,305	118	6,867	131	40
41			9,296	46	8,719	69	8,282	85	8,176	88	8,071	91	7,666	105	7,187	120	6,736	131	41
42			9,250	48	8,650	70	8,197	86	8,088	90	7,980	94	7,561	106	7,067	120	6,605	132	42
43			9,202	50	8,580	73	8,111	88	7,998	91	7,886	95	7,455	108	6,947	121	6,473	133	43
44			9,152	53	8,507	75	8,023	90	7,907	94	7,791	97	7,347	110	6,826	123	6,340	133	44
45			9,099	54	8,432	77	**7,933**	92	7,813	96	**7,694**	99	7,237	111	6,703	124	6,207	135	45
46			9,045	58	8,355	79	7,841	95	7,717	98	7,595	102	7,126	113	6,579	126	6,072	135	46
47			8,987	60	8,276	83	7,746	98	7,619	101	7,493	104	7,013	116	6,453	128	5,937	137	47
48			8,927	64	8,193	86	7,648	100	7,518	104	7,389	106	6,897	118	6,325	129	5,800	138	48
49			8,863	67	8,107	89	7,548	104	7,414	107	7,283	111	6,779	121	6,196	132	5,662	139	49
50			8,796	72	8,018	93	7,444	107	7,307	110	7,172	113	6,658	124	6,064	134	5,523	142	50

51		8,724	76	7,925	98	7,337	112	7,197	115	7,059	117	6,534	127	5,930	136	5,381	143	51
52		8,648	81	7,827	102	7,225	115	7,082	118	6,942	122	6,407	131	5,794	140	5,238	145	52
53		8,567	87	7,725	108	7,110	121	6,964	124	6,820	135	6,276	135	5,654	143	5,093	148	53
54		8,480	94	7,617	113	6,989	125	6,840	128	6,695	131	6,141	139	5,511	146	4,945	150	54
55		8,386	100	7,504	120	**6,864**	132	6,712	134	**6,564**	136	6,002	143	5,365	149	4,795	153	55
56		8,286	108	7,384	126	6,732	137	6,578	139	6,428	142	5,859	149	5,216	154	4,642	156	56
57		8,178	116	7,258	134	6,595	144	6,439	146	6,286	148	5,710	154	5,062	158	4,486	159	57
58		8,062	126	7,124	142	6,451	151	6,293	153	6,138	154	5,556	159	4,904	162	4,327	162	58
59		7,936	135	6,982	150	6,300	159	6,140	160	5,984	162	5,397	165	4,742	166	4,165	165	59
60		7,801	145	6,832	160	6,141	166	5,980	168	5,822	168	5,232	171	4,576	171	4,000	168	60
61		7,656	158	6,672	169	5,975	175	5,812	175	5,654	176	5,061	177	4,405	176	3,832	171	61
62		7,498	169	6,503	180	5,800	183	5,637	184	5,478	184	4,884	184	4,229	180	3,661	174	62
63		7,329	182	6,323	189	5,617	191	5,453	192	5,294	192	4,700	190	4,049	185	3,487	178	63
64		7,147	195	6,134	201	5,426	202	5,261	201	5,102	200	4,510	196	3,864	189	3,309	179	64
65		6,952	209	**5,933**	212	5,224	209	5,060	208	4,902	208	**4,314**	202	3,675	193	3,130	182	65
66		6,743	223	5,721	222	5,015	219	4,852	218	4,694	216	4,112	209	3,482	196	2,948	184	66
67		6,520	237	5,499	234	4,796	227	4,634	225	4,478	223	3,903	213	3,286	200	2,764	185	67
68		6,283	251	5,265	244	4,569	235	4,409	232	4,255	229	3,690	218	3,086	202	2,579	185	68
69		6,032	265	5,021	254	4,334	242	4,177	239	4,026	236	3,472	222	2,884	203	2,394	185	69
70		5,767	278	4,767	263	4,092	249	3,938	245	3,790	241	3,250	224	2,681	204	2,209	183	70
71		5,489	291	4,504	271	3,843	254	3,693	249	3,549	245	3,026	226	2,477	203	2,026	181	71
72		5,198	302	4,233	278	3,589	257	3,444	252	3,304	247	2,800	227	2,274	201	1,845	177	72
73		4,896	312	3,955	283	3,332	260	3,192	254	3,057	248	2,573	225	2,073	198	1,668	173	73
74		4,584	321	3,672	289	3,072	260	2,938	254	2,809	247	2,348	224	1,875	195	1,495	167	74
75		**4,263**	322	3,385	285	2,812	259	2,684	252	2,562	245	2,124	216	**1,680**	186	1,328	159	75
76		3,941	322	3,098	286	2,553	255	2,432	247	2,317	242	1,908	212	1,494	179	1,169	151	76
77		3,613	328	2,812	283	2,298	249	2,185	241	2,075	233	1,696	203	1,315	171	1,018	142	77
78		3,285	325	2,529	276	2,049	240	1,944	232	1,842	222	1,493	194	1,144	160	876	132	78
79		2,960	319	2,253	267	1,809	230	1,712	222	1,620	213	1,299	182	984	149	744	120	79
80		2,641	324	1,986	266	1,579	227	1,490	216	1,407	208	1,117	175	835	140	624	112	80
81		2,317	308	1,720	249	1,352	208	1,274	200	1,199	191	942	159	695	126	512	99	81
82		2,009	295	1,471	235	1,144	194	1,074	185	1,008	176	783	145	569	112	413	86	82
83		1,714	264	1,236	207	950	169	889	160	832	152	638	123	457	94	327	72	83
84		1,450	237	1,029	182	781	147	729	139	680	132	515	106	363	80	255	60	84
85	1,733	1,213	212	847	159	634	127	590	120	548	113	409	89	283	65	**195**	48	85
86		1,001	185	688	138	507	108	470	101	435	95	320	74	218	54	147	39	86
87		816	162	550	118	399	91	369	86	340	80	246	61	164	44	108	30	87
88		654	141	432	101	308	77	283	71	260	67	185	50	120	35	78	24	88
89		513	122	331	85	231	63	212	59	193	54	135	40	85	27	54	18	89
90		391	103	246	71	168	51	153	47	139	43	95	31	58	20	36	14	90
91		288	86	175	56	117	40	106	37	96	34	64	24	38	15	22	9	91
92		202	68	119	43	77	30	69	27	62	24	40	17	23	10	13	6	92
93		134	52	76	32	47	20	42	18	38	17	23	11	13	7	7	4	93
94		82	36	44	21	27	14	24	12	21	11	12	6	6	3	3	2	94
95		46	24	23	12	13	7	12	7	10	6	6	4	3	2	1	1	95
96		22	13	11	7	6	4	5	3	4	3	2	1	1	1	..		96
97		9	6	4	3	2	1	2	2	1	1	1	1	..		..		97
98		3	3	1	1	1	1	..		..		..		..		..		98

Life-lines giving Equal Values of Policies at 3 per-cent Interest.—Values of l_x (commencing with $l_{15}=10,000$) calculated from the formula $p'_{x+n}=p_{x+n}\left(1 \mp \frac{k}{a_{x+n}}\right)$.



Normal Life-line, Experience of Scottish Offices, 1863.

+·01 &c. Life-lines derived from $k=·01$, &c.

-·01 &c. " " $k=-·01$, &c.

Life-line giving larger values of policies.

" " smaller "

The abscissa represents the age, and the ordinate represents $l_x \div 1,000$.

On a Method of Approximating to the Rate of Interest in an Annuity-Certain. *By* D. J. McG. McKenzie, A.I.A.

We have received from Mr. McKenzie a paper explaining and illustrating a method of dealing with the above problem, which will probably be new to most of our readers. This problem, Mr. McKenzie remarks, has been frequently discussed in the pages of this *Journal,* but he thinks that the greater simplicity of his operations, and the greater precision in the results he obtains, form a sufficient justification for his communication.

The method consists in finding the value of an annuity at two different rates of interest very close to each other; and when this has been done with proper precautions, the required rate of interest can be found with very great exactness by a simple interpolation by first differences.

Example 1.—The present value of an annuity of £40 for 60 years being £500, required the rate of interest: (Baily's *Doctrine of Interest,* page 131).

$500 \div 40 = 12{\cdot}5$; so that we have, in the present case, $a_{\overline{60}|} = 12{\cdot}5$. By inspection of the annuity-values given in Oakes's *Tables of Compound Interest,* it is found that the required rate of interest lies between $7\frac{7}{8}$ and 8 per-cent. Thus:

at $7\frac{7}{8}$ per-cent $a_{\overline{60}|}$	$=12{\cdot}56398$	. . . .	$12{\cdot}56398$
„ 8 „ „	$=12{\cdot}37655$	at the required rate	$12{\cdot}50000$
Difference	$\cdot18743$	. . . .	$\cdot06398$

Hence the first approximation to the rate per-cent is

$$7{\cdot}875 + \tfrac{6398}{18743} \cdot \tfrac{1}{8} = 7{\cdot}91767, \text{ or } i = \cdot0791767 \text{ approximately.}$$

Since the value of a perpetuity is $1 : i$, the arguments in a table of reciprocals may be regarded as the present values of perpetuities of which the reciprocals are the rates of interest. From Barlow's table of reciprocals we find $12{\cdot}63 = 1 : \cdot07917656$, $12{\cdot}64 = 1 : \cdot07911392$, and we have now to find the value of an annuity for 60 years at these two rates of interest. This can be most conveniently done by means of the formula

$$a_{\overline{n}|} = (1 - v^n) : i = a_\infty (1 - v^n) = a_\infty - a_\infty v^n.$$

Mr. McKenzie states that he found this method of calculating the value of an annuity-certain first in John Newton's introduction to his Logarithms (*Trigonometria Britannica,* London, 1658); that it is also exemplified in John Hill's *Arithmetic* (4th Edition, London, 1723); and that it is mentioned by Fédor

Thoman in his *Theory of Compound Interest and Annuities* (London, 1859), page 19.

In applying the above formula we require to find $\log(1+i)$ to 9 or 10 places of decimals, and this is very readily got by means of Mr. Gray's Tables,* observing that

$$1+a_\infty=1+1:i=(1+i):i=a_\infty(1+i),$$

whence $$\log(1+i)=\log(1+a_\infty)-\log a_\infty.$$

In the present case $a_\infty=12{\cdot}63$ and $12{\cdot}64$, $1+a_\infty=13{\cdot}63$ and $13{\cdot}64$. This explanation will probably be sufficient to enable the reader to understand Mr. McKenzie's process, which he arranges as follows:—

$1+a_\infty=13{\cdot}63$ log $=$	1·134495856	$13{\cdot}64$ log $=$	1·134814370
$a_\infty=12{\cdot}63$ log $=$	1·101403351	$12{\cdot}64$ log $=$	1·101747074
$\log 1{\cdot}07917656=$	$0{\cdot}033092505\times 60$	$\log(1+i)=$	$0{\cdot}033067296\times 60$
$\log(1+i)^{60}=$	1·9855503	$\log(1+i)^{60}=$	1·9840378
$\log(1+i)^{-60}=$	8·0144497	$\log(1+i)^{-60}=$	8·0159622
$\log\{(1+i)^{60}-1\}=$	1·9810371†	$\log\{(1+i)^{60}-1\}=$	1·9795087†
$\log a_\infty=$	1·1014034	$\log a_\infty=$	1·1017471
$\log a_{\overline{60}\vert}=$	1·0968902	$\log a_{\overline{60}\vert}=$	1·0972180
$a_{\overline{60}\vert}=$	12·49943	$a_{\overline{60}\vert}=$	12·50887

12·50887		12·50887		·07917656		
12·49943		12·50000		·07911392		
944	:	887	: :	6264	:	5885
						·07911392

The rate required is exactly ·07917277

Proof.—$\log 1{\cdot}07917277=0{\cdot}0330909786\times 60$

$$\log(1+i)^{60}=1{\cdot}9854587$$
$$\log(1+i)^{-60}=8{\cdot}0145413$$
$$\log\{(1+i)^{60}-1\}=1{\cdot}9809445$$
$$\operatorname{colog} i=\log a_\infty=1{\cdot}1014242$$
$$\log a_{\overline{60}|}=1{\cdot}0969100$$
$$a_{\overline{60}|}=12{\cdot}5$$

Mr. McKenzie proceeds:—

In applying Mr. Gray's tables, as above, to find $\log(1+i)$, unity is always understood to be prefixed to the argument column,

* *Tables for the formation of Logarithms and Anti-logarithms to twelve places*, 1865, and *Tables for the formation of Logarithms and Anti-logarithms to twenty-four or any less number of places*, 1876. Laytons, London.

† $\log\{(1+i)^{60}-1\}$ is here obtained from $\log(1+i)^{60}$ by the use of Wittstein's table of Gauss's logarithms (Hanover, 1866), a most useful companion to all seven-place tables.

the logarithms being found in column I immediately adjoining. We have here, therefore, as arguments, the numbers 1,000 to 2,000, answering as perpetuities for all rates of interest from 10 to 5 per-cent. Multiplying the numbers in the argument column by 2, and adding log 2 to the tabular logs, $\log(1+i)$ is found by the formula for all rates from $2\frac{1}{2}$ to 5 per-cent. For rates from 0 to $2\frac{1}{2}$ per-cent $\log(1+i)$ must be computed by the direct process given by Mr. Gray in the introduction to the tables. Where the ten-place tables of Vlacq or Vega are at hand, $\log(1+i)$ may be obtained by means of the formula in all the cases.

Example 2.—The present value of an annuity of 1 for 50 years being 22·61794, required the rate of interest.

By Oakes's tables, we find

at $3\frac{5}{8}$ per-cent $a_{\overline{50}\vert}$	$=22{\cdot}93608$	. . . .	22·93608
„ $3\frac{3}{4}$ „ „	$=22{\cdot}43449$	at the required rate	22·61794
Difference	·50159	. . . .	·31814

The first approximation to the rate per-cent is

$$3{\cdot}625+\tfrac{31814}{50194}\cdot\tfrac{1}{8}=3{\cdot}70428, \text{ or } i={\cdot}0370428,$$

the approximate rate thus found being always too great.

Entering Barlow's table with the approximate rate, we find $a_\infty 26{\cdot}98=1:{\cdot}03706449$, $a_\infty 27{\cdot}=1:{\cdot}03703704$, and $a_{\overline{50}|}$ is to be computed for these two rates of interest as follows :—

In this instance, the numbers in Mr. Gray's table are multiplied by 2, and log 2 is added to the tabular logs.

Here $a_\infty=26{\cdot}98$ and 27, $1+a_\infty=27{\cdot}98$ and 28.

$$\log 1+a_\infty=\log 13{\cdot}99+\log 2=\log 27{\cdot}98=1{\cdot}446847710 \qquad \log 28=1{\cdot}447158031$$
$$\log a_\infty=\log 13{\cdot}49+\log 2=\log 26{\cdot}98=1{\cdot}431041945 \qquad \log 27=1{\cdot}431363764$$
$$\log 1{\cdot}03706449=0{\cdot}015805765\times 50 \qquad \log(1+i)=0{\cdot}015794267\times 50$$
$$\log(1+i)^{50}=0{\cdot}7902883 \qquad \log(1+i)^{50}=0{\cdot}7897134$$
$$\log a_\infty=1{\cdot}4310419 \qquad \log a_\infty=1{\cdot}4313638$$
$$\log(a_\infty.v^{50})=0{\cdot}6407536 \qquad \log(a_\infty.v^{50})=0{\cdot}6416504$$
$$a_\infty.v^{50}=4{\cdot}372739 \qquad a_\infty.v^{50}=4{\cdot}381778$$
$$a_\infty=26{\cdot}98 \qquad a_\infty=27$$
$$a_{\overline{50}|}=22{\cdot}607261 \qquad a_{\overline{50}|}=22{\cdot}618222$$

22·618222		22·618222		·03706449		
22·607261		22·617940		·03703704		
10961	:	282	: :	2745	:	71
						·03703704
			The required rate is exactly			·03703775

$$\begin{aligned}
\textit{Proof.}\text{—}\log 1{\cdot}03703775 &= \underline{0{\cdot}0157945658 \times 50} \\
\log (1+i)^{50} &= 0{\cdot}7897283 \\
\log (1+i)^{-50} &= 9{\cdot}2102717 \\
\log \{(1+i)^{50}-1\} &= 0{\cdot}7128259 \\
\text{Colog}\, i = \log a_\infty &= \underline{1{\cdot}4313554} \\
\log a_{\overline{50|}} &= \underline{1{\cdot}3544530} \\
a_{\overline{50|}} &= 22{\cdot}61794
\end{aligned}$$

Example 3.—The present value of an annuity of 1 for 80 intervals is 42·13476, required the rate of interest.

We find, from Oakes's table,

at $1\frac{3}{4}$ per-cent $a_{\overline{80|}}$	$=42{\cdot}87994$	. . . .	$42{\cdot}87994$
,, $1\frac{7}{8}$,, ,,	$=41{\cdot}26662$	at the required rate	$42{\cdot}13476$
Difference	$1{\cdot}61332$	. . . .	$\cdot 74518$

The first approximation to the rate per-cent is

$1{\cdot}75+\frac{74518}{161332}\cdot\frac{1}{8}=1{\cdot}80773$, or $i=\cdot 0180773$ approximately.

Bearing in mind that the approximate rate is too great, we enter Barlow's table with it, and find $a_\infty 55{\cdot}33=1:\cdot 01807338$, $55{\cdot}35=1:\cdot 01806685$.

In this example we take $\log(1+i)$ to eight places, from the tables of Babbage or Hutton. If the number of intervals by which it is to be multiplied is not great, the error from working with $\log(1+i)$ to eight places is inconsiderable. In the present instance the rate happens to come out correct, but it will not always be found so exact, if $\log(1+i)$ is taken to eight places only.

$$\begin{aligned}
\log 1{\cdot}01807338 &= \underline{0{\cdot}00777908 \times 80} \\
\log (1+i)^{80} &= 0{\cdot}6223264 \\
\log 55{\cdot}33 = \log a_\infty &= \underline{1{\cdot}7429607} \\
\log (a_\infty . v^{80}) &= \underline{1{\cdot}1206343} \\
a_\infty . v^{80} &= 13{\cdot}20183 \\
a_\infty &= \underline{55{\cdot}33} \\
a_{\overline{80|}} &= 42{\cdot}12817
\end{aligned}
\qquad
\begin{aligned}
\log 1{\cdot}01806685 &= \underline{0{\cdot}00777629 \times 80} \\
\log (1+i)^{80} &= 0{\cdot}6221032 \\
\log 55{\cdot}35 = \log a_\infty &= \underline{1{\cdot}7431176} \\
\log (a_\infty . v^{80}) &= \underline{1{\cdot}1210144} \\
a_\infty . v^{80} &= 13{\cdot}21339 \\
a_\infty &= \underline{55{\cdot}35} \\
a_{\overline{80|}} &= 42{\cdot}13661
\end{aligned}$$

42·13661		42·13661		·01807338		
42·12817		42·13476		·01806685		
844	:	185	::	653	:	143
						·01806685
				The required rate is		·01806828

$$\begin{aligned}
\textit{Proof.}\text{—}\log 1{\cdot}01806828 &= \underline{0{\cdot}0077769063 \times 80} \\
\log (1+i)^{80} &= 0{\cdot}6221525 \\
\log (1+i)^{-80} &= 9{\cdot}3778475 \\
\log \{(1+i)^{80}-1\} &= 0{\cdot}5037099 \\
\text{Colog}\, i = \log a_\infty &= \underline{1{\cdot}7430832} \\
\log a_{\overline{80|}} &= \underline{1{\cdot}6246406} \\
a_{\overline{80|}} &= 42{\cdot}13477
\end{aligned}$$

Example 4.—A loan is issued at the price of 78 per-cent, redeemable at par in 30 years, by an accumulative sinking fund. The annuity is 6·505144, and $a_{\overline{30}|}$ is 11·99051, the true value. (*J.I.A.* xix, 89-94.) Required the rate of interest.

We refer to Oakes's tables as before, and find

at $7\frac{1}{4}$ per-cent $a_{\overline{30}|}$	$=12·10366$	. . . .	12·10366
„ $7\frac{3}{8}$ „ „	$=11·95555$	at the required rate	11·99051
Difference	·14811	. . . .	·11315

Hence the approximate rate per-cent is

$$7·25+\tfrac{11315}{14811}\cdot\tfrac{1}{8}=7·34549, \text{ or } i=·0734549 \text{ nearly.}$$

Entering Barlow's table of reciprocals with this value, we find $a_\infty 13·61=1:·07347539$, $a_\infty 13·62=1:·07342144$. We operate as before—

$$\begin{array}{rlrl}
1+a_\infty=14·62 \quad \log= & 1·164947373 & 14·61 \quad \log= & 1·164650216 \\
a_\infty=13·62 \quad \log= & \underline{1·134177108} & 13·61 \quad \log= & \underline{1·133858125} \\
\log(1+i)= & \underline{0·030770265}\times 30 & \log(1+i)= & \underline{0·030792091}\times 30 \\
\log(1+i)^{30}= & 0·9231080 & \log(1+i)^{30}= & 0·9237627 \\
\log a_\infty= & \underline{1·1341771} & \log a_\infty= & \underline{1·1338581} \\
\log(a_\infty.v^{30})= & \underline{0·2110691} & \log(a_\infty.v^{30})= & \underline{0·2100954} \\
a_\infty.v^{30}= & 1·625808 & a_\infty.v^{30}= & 1·622167 \\
a_\infty= & \underline{13·62} & a_\infty= & \underline{13·61} \\
a_{\overline{30}|}= & 11·994192 & a_{\overline{30}|}= & 11·987833
\end{array}$$

$$\begin{array}{rcrcrcr}
11·994192 & & 11·994192 & & ·07347539 & & \\
\underline{11·987833} & & \underline{11·990510} & & \underline{·07342144} & & \\
6359 & : & 3682 & :: & 5395 & : & 3123 \\
 & & & & & & \underline{·07342144} \\
 & & & & \text{The exact rate is} & & ·07345267
\end{array}$$

$$\begin{array}{rl}
\textit{Proof.}\text{—}\log 1·07345267= & \underline{0·0307829005}\times 30 \\
\log(1+i)^{30}= & 0·9234870 \\
\log(1+i)^{-30}= & 9·0765130 \\
\log\{(1+i)^{30}-1\}= & 0·8683322 \\
\text{Colog } i=\log a_\infty= & \underline{1·1339924} \\
\log a_{\overline{30}|}= & \underline{1·0788376} \\
a_{\overline{30}|}= & 11·99051 \text{ , the true value.}
\end{array}$$

Example 5.—£11,835 is the value of £789 per annum for 57 years, together with an additional payment of £394·5 at the end of the term, required the rate of interest.

Here $a_{\overline{57}|}+\frac{1}{2}v^{57}=15$.

Referring to Oakes's table, and adding the values, we have

at $6\frac{3}{8}$ per-cent $a_{\overline{57}|}+\frac{1}{2}v^{57}$	$=15·23795$	. . . .	15·23795
„ $6\frac{1}{2}$ „ „	$=14·97365$	at the required rate	15·00000
Difference	·26430	. . . .	·23795

Hence the approximate rate per-cent is

$$6{\cdot}375+\tfrac{23795}{26430}\cdot\tfrac{1}{8}=6{\cdot}48753, \text{ or } i={\cdot}0648753 \text{ approximately.}$$

Entering Barlow's table of reciprocals with this value, we find $a_\infty 15{\cdot}41=1:{\cdot}06489293$, $a_\infty 15{\cdot}42=1:{\cdot}06485084$. Working as before, we have

$$\begin{array}{rlrl}
\text{at } i={\cdot}06485084, & a_{\overline{57}|}=14{\cdot}99083, & \text{at } i={\cdot}06489293, & a_{\overline{57}|}=14{\cdot}98208 \\
& \tfrac{1}{2}v^{57}=\quad{\cdot}01392, & & \tfrac{1}{2}v^{57}=\quad{\cdot}01388 \\
& a_{\overline{57}|}+\tfrac{1}{2}v^{57}=15{\cdot}00475, & & a_{\overline{57}|}+\tfrac{1}{2}v^{57}=14{\cdot}99596
\end{array}$$

$$\begin{array}{rcrcrcr}
15{\cdot}00475 & & 15{\cdot}00475 & & {\cdot}06489293 & & \\
14{\cdot}99596 & & 15{\cdot}00000 & & {\cdot}06485084 & & \\
\hline
879 & : & 475 & :: & 4209 & : & 2275 \\
 & & & & & & {\cdot}06485084 \\
\cline{7-7}
 & & & & \text{The required rate is} & & {\cdot}06487359
\end{array}$$

Proof.—$11835\times{\cdot}06487359=767{\cdot}77893765$, Int. on Cap. for 1 year.

$$\begin{array}{rl}
789 \qquad\quad & \text{, Annuity.} \\
\hline
21{\cdot}22106235, & \text{Sinking Fund.}
\end{array}$$

$$\begin{array}{rl}
\log 1{\cdot}06487359= & 0{\cdot}0272980562\times 57 \\
\log(1+i)^{57}= & 1{\cdot}5559892 \\
\log\{(1+i)^{57}-1\}= & 1{\cdot}5437458 \\
\text{Colog } i=\log a_\infty= & 1{\cdot}1879320 \\
\text{log Sinking Fund}= & 1{\cdot}3267671 \\
\log(s_{\overline{57}|}+\text{log S. Fund})= & 4{\cdot}0584449 \\
s_{\overline{57}|}\,.\,\text{Sinking Fund}= & 11440{\cdot}50 \\
\text{Additional Payment}= & 394{\cdot}50 \\
\hline
\text{Capital repaid}= & 11835{\cdot} \\
\text{Original Capital}= & 11835{\cdot}
\end{array}$$

If the reciprocals and perpetuities are extended, a still greater degree of accuracy may be obtained. The following example is believed to exhibit a greater degree of precision than is likely to be required in practice. In it ten-place reciprocals are used.

Example 6.—The present value of an annual payment of £176 for 31 years is £2,680. What is the rate of interest?

$2680\div176=15{\cdot}227$, so that we have here $a_{\overline{31}|}=15{\cdot}227$.

Again referring to Oakes's table, we find

$$\begin{array}{llrlr}
\text{at } 5\tfrac{1}{8} \text{ per-cent} & a_{\overline{31}|}= & 15{\cdot}36819 & \ldots\ldots\ldots & 15{\cdot}36819 \\
\text{,, } 5\tfrac{1}{4} \quad \text{,,} & \text{,,} \quad = & 15{\cdot}14860 & \text{at the required rate} & 15{\cdot}22727 \\
\hline
\quad \text{Difference} & & {\cdot}21959 & \ldots\ldots\ldots & {\cdot}14092
\end{array}$$

So that the first approximation to the rate per-cent is

$$5{\cdot}125+\tfrac{14092}{21959}\cdot\tfrac{1}{8}=5{\cdot}2052, \text{ or } i={\cdot}052052 \text{ approximately.}$$

By inspection of Oakes's table of reciprocals, and by extending the values selected, we find $a_\infty = 19{\cdot}21 = 1 : {\cdot}0520562207 2$, $a_\infty = 19{\cdot}213 = 1 : {\cdot}05204809243$. The required rate lies between these two values of i, and we have to compute $a_{\overline{31}|}$ for each of them, as follows:—

$1+a_\infty = 20{\cdot}21$ $\log =$	1·30556,63135,15	$20{\cdot}213$ $\log =$	1·30563,07759,97
$a_\infty = 19{\cdot}21$ $\log =$	1·28352,73648,62	$19{\cdot}213$ $\log =$	1·28359,51827,98
$\log(1+i) =$	0·02203,89486,53 × 31	$\log(1+i) =$	0·02203,55931,99 × 31
$\log(1+i)^{31} =$	0·68320,74082	$\log(1+i)^{31} =$	0·68310,33892
$\log a_\infty =$	1·28352,73649	$\log a_\infty =$	1·28359,51828
$\log(a_\infty . v^{31}) =$	0·60031,99567	$\log(a_\infty . v^{31}) =$	0·60049,17936
$a_\infty . v^{31} =$	3·98400,57505	$a_\infty . v^{31} =$	3·98558,24115
$a_\infty =$	19·21	$a_\infty =$	19·213
$a_{\overline{31}|} =$	15·22599,42495	$a_{\overline{31}|} =$	15·22741,75885

15·22741,75885	15·22741,75885	·052056,22072	
15·22599,42495	15·22727,27273	·052048,09243	
142,33390	: 14,48612	:: 8,12829	: 82726
			·052048,09243
		The required rate is exactly	·052048,91969

Proof.—$\log 1{\cdot}052048{,}91969 =$	0·02203,59347,40 × 31
$\log(1+i)^{31} =$	0·68311,39769
$\log(1+i)^{-31} =$	9·31688,60231
$(1+i)^{-31} =$	0·20743,69045
$1-(1+i)^{-31} =$	0·79256,30955
$\log\{1-(1+i)^{-31}\} =$	9·89903,38462
$\log i =$	8·71641,17199
$\log a_{\overline{31}|} =$	1·18262,21263
$a_{\overline{31}|} =$	15·22727,273

The above example is from Thoman's *Theory of Compound Interest*, pp. 47, 48. Thoman's solution differs from the above by 3 in the tenth decimal place.

The same method applies to annuities payable in fractional intervals of a year in order to determine the rate of interest.

For such minute intervals as months, fortnights, or weeks, there are no tables of annuity-values published at rates of interest close enough to serve for a preliminary interpolation. The tables of Colonel Oakes, will, however, answer every requirement in connection with the table of momently factors, and rates of interest, here given (*vide* Appendix, p. 417), which extends from $2\frac{1}{2}$ to 10 per-cent, proceeding by subdivisions of one-eighth. By multiplying annuity-values payable yearly, by the momently factors, they are transformed into annuity-values payable momently; and if these

again are multiplied by any number m, the annuity-values so produced, will always be very approximate to the annuity-values which would be produced by rates of interest equal to $\frac{1}{m}$th of the momently rates, whatever values may be assigned to m. We are thus furnished with a means of conveniently entering the table of reciprocals for any fractional rate of interest whatever. For example:—

$$\begin{array}{rr}
\text{At 5 per-cent } a_{\overline{10}|} = & 7{\cdot}72173 \\
\text{Factor inverted} = & \underline{7974201} \\
 & 7{\cdot}72173 \\
 & {\cdot}15443 \\
 & 3089 \\
 & 540 \\
 & 69 \\
 & \underline{5} \\
\bar{a}_{\overline{10}|} = & 7{\cdot}91319 \\
\times & \underline{4} \\
4.\bar{a}_{\overline{10}|} = & 31{\cdot}65276
\end{array}$$

$\text{Log}_e(1+i) \times \frac{1}{4} = {\cdot}048790{,}164 \times \frac{1}{4} = {\cdot}01219754$. At this rate of interest $a_{\overline{40}|} = 31{\cdot}50407$, showing the rate of interest involved in $4.\bar{a}_{\overline{10}|}$ to be very approximate to, but slightly less than, $\log_e(1+i) \times \frac{1}{4}$, the true rate involved in $31{\cdot}65276$ being ${\cdot}01194536$.

$$\begin{array}{rr}
\text{At 10 per-cent } a_{\overline{10}|} = & 6{\cdot}14457 \\
\text{Factor inverted} = & \underline{6029401} \\
 & 6{\cdot}14457 \\
 & {\cdot}24578 \\
 & 5530 \\
 & 123 \\
 & \underline{4} \\
\bar{a}_{\overline{10}|} = & 6{\cdot}44692 \\
\times & \underline{4} \\
4.\bar{a}_{\overline{10}|} = & 25{\cdot}78768
\end{array}$$

$\text{Log}_e(1+i) \times \frac{1}{4} = {\cdot}095310{,}180 \times \frac{1}{4} = {\cdot}02382755$. At this rate of interest $a_{\overline{40}|} = 25{\cdot}60578$, showing the rate of interest involved in $4.\bar{a}_{\overline{10}|}$ to be very approximate to, but slightly less than, $\log_e(1+i) \times \frac{1}{4}$, the true rate involved in $25{\cdot}78768$ being found to be ${\cdot}02341200$.

Example 7.—The present value of £1. 1*s*. 0*d*. payable monthly for 12 years is £100, required the monthly rate of interest?

Here $12 \times 12 = 144$, and $a_{\overline{144}|} = 100 : 1{\cdot}05 = 95{\cdot}23810$. We require to find the value of i which will give $a_{\overline{144}|} = 95{\cdot}23810$.

A few trials with Oakes's table and the momently factors as shown in the preceding examples, will discover the limits within which the required rate is contained; for example:—

At $7\frac{5}{8}$ per-cent $a_{\overline{12}|} = 7{\cdot}68472$, Factor $= 1{\cdot}037658$, $m = 12$, $\log_e(1+i)^{\frac{1}{12}} = {\cdot}00612356$, and $7{\cdot}68472 \times 1{\cdot}037658 \times 12 = 95{\cdot}68942$, i approximately $= {\cdot}00612356$;

at $7\frac{3}{4}$ per-cent $a_{\overline{12}|} = 7{\cdot}63468$, Factor $= 1{\cdot}038268$, $m = 12$, $\log_e(1+i)^{\frac{1}{12}} = {\cdot}006220295$, and $7{\cdot}63468 \times 1{\cdot}038268 \times 12 = 95{\cdot}12207$, i approximately $= {\cdot}006220295$.

Remembering that the true rates of interest are always rather less than the approximate rates, we enter the table of reciprocals

with the above approximate rates, and select $a_\infty = 161·9 = 1 : ·006176652$, $a_\infty = 162·1 = 1 : ·006169031$. Operating as in the previous examples, we have

$1+a_\infty = 162·9$ log $=$	2·21192,10843	163·1 log $=$	2·21245,39610
$a_\infty = 161·9$ log $=$	2·20924,68488	162·1 log $=$	2·20978,30148
$\log(1+i) =$	0·00267·42355 × 144	$\log(1+i) =$	0·00267,09462 × 144
$\log(1+i)^{144} =$	0·3850899	$\log(1+i)^{144} =$	0·3846163
$\log(1+i)^{-144} =$	9·6149101	$\log(1+i)^{-144} =$	9·6153837
$\log\{(1+i)^{144}-1\} =$	0·1544582	$\log\{(1+i)^{144}-1\} =$	0·1536525
$\log a_\infty =$	2·2092468	$\log a_\infty =$	2·2097830
$\log a_{\overline{144}\rvert} =$	1·9786151	$\log a_{\overline{144}\rvert} =$	1·9788192
$a_{\overline{144}\rvert} =$	95·19521	$a_{\overline{144}\rvert} =$	95·23995

95·23995		95·23995		·006176652		
95·19521		95·23810		·006169031		
4474	:	185	::	7621	:	315
						·006169031
				The rate very nearly is		·006169346

Proof.—$\log 1·006169346 =$ 0·00267,10820 × 144

$\log(1+i)^{144} =$ 0·3846358

$\log(1+i)^{-144} =$ 9·6153642

$\log\{(1+i)^{144}-1\} =$ 0·1536857

Colog $i = \log a_\infty =$ 2·2097609

$\log a_{\overline{144}\rvert} =$ 1·9788108

$a_{\overline{144}\rvert} =$ 95·23812

True value $=$ 95·23810

Error $=$ ·00002

Example 8.—The present value of £1 per fortnight, payable for 10 years (of 26 fortnights each), is £194, required the fortnightly rate of interest?

The number of intervals here is $10 \times 26 = 260$, and $a_{\overline{260}\rvert} = 194·00000$, $\log a_{\overline{260}\rvert} = 2·2878017$.

at $6\frac{1}{4}$ per-cent $\log a_{\overline{10}\rvert} =$	0·8617548	at $6\frac{3}{8}$ per-cent $\log a_{\overline{10}\rvert} =$	0·8592026
log Factor $=$	0·0132310	log Factor $=$	0·0134889
$\log 26 =$	1·4149733	$\log 26 =$	1·4149733
$\log(26 . \bar{a}_{\overline{10}\rvert}) =$	2·2899591	$\log(26 . \bar{a}_{\overline{10}\rvert}) =$	2·2876648
$\log_e(1+i) \times \frac{1}{26} =$	·0023317	$\log_e(1+i) \times \frac{1}{26} =$	·0023769
	&c.		&c.

Log $a_{\overline{260}\rvert}$ being $= 2·2878017$, it is evident that the rate of interest sought for must be contained within the limits of the above two approximate rates. We enter the table of reciprocals with them, selecting $a_\infty = 422·6 = 1 : ·002366304$, $a_\infty = 422·4 = 1 : ·002467424$. In the present example it is more convenient to compute $\log(1+i)$

by the direct process given in the introduction to Mr. Gray's tables, and, as the multiplier is large, we require to use 10 places in order that the seventh may be true after the multiplication is effected.

$$\begin{array}{rl} \log 1{\cdot}002366304 = & 0{\cdot}0010264588 \times 260 \\ \log (1+i)^{260} = & 0{\cdot}2668793 \\ \log a_\infty = \log 422{\cdot}6 = & 2{\cdot}6259295 \\ \log (a_\infty . v^{260}) = & 2{\cdot}3590502 \\ a_\infty . v^{260} = & 228{\cdot}5863 \\ a_\infty = & 422{\cdot}6 \\ a_{\overline{260}|} = & 194{\cdot}0137 \end{array} \qquad \begin{array}{rl} \log 1{\cdot}002367424 = & 0{\cdot}0010269440 \times 260 \\ \log (1+i)^{260} = & 0{\cdot}2670054 \\ \log a_\infty = \log 422{\cdot}4 = & 2{\cdot}6257239 \\ \log (a_\infty . v^{260}) = & 2{\cdot}3587185 \\ a_\infty . v^{260} = & 228{\cdot}4118 \\ a_\infty = & 422{\cdot}4 \\ a_{\overline{260}|} = & 193{\cdot}9882 \end{array}$$

$$\begin{array}{rcrcrcr} 194{\cdot}0137 & & 194{\cdot}0137 & & {\cdot}002367424 & & \\ 193{\cdot}9882 & & 194{\cdot}0000 & & {\cdot}002366304 & & \\ \hline 255 & : & 137 & :: & 1120 & : & 602 \\ & & & & & & {\cdot}002366304 \\ & & & & \text{The exact rate is} & & {\cdot}002366906 \end{array}$$

$$\begin{array}{rl} \textit{Proof.}\text{—}\log 1{\cdot}002366906 = & 0{\cdot}0010267196 \times 260 \\ \log (1+i)^{260} = & 0{\cdot}2669471 \\ \log (1+i)^{-260} = & 9{\cdot}7330529 \\ \log \{(1+i)^{260}-1\} = & 9{\cdot}9289298 \\ \text{Colog } i = \log a_\infty = & 2{\cdot}6258190 \\ \log a_{\overline{260}|} = & 2{\cdot}2878017 \\ a_{\overline{260}|} = & 194{\cdot}0000 \quad \text{, the true value.} \end{array}$$

It may here be mentioned that the ratio between i and $\log_e(1+i)$ is always uniform at the same rates of interest, for any subdivisions of each, for example, $\cdot01219754 \times 1\cdot024797 = \cdot05 \times \frac{1}{4} = \cdot0125 = \frac{1}{4} \times i$ at 5 per-cent, also $\cdot02382755 \times 1\cdot049206 = \cdot10 \times \frac{1}{4} = \cdot025 = \frac{1}{4} \times i$ at 10 per-cent.* But the annuity-values obtained from $a_{\overline{n}|} \times \frac{i}{\log_e(1+i)} \times m$, are the annuity-values at rates of interest $= \{(1+i)^{\frac{1}{m}} - 1\}$ payable momently. We may subdivide i by taking the mth part of it, but we can subdivide $(1+i)$ only by extracting the mth root of it. From this it will be seen that the ratios involved in the annuity-values obtained from $a_{\overline{n}|} \times \frac{i}{\log_e(1+i)} \times m$, diminish according as m is increased, and they thus preserve the closeness of their approximation to annuity-values obtained from rates of interest equal to $\log_e(1+i) \div m$. For example, (referring to Ex. 8), at $6\frac{1}{4}$ per-cent $\{(1+i)^{\frac{1}{26}} - 1\} = \cdot002334437$, and at the same yearly rate of interest $\log_e(1+i)^{\frac{1}{26}} = \cdot002331716$.

* *Vide* the examples on p. 412.

Computing directly the annuity-value payable momently from these two fractional rates, we have

$$\log (1+i)^{260 \times \frac{1}{26}} = \log (1+i)^{10} = 0{\cdot}2632894 = \log 1{\cdot}002334437^{260}$$
$$\log (1+i)^{-10} = 9{\cdot}7367106$$
$$\log \{(1+i)^{10} - 1\} = 9{\cdot}9209242$$
$$\text{Colog } \{\log_e (1+i)^{\frac{1}{26}}\} = 2{\cdot}6323243 = \text{Colog } {\cdot}002331716$$
$$\log \bar{a}_{\overline{260}|} = \log (26 . \bar{a}_{\overline{10}|}) = 2{\cdot}2899591, \text{ as before.}$$

But the ratio between i and $\log_e (1+i)$ has diminished, for, at $6\frac{1}{4}$ per-cent payable yearly, it is $= \frac{{\cdot}0625}{{\cdot}0606246 2} = 1{\cdot}0309343$, while at the same rate payable fortnightly, it is $= \frac{{\cdot}002334437}{{\cdot}002331716} = 1{\cdot}001167$.

What definite meaning may we attach to rates of interest payable momently which we know to be impossible? It is suggested that they are the sums of the particles of interest, which, if improved in their respective ratios as they accrue, would exactly amount to the effective rates at the end of their intervals.*

The method of finding the rate of interest involved in amounts of annuities, is the same as that already exemplified, except that the proportional part is now to be subtracted from the larger rate.

Example 9.—£39,636 is the amount of an annuity of £50 for 68 years, required the rate of interest?

39636 : 50 = 792·72, so that we have $s_{\overline{68}|} = 792{\cdot}72$. Inspection of Oakes's table shows the rate to lie between $5\frac{3}{4}$ and $5\frac{7}{8}$ per-cent. The first approximation is as follows, namely,

at $5\frac{7}{8}$ per-cent $s_{\overline{68}|}$	$= 808{\cdot}91$	. . . .	808·91
„ $5\frac{3}{4}$ „ „	$= 761{\cdot}36$	at the required rate	792·72
Difference	47·55	. . . .	16·19

whence the approximate rate per-cent is $5{\cdot}875 - \frac{1619}{4755} . \frac{1}{8} = 5{\cdot}8324$.

Entering the table of reciprocals with this rate, we find $a_\infty\, 17{\cdot}14 = 1 : {\cdot}05834306$, $a_\infty = 17{\cdot}15 = 1 : {\cdot}05830904$, and observing that we have now to find the amount of an annuity by the formula $s_{\overline{n}|} = a_\infty (1+i)^n - a_\infty$, we operate as formerly—

$1 + a_\infty = 18{\cdot}14 \quad \log = 1{\cdot}25863{,}72827$	$18{\cdot}15 \quad \log = 1{\cdot}25887{,}66294$
$a_\infty = 17{\cdot}14 \quad \log = 1{\cdot}23401{,}08176$	$17{\cdot}15 \quad \log = 1{\cdot}23426{,}41244$
$\log (1+i) = 0{\cdot}02462{,}64651 \times 68$	$\log (1+i) = 0{\cdot}02461{,}25050 \times 68$
$\log (1+i)^{68} = 1{\cdot}6745996$	$\log (1+i)^{68} = 1{\cdot}6736503$
$\log a_\infty = 1{\cdot}2340108$	$\log a_\infty = 1{\cdot}2342641$
$\log a_\infty (1+i)^{68} = 2{\cdot}9086104$	$\log a_\infty (1+i)^{68} = 2{\cdot}9079144$
$a_\infty (1+i)^{68} = 810{\cdot}2338$	$a_\infty (1+i)^{68} = 808{\cdot}9364$
$a_\infty = 17{\cdot}14$	$a_\infty = 17{\cdot}15$
$s_{\overline{68}|} = 793{\cdot}0938$	$s_{\overline{68}|} = 791{\cdot}7864$

* I am indebted to Mr. George King for the suggestion to compute the momently factors. The particular application of them given in the present paper is original.

$$\begin{array}{rcrcrcr} 793{\cdot}0938 & & 793{\cdot}0938 & & {\cdot}05834306 & & \\ \underline{791{\cdot}7864} & & \underline{792{\cdot}7200} & & \underline{{\cdot}05830904} & & {\cdot}05834306 \\ 1{\cdot}3074 & : & {\cdot}3738 & :: & 3402 & : & -\underline{\quad 973} \\ & & & & & & {\cdot}05833333, \text{ the exact rate.} \end{array}$$

$$\begin{aligned} \textit{Proof.}\text{—}\log 1{\cdot}05833333 &= \underline{0{\cdot}02462{,}24736} \times 68 \\ \log (1+i)^{68} &= \underline{1{\cdot}6743282} \\ \log\{(1+i)^{68}-1\} &= 1{\cdot}6650366 \\ \text{Colog } i = \log a_\infty &= \underline{1{\cdot}2340832} \\ \log s_{\overline{68}|} &= 2{\cdot}8991198 \\ s_{\overline{68}|} &= 792{\cdot}7200, \text{ the true amount.} \end{aligned}$$

Example 10.—The amount of an annuity of £20 for 30 years is £1,600, required the rate of interest? (Baily's *Doctrine of Interest*, p. 122.)

1,600 : 20 = 80. We have here $s_{\overline{30}|}=80$. Referring to Oakes's table, we have

$$\begin{array}{llll} \text{at } 6\tfrac{1}{8} \text{ per-cent} & s_{\overline{30}|} = 80{\cdot}82 & \ldots\ldots\ldots & 80{\cdot}82 \\ \text{,, } 6 \quad\text{,,} & \text{,,} \quad = \underline{79{\cdot}06} & \text{at the required rate} & \underline{80{\cdot}00} \\ \quad \text{Difference} & \quad 1{\cdot}76 & \ldots\ldots\ldots & {\cdot}82 \end{array}$$

And the approximate rate per-cent is $6{\cdot}125-\frac{82}{176}\cdot\frac{1}{8}=6{\cdot}0668$.

Entering the table of reciprocals with this value, we find $a_\infty 16{\cdot}48=1:{\cdot}06067961$, $a_\infty 16{\cdot}49=1:{\cdot}06064281$. Working as before, we have

$$\begin{aligned} 1+a_\infty = 17{\cdot}48 \quad \log &= 1{\cdot}24254{,}1428 \\ a_\infty = 16{\cdot}48 \quad \log &= \underline{1{\cdot}21695{,}7207} \\ \log (1+i) &= 0{\cdot}02558{,}4221 \times 30 \\ \log (1+i)^{30} &= 0{\cdot}7675266 \\ \log a_\infty &= \underline{1{\cdot}2169572} \\ \log a_\infty (1+i)^{30} &= 1{\cdot}9844838 \\ a_\infty (1+i)^{30} &= 96{\cdot}49033 \\ a_\infty &= \underline{16{\cdot}48} \\ s_{\overline{30}|} &= 80{\cdot}01033 \end{aligned} \qquad \begin{aligned} 17{\cdot}49 \quad \log &= 1{\cdot}24278{,}9809 \\ 16{\cdot}49 \quad \log &= \underline{1{\cdot}21722{,}0656} \\ \log (1+i) &= 0{\cdot}02556{,}9153 \times 30 \\ \log (1+i)^{30} &= 0{\cdot}7670746 \\ \log a_\infty &= \underline{1{\cdot}2172207} \\ \log a_\infty (1+i)^{30} &= 1{\cdot}9842953 \\ a_\infty (1+i)^{30} &= 96{\cdot}44846 \\ a_\infty &= \underline{16{\cdot}49} \\ s_{\overline{30}|} &= 79{\cdot}95846 \end{aligned}$$

$$\begin{array}{rcrcrcr} 80{\cdot}01033 & & 80{\cdot}01033 & & {\cdot}06067961 & & \\ \underline{79{\cdot}95846} & & \underline{80{\cdot}00000} & & \underline{{\cdot}06064281} & & {\cdot}06067961 \\ 5187 & : & 1033 & :: & 3680 & : & -\underline{\quad 733} \\ & & & & & & {\cdot}06067228, \text{ the exact rate.} \end{array}$$

$$\begin{aligned} \textit{Proof.}\text{—}\log 1{\cdot}06067228 &= \underline{0{\cdot}02558{,}12190} \times 30 \\ \log (1+i)^{30} &= \underline{0{\cdot}7674366} \\ \log\{(1+i(^{30}-1\} &= 0{\cdot}6860803 \\ \text{Colog } i = \log a_\infty &= \underline{1{\cdot}2170097} \\ \log s_{\overline{30}|} &= 1{\cdot}9030900 \\ s_{\overline{30}|} &= 80{\cdot}00000 \quad, \text{ the true amount.} \end{aligned}$$

Baily gives ·060672284 as true in the last place, and he remarks, at p. 123, in reference to his method of finding the rate of interest in amounts of annuities-certain,—"Though the operations may appear tedious and intricate, there is no mode by which they can be simplified or abridged."

APPENDIX.

Rates of Interest, payable momently, with Factors for transforming Annuity-values, payable yearly, into the like payable momently.

Yearly Rates.	Momently Rates.	Momently Factors.	Yearly Rates.	Yearly Rates.	Momently Rates.	Momently Factors.	Yearly Rates.
$100i$	$\text{Log}_e(1+i)$	$\dfrac{i}{\text{Log}_e(1+i)}$	$100i$	$100i$	$\text{Log}_e(1+i)$	$\dfrac{i}{\text{Log}_e(1+i)}$	$100i$
2½	·024692,612	1·01244,86	2½	6¼	·060624,622	1·03093,43	6¼
2⅝	·025911,382	1·01306,83	2⅝	6⅜	·061800,401	1·03154,67	6⅜
2¾	·027128,668	1·01368,78	2¾	6½	·062974,799	1·03215,89	6½
2⅞	·028344,474	1·01430,71	2⅞	6⅝	·064147,820	1·03277,09	6⅝
3	·029558,802	1·01492,61	3	6¾	·065319,467	1·03338,26	6¾
3⅛	·030771,659	1·01554,49	3⅛	6⅞	·066489,742	1·03399,41	6⅞
3¼	·031983,046	1·01616,34	3¼	7	·067658,649	1·03460,54	7
3⅜	·033192,967	1·01678,17	3⅜	7⅛	·068826,191	1·03521,64	7⅛
3½	·034401,426	1·01739,97	3½	7¼	·069992,373	1·03582,71	7¼
3⅝	·035608,428	1·01801,74	3⅝	7⅜	·071157,196	1·03643,77	7⅜
3¾	·036813,973	1·01863,50	3¾	7½	·072320,661	1·03704,80	7½
3⅞	·038018,068	1·01925,22	3⅞	7⅝	·073482,776	1·03765,81	7⅝
4	·039220,713	1·01986,93	4	7¾	·074643,543	1·03826,79	7¾
4⅛	·040421,915	1·02048,61	4⅛	7⅞	·075802,965	1·03887,76	7⅞
4¼	·041621,675	1·02110,26	4¼	8	·076961,039	1·03948,70	8
4⅜	·042819,997	1·02171,89	4⅜	8⅛	·078117,778	1·04009,61	8⅛
4½	·044016,885	1·02233,49	4½	8¼	·079273,182	1·04070,50	8¼
4⅝	·045212,343	1·02295,08	4⅝	8⅜	·080427,248	1·04131,38	8⅜
4¾	·046406,373	1·02356,63	4¾	8½	·081579,986	1·04192,22	8½
4⅞	·047598,979	1·02418,16	4⅞	8⅝	·082731,396	1·04253,05	8⅝
5	·048790,164	1·02479,67	5	8¾	·083881,486	1·04313,84	8¾
5⅛	·049979,933	1·02541,15	5⅛	8⅞	·085030,249	1·04374,62	8⅞
5¼	·051168,288	1·02602,61	5¼	9	·086177,698	1·04435,37	9
5⅜	·052355,231	1·02664,05	5⅜	9⅛	·087323,829	1·04496,10	9⅛
5½	·053540,767	1·02725,46	5½	9¼	·088468,649	1·04556,81	9¼
5⅝	·054724,900	1·02786,85	5⅝	9⅜	·089612,159	1·04617,50	9⅜
5¾	·055907,632	1·02848,21	5¾	9½	·090754,364	1·04678,16	9½
5⅞	·057088,967	1·02909,55	5⅞	9⅝	·091895,264	1·04738,80	9⅝
6	·058268,907	1·02970,87	6	9¾	·093034,866	1·04799,42	9¾
6⅛	·059447,458	1·03032,16	6⅛	9⅞	·094173,170	1·04860,01	9⅞
				10	·095310,180	1·04920,59	10

I give an example of the formation of $\log_e(1+i)$ from $\log_{10}(1+i)$ by means of a manuscript table of the first thousand multiples of M, the modulus to base 10, to 33 places, by Mr. Peter Gray :—

$\log_{10}(1{\cdot}03)=$	012,837,224,705,172,205,171
M × 029 =	012,594,539,975,194,303,002
	242,684,729,977,902,169
M × 558 = .	242,336,320,902,014,520
	348,409,075,887,649
M × 802 = . . .	348,304,174,486,408
	104,901,401,241
M × 241 = ...	104,664,970,139
	236,431,102
M × 544 = ...	236,256,198
	174,904
M × 402 = ...	174,586
	318
M × 733 = ...	318

$\log_e(1+i)=029{,}558{,}802{,}241{,}544{,}402{,}733.$

Further examples of the use of this table, which it is very desirable to have printed, are given by Mr. Gray in vol. xxii of the *Journal of the Institute*, pp. 351, 352.

By means of this table, and Wolfram's table of natural logs, appended to the *Thesaurus* of Vega, $\log s_{10}$ for all numbers from 1 to 10,000, are readily known to 33 places.

SUPPLEMENT.

Since the preceding paper was written, it occurred to me to apply the tables of Henry Goodwyn to form annuity-values, and to find the rate of interest.

These tables,—namely, *A Tabular Series of Decimal Quotients for all the Proper Vulgar Fractions, of which, when in their lowest terms, neither the Numerator nor the Denominator is greater than* 1000, Part I (all published), and *A Table of the Circles arising from the Division of a Unit, or any other whole number, by all the Integers from* 1 *to* 1024,—both London, 1823, may, very conveniently, be used instead of reciprocals, when it is required to find a rate to many places, on account of the facility whereby the necessary elements may be obtained from them. In the *Tabular Series* we have upwards of 30,000 vulgar fractions, with their equivalent decimals, from $\frac{1}{1000}={\cdot}001$ to $\frac{99}{991}={\cdot}\dot{0}9989909$,* &c., and by means of the *Table of Circles* the decimal quotients may be extended, without computation, as far as we please.

Denoting by D the denominators, and by N the numerators, of the fractions, we have $i=\text{N}:\text{D}$, $a_\infty=\text{D}:\text{N}$, $(1+i)=(\text{D}+\text{N}):\text{D}$, consequently, $\log i=\log \text{N}-\log \text{D}$, $\log a_\infty=\log \text{D}-\log \text{N}=\operatorname{colog} i$, $\log(1+i)=\log(\text{D}+\text{N})-\log \text{D}$; and as the terms of the tabulated fractions are less than 1000, their logs may be obtained to twenty

* There are 990 figures in the period of this circulating decimal.

places, if necessary, from the tables of Hutton or Callet. To obtain the anti-logs, recourse must be had to the tables of Mr. Peter Gray. The anti-logs are most easily formed by taking the first three figures from Hutton or Callet, and the remaining triads from Gray, which may then be multiplied together by means of Crelle's table without mental effort. For a numerical illustration, let us take Example 6 of the preceding paper. We there found $a_{\overline{31}|} = 15{\cdot}2\dot{2}\dot{7}$, and $i = {\cdot}052052$ approximately. From Goodwyn's tables we obtain $\frac{33}{634} = {\cdot}052050{,}47318{,}61 = i$, $a_\infty = 19{\cdot}21212{,}12121{,}21$, and $\frac{47}{903} = {\cdot}052048{,}72646{,}73 = i$, $a_\infty = 19{\cdot}21276{,}59574{,}47$. With these elements we operate as before.

Example of Annuity-values and Rate of Interest to twelve places.

D + N = 950, log =	2·97772,36052,88848	667, log =	2·82412,58339,16549
D 903, log =	2·95568,77503,13506	634, log =	2·80208,92578,81733
$\log(1+i) =$	0·02203,58549,75342 × 31		0·02203,65760,34816 × 31
$\log(1+i)^{31} =$	0·68311,15042,36		0·68313,38570,79
$\log a_\infty =$	1·28358,98923,78		1·28357,53180,04
$\log(a_\infty . v^{31}) =$	0·60047,83881,42		0·60044,14609,25
$a_\infty . v^{31} =$	3·98545,93892,18		3·98512,05277,49
$a_\infty =$	19·21276,59574,47		19·21212,12121,21
$a_{\overline{31}\|} =$	15·22730,65682,29		15·22700,06843,72

·22730,65682,29	·22730,65682,29	·052050,47318,61	
·22700,06843,72	·22727,27272,73	·052048,72646,73	
30,58838,57	: 3,38409,56	: : 1,74671,88	: 19324,54
			·052048,72646,73

The rate to twelve significant figures, is very nearly = ·052048,91971,27

Proof.—log 1·052048,91971,27 =	0·02203,59347,4863 × 31
$\log(1+i)^{31} =$	0·68311,39772,08
$\log v^{31} =$	$\bar{1}$·31688,60227,92
$v^{31} =$	0·20743,69044,00
$1 - v^{31} =$	0·79256,30956,00
$\log(1 - v^{31}) =$	$\bar{1}$·89903,38462,59
Colog $i = \log a_\infty =$	1·28358,82799,42
$\log a_{\overline{31}\|} =$	1·18262,21262,01
$a_{\overline{31}\|} =$	15·22727,27268
True value =	15·22727,27273
Error =	·00000,00005

It will be observed that the value of i here found is slightly too great, but it gives an annuity-value differing from the true one by

only 5 in the twelfth figure. It differs by a unit in the tenth figure from the rate found by Thoman in his Example No. 59, which, although less exact than that above found, proves the remarkable closeness of the result obtained by the method of approximation given by that eminent mathematician in pp. 47, 48, of his *Theory of Compound Interest and Annuities*.

It may be mentioned, in justice to his memory, that an erroneous numerical solution to Example 104, is substituted, in the 2nd edition, 1872, and in the 3rd edition, 1877, in place of the correct one given by Thoman in the 1st edition, 1859.* Other alterations may also be noted, which, apart from the correction of obvious misprints, it is probable the distinguished author, if living, would not have sanctioned.

A table, occupying two 8vo. pages, by M. Achard, to find i by a simple operation, without tables of annuity-values, giving values of i usually correct to four figures, may be seen in M. Charlon's *Théorie Mathématique des Opérations Financières*, 2nd edition, Paris, 1878.

The Theory of Constant Coefficients. By the late John Naylor.

[This paper is extracted from a manuscript Treatise on Life Contingencies, written many years ago (probably not less than 50) and recently acquired by the Institute. It appears from this treatise that Mr. Naylor made extensive use of the formulas here obtained, in calculating the values of complicated contingencies.]

(1) In the following investigation, the accentuated characteristics, $\Sigma' u_x$, $\Delta' u_x$, $\Delta'^2 u_x \ldots \Delta'^n u_x$, respectively denote the integral and the successive orders of differences of the function u_x, when the increment of the variable x is h; whilst the unaccentuated characteristics, Σu_x, Δu_x, $\Delta^2 u_x \ldots \Delta^n u_x$, have corresponding significations when the increment of the variable is unity. Thus, if $u_x = x(x-h)$,

$$\Sigma' u_x = \Sigma' x(x-h) = \frac{x(x-h)(x-2h)}{3h} + \text{const.};$$

* A French translation from the English edition of 1859, with additional matter and amplification of many of the formulas, was published at Paris in 1878 by Gauthier-Villars. This excellent edition contains a prefatory memoir of M. Thoman, by Monsieur J. Bertrand, Secrétaire Perpétuel de l'Académie des Sciences, and author of very valuable works on the Differential and Integral Calculus.

whilst $\Sigma u_x = \Sigma x(x-h) = \Sigma\{x(x-1)-(h-1)x\}$

$$= \tfrac{1}{3}x(x-1)(x-2) - \tfrac{1}{2}(h-1)x(x-1) + \text{const.}$$

So also $\Delta' u_x = u_{x+h} - u_x = (x+h)x - x(x-h) = 2hx$,

whilst $\Delta u_x = u_{x+1} - u_x = (x+1)(x+1-h) - x(x-h) = 2x+1-h$;

and $\Delta'^2 u_x = \Delta' u_{x+h} - \Delta' u_x = 2h(x+h) - 2hx = 2h^2$,

whilst $\Delta^2 u_x = \Delta u_{x+1} - \Delta u_x = 2(x+1)+1-h-(2x+1-h) = 2$.

(2) If $S_m u_x = u_0 + u_1 + u_2 + \ldots + u_m$, that is, if $S_m u_x$ represent the sum of a series obtained by substituting in u_x, 0, 1, 2, ... m successively for x, we have, according to the principles of the Calculus of Differences, $S_m u_x$, or the sum of the series in question, equal to $\Sigma u_x - \Sigma u_{x'}$, $(m+1)$ being substituted for x, and 0 for x', after the integration. In every case in which the theory about to be investigated is intended to be applied, the integral $\Sigma u_{x'}$ will contain the factor x', and will consequently vanish after the substitution. The sum of the series, $S_m u_x$, may always therefore be obtained by integrating the function u_x, and substituting $(m+1)$ for x after the integration.

(3) If $\phi' t$ be the generating function of u_x, $t^x \phi' t$ will be the generating function of u_0. Let $\phi t = t^x \phi' t$; then we have the generating function of $u_x = \phi t \dfrac{1}{t^x}$, and the generating function of $S_m u_x$, or of $u_0 + u_1 + u_2 \ldots + u_x + u_{x+1} + \ldots + u_m$,

$$= \phi t \left\{1 + \frac{1}{t} + \frac{1}{t^2} + \ldots + \frac{1}{t^x} + \frac{1}{t^{x+1}} + \ldots + \frac{1}{t^m}\right\} = \phi t \, S_m \frac{1}{t^x};$$

and, substituting $\left(1 + \dfrac{1}{t^h} - 1\right)^{\frac{1}{h}}$ for $\dfrac{1}{t}$,

$= \phi t \, S_m \left(1 + \dfrac{1}{t^h} - 1\right)^{\frac{x}{h}}$. $\therefore$ substituting in $\left(1 + \dfrac{1}{t^h} - 1\right)^{\frac{x}{h}}$, 0, 1, 2, ... m successively for x, and developing in powers of $\left(\dfrac{1}{t^h} - 1\right)$, we have the generating function of $S_m u_x$ equal to

$$\phi t \left\{A_0 + A_1\left(\frac{1}{t^h}-1\right) + A_2\left(\frac{1}{t^h}-1\right)^2 + \ldots + A_r\left(\frac{1}{t^h}-1\right)^r + \ldots\right\} \quad . \quad . \quad (a)$$

But this last expression is, according to the theory of generating functions, equal to the generating function of

$$A_0 u_0 + A_1 \Delta' u_0 + A_2 \Delta'^2 u_0 + \ldots + A_r \Delta'^r u_0 + \ldots;$$

hence, as quantities which have the same generating functions must necessarily be equal, we have

$$S_m u_x = A_0 u_0 + A_1 \Delta' u_0 + A_2 \Delta'^2 u_0 + \ldots + A_r \Delta'^r u_0 + \quad \ldots \quad (\beta)$$

(4) In order to find the values of the coefficients A_0, A_1, $A_2 \ldots A_r$, we observe that A_r is the coefficient of $\left(\frac{1}{t^h}-1\right)^r$ in the development of $S_m\left(1+\frac{1}{t^h}-1\right)^{\frac{x}{h}}$, after substituting $0, 1, 2, \ldots m$ successively for x; and consequently is, according to the binomial theorem, equal to

$$S_m \frac{\frac{x}{h}\left(\frac{x}{h}-1\right)\left(\frac{x}{h}-2\right)\ldots\left(\frac{x}{h}-r+1\right)}{1.2.3\ldots\ldots r}$$

$$= \frac{1}{1.2.3\ldots rh^r} S_m x(x-h)(x-2h) \ldots \{x-(r-1)h\};$$

that is, by § (2),

$$A_r = \frac{1}{1.2.3\ldots rh^r} \Sigma x(x-h)(x-2h) \ldots \{x-(r-1)h\} \quad \ldots \quad (\gamma)$$

substituting $(m+1)$ for x after the integration.

(5) In the application of this theory, m will always be taken so as to be a multiple of h, and consequently $m=nh$, n being a whole positive number, whilst u_x will always be assumed to be a rational integral function of x, with the nth order of differences constant. Hence the equation (β) will become

$$S_m u_x = A_0 u_0 + A_1 \Delta' u_0 + A_2 \Delta'^2 u_0 + \ldots + A_r \Delta'^r u_0 + \ldots + A_n \Delta'^n u_0 \quad \ldots \quad (\delta)$$

and the values of the coefficients A_0, A_1, A_2, $\ldots A_r \ldots A_n$, will be determinable by the formula (γ).

(6) If in the equation (δ) we substitute for $\Delta' u_0$, $\Delta'^2 u_0$, . . $\Delta'^n u_0$, their respective values, $u_h - u_0$, $u_{2h} - 2u_h + u_0$, ... $u_{nh} - nu_{(n-1)h} + \frac{n.n-1}{1.2} u_{(n-2)h} - \ldots$, we have

$$
\begin{aligned}
S_m u_x = {} & A_0\{ \ldots\ldots\ldots\ldots u_0 \} \\
& + A_1\{ \ldots\ldots\ldots\ldots u_h - u_0 \} \\
& + A_2\{ \ldots\ldots\ldots\ldots u_{2h} - 2u_h + u_0 \} \\
& + A_3\{ \ldots\ldots\ldots\ldots u_{3h} - 3u_{2h} + 3u_h - u_0 \} \\
& + A_4\{ \ldots\ldots\ldots\ldots u_{4h} - 4u_{3h} + 6u_{2h} - 4u_h + u_0 \} \\
& \ldots\ldots\ldots\ldots \\
& + A_{n-2}\Big\{ \ldots\ldots u_{(n-2)h} - \frac{n-2}{1} u_{(n-3)h} + \ldots + (-1)^n \Big[\frac{n-2.n-3.n-4.n-5}{1.2.3.4} u_{4h} - \frac{n-2.n-3.n-4}{1.2.3} u_{3h} \\
& \qquad + \frac{n-2.n-3}{1.2} u_{2h} - \frac{n-2}{1} u_h + u_0 \Big] \Big\} \\
& + A_{n-1}\Big\{ \ldots\ldots u_{(n-1)h} - \frac{n-1}{1} u_{(n-2)h} + \frac{n-1.n-2}{1.2} u_{(n-3)h} - \ldots + (-1)^n \Big[\frac{n-1.n-2.n-3.n-4}{1.2.3.4} u_{4h} - \frac{n-1.n-2.n-3}{1.2.3} u_{3h} \\
& \qquad + \frac{n-1.n-2}{1.2} u_{2h} - \frac{n-1}{1} u_h + u_0 \Big] \Big\} \\
& + A_n \Big\{ u_{nh} - \frac{n}{1} u_{(n-1)h} + \frac{n.n-1}{1.2} u_{(n-2)h} - \frac{n.n-1.n-2}{1.2.3} u_{(n-3)h} + \ldots + (-1)^n \Big[\frac{n.n-1.n-2.n-3}{1.2.3.4} u_{4h} - \frac{n.n-1.n-2}{1.2.3} u_{3h} \\
& \qquad + \frac{n.n-1}{1.2} u_{2h} - \frac{n}{1} u_h + u_0 \Big] \Big\}.
\end{aligned}
$$

That is, $S_m u_x = k_0 u_0 + k_1 u_h + k_2 u_{2h} + \ldots + k_r u_{rh} + \ldots + k_n u_{nh}$ . . . . (ϵ), where

$$k_0 = A_0 - A_1 + A_2 - A_3 + A_4 - A_5 + \ldots + (-1)^n\{A_{n-2} - A_{n-1} + A_n\}$$
$$k_1 = \ldots A_1 - 2A_2 + 3A_3 - 4A_4 + 5A_5 - \ldots + (-1)^n\{(n-2)A_{n-2} - (n-1)A_{n-1} + nA_n\}$$
$$k_2 = \ldots\ldots A_2 - 3A_3 + 6A_4 - 10A_5 + \ldots + (-1)^n\left\{\frac{n-2.n-3}{1.2}A_{n-2} - \frac{n-1.n-2}{1.2}A_{n-1} + \frac{n.n-1}{1.2}A_n\right\}$$
$$k_3 = \ldots\ldots\ldots A_3 - 4A_4 + 10A_5 - \ldots + (-1)^n\left\{\frac{n-2.n-3.n-4}{1.2.3}A_{n-2} - \frac{n-1.n-2.n-3}{1.2.3}A_{n-1} + \frac{n.n-1.n-2}{1.2.3}A_n\right\}$$
$$k_4 = \ldots\ldots\ldots\ldots A_4 - 5A_5 + \ldots + (-1)^n\left\{\frac{n-2.n-3.n-4.n-5}{1.2.3.4}A_{n-2} - \frac{n-1.n-2.n-3.n-4}{1.2.3.4}A_{n-1} + \frac{n.n-1.n-2.n-3}{1.2.3.4}A_n\right\}$$
$$\ldots\ldots\ldots\ldots\ldots\ldots$$
$$k_{n-2} = \ldots\ldots\ldots\ldots A_{n-2} - \frac{n-1}{1}A_{n-1} + \frac{n.n-1}{1.2}A_n$$
$$k_{n-1} = \ldots\ldots\ldots\ldots A_{n-1} - nA_n$$
$$k_n = \ldots\ldots\ldots\ldots A_n$$

Hence it is evident that k_r, the coefficient of u_{rh} in the formula (ϵ), $= aA_r - \beta A_{r+1} + \gamma A_{r+2} \ldots + (-1)^{n-r}\pi A_n$,
where a is the coefficient of u_{rh} in $\Delta'^r u_0$, that is, in $u_{rh} - ru_{(r-1)h} + \ldots\ldots \therefore\ a = 1$

$-\beta$,, ,, u_{rh} in $\Delta'^{r+1}u_0$, or in $u_{(r+1)h} - (r+1)u_{rh} + \ldots\ldots \therefore\ \beta = r+1$

γ ,, ,, u_{rh} in $\Delta'^{r+2}u_0$, or in $u_{(r+2)h} - (r+2)u_{(r+1)h} + \frac{r+2.r+1}{1.2}u_{rh} - \ldots\ldots \therefore\ \gamma = \frac{r+1.r+2}{1.2}$

$\ldots\ldots\ldots\ldots$

$(-1)^{n-r}\pi$,, u_{rh} in $\Delta'^n u_0$, or in $u_{nh} - nu_{(n-1)h} + \ldots (-1)^{n-r}\frac{n.n-1\ldots r+2.r+1}{1.2\ldots\ldots n-r-1.n-r}u_{rh}$

$$\therefore\ \pi = \frac{r+1.r+2\ldots n-1.n}{1.2\ldots n-r-1.n-r}.$$

Consequently, $$k_r = A_r - (r+1)A_{r+1} + \frac{(r+1)(r+2)}{1.2}A_{r+2} - \ldots + (-1)^{n-r}\frac{(r+1)(r+2)\ldots(n-1)n}{1.2\ldots(n-r-1)(n-r)}A_n \quad . \quad . \quad (\zeta)$$

(7) In the foregoing investigation, it is evident that, when n and h are given, the coefficients A_0, A_1, $A_2 \ldots . A_n$, and consequently k_0, k_1, $k_2 \ldots . k_n$, are given, being given functions of n and h, and altogether independent of the quantities u_0, u_h, $u_{2h} \ldots . u_{nh}$, which may be of any finite magnitude, although, being rational integral functions of x with the nth order of differences constant, they must necessarily be of the form $a+bx+cx^2+\ldots .+gx^n$. For this reason, $k_0, k_1, k_2, \ldots . k_n$, are called Constant Coefficients.

(8) Since
$$\begin{aligned} S_m u_x &= u_0+u_1+u_2+\ldots .+u_{m-2}+u_{m-1}+u_m \\ &= k_0u_0+k_1u_h+k_2u_{2h}+\ldots .+k_{n-2}u_{(n-2)h} \\ &\qquad +k_{n-1}u_{(n-1)h}+k_nu_{nh} \quad . \quad . \quad . \quad . \qquad (\epsilon) \end{aligned}$$
we have
$$\begin{aligned} S_m v_x &= v_0+v_1+v_2+\ldots .+v_{m-2}+v_{m-1}+v_m \\ &= k_0v_0+k_1v_h+k_2v_{2h}+\ldots .+k_{n-2}v_{(n-2)h} \\ &\qquad +k_{n-1}v_{(n-1)h}+k_nv_{nh}, \end{aligned}$$
and making $v_x=u_{m-x}$, we have
$$\begin{aligned} S_m u_{m-x} &= u_m+u_{m-1}+u_{m-2}+\ldots .+u_2+u_1+u_0 \\ &= k_0u_{nh}+k_1u_{(n-1)h}+k_2u_{(n-2)h}+\ldots .+k_{n-2}u_{2h} \\ &\qquad +k_{n-1}u_h+k_nu_0 \quad . \quad . \quad . \quad . \quad . \quad . \qquad (\epsilon') \end{aligned}$$
and subtracting from each other the identical equations (ϵ) and (ϵ'), we have
$$\begin{aligned} &(k_n-k_0)(u_{nh}-u_0)+(k_{n-1}-k_1)(u_{(n-1)h}-u_h) \\ &\qquad +(k_{n-2}-k_2)(u_{(n-2)h}-u_{2h})+\ldots .=0 \quad . \quad . \quad . \qquad (\eta) \end{aligned}$$
whatever be the values of $u_0, u_h \ldots ., u_{nh}$. Now, since u_x is a rational integral function of the form $a+bx+cx^2+\ldots .+gx^n$, the coefficients $a, b, c, \ldots . g$ having any values whatever, it is evident that these coefficients may be so taken that, whilst $u_{nh}-u_0$ is of finite magnitude, $u_{(n-1)h}-u_h$, $u_{(n-2)h}-u_{2h}, \ldots .$ will be each $=0$, in which case the equation (η) will become
$$(k_n-k_0)(u_{nh}-u_0)=0, \ \therefore \ k_n-k_0=0, \text{ and } k_0=k_n \quad . \quad . \qquad (\vartheta)$$
In like manner it may be shown that
$$k_1=k_{n-1}, \ k_2=k_{n-2}, \text{ and generally that } k_r=k_{n-r} \quad . \quad . \qquad (\vartheta')$$
r being any whole number not greater than n.

(9) It is thus evident that, if we have a series
$$S_mu_x=u_0+u_1+\ldots .+u_h+u_{h+1}+\ldots .+u_{2h}+u_{2h+1}+\ldots .+u_{nh},$$
u_x being a rational integral function of x with the nth order of differences constant, then by multiplying the equidistant terms

$u_0, u_h, u_{2h} \ldots . u_{nh}$ by the constant coefficients $k_0, k_1, k_2 \ldots . k_n$, we should obtain the exact sum of the series.

Hence, if $(n+1)$ equidistant terms only of a series be given, we may, by multiplying each of these by the proper constant coefficient, obtain the sum of the entire series, *precisely the same* as if the intermediate terms were interpolated by the method of differences (taking in each case n orders of differences into account) and the whole added together. So also, if u_x be a function of any nature whatever, and $(n+1)$ equidistant terms, $u_0, u_h, u_{2h} \ldots . u_{nh}$, of the series $S_m u_x$ be given, then, if the intermediate terms could, with a sufficient degree of accuracy, be interpolated by the method of differences (taking the nth difference into account), the sum of the series might be obtained without such interpolation, by merely multiplying each of the given equidistant terms by a known constant coefficient; and the result would be precisely the same as if the interpolation were actually effected and the terms all added together. Now, as the function u_x, in every case in which it is intended to apply the theory, is always of such a nature that the interpolation in question may safely be effected, that is, none of the terms so interpolated would be affected with any material error; and as such small errors, if any, would be sometimes in excess and sometimes in defect, and consequently in summing up the series, destroy each other; it is evident that the final error, or the error *in the sum* of the series obtained by multiplying a few of the equidistant terms, each by a proper constant coefficient, would be so small, when the system is judiciously applied, as not significantly to affect the result.

(10) It is to be observed that, in order to obtain the sum of a series by means of the formula

$$S_m u_x = k_0 u_0 + k_1 u_h + \ldots . + k_n u_{nh} \quad . \quad . \quad . \quad . \quad (\epsilon),$$

m is always taken $=nh$; that is, m must be a multiple of n and h. But in the great majority of cases to which the system is intended to be applied, $S_m u_x$ is continued until u_x ceases to have a significant value, that is, $u_m = 0$. Hence the series may be assumed to be continued indefinitely, each of the terms beyond the mth being$=0$, and in fact to contain $(m'+1)$ terms, in which m' may be any number taken at pleasure,—not less than m; and consequently the formula (ϵ) may always correctly be applied, if nh be taken $=$ or $>$ m.

(11) In the practical application however of the system, it is necessary to adopt given values of n and h, that is, it becomes

necessary to resort to tables of constant coefficients, computed first by giving a numerical value to n and afterwards to h.

If n be taken $=3$, or the third order of differences of the function u_x be considered constant, we have $m=3h$; if $n=4$, or the fourth order be assumed constant, we have $m=4h$; if $n=5$, or the fifth order be taken as constant, we have $m=5h$. The values 3, 4, and 5, are all the values of n which have been adopted in the construction of the annexed tables.

Having taken these values of n, and obtained (in each of the three cases) expressions of the values of k_0, k_1, &c., ... in terms of h only, we have next assumed values of h from $h=5$ to $h=10$, calculated the numerical values of k_0, k_1, &c., ..., and arranged the results in tables. Table I exhibits the several values obtained when $m=3h$, Table II the corresponding values when $m=4h$, and Table III similar values when $m=5h$.

(12) It is thus evident that, in the practical application of the system, as the values of n and h are limited, nh cannot always (as directed in § 10) be taken equal to or greater than m. But, since we have generally

$$\begin{aligned} S_{2m}u_x &= u_0+u_1+\ldots\ldots+u_m+u_m+u_{m+1}+\ldots\ldots+u_{2m}-u_m \\ &= S_m u_x + S_m u_{m+x} - u_m, \end{aligned}$$

and
$$\begin{aligned} S_m u_x &= k_0u_0+k_1u_h+\ldots\ldots+k_{n-1}u_{(n-1)h}+k_nu_{nh} \quad \ldots \quad (\epsilon) \\ &= k_0u_0+k_1u_h+\ldots\ldots+k_1u_{(n-1)h}+k_0u_{nh} \quad \ldots\ldots \quad (\vartheta') \end{aligned}$$

$$\therefore\ S_m u_{m+x} = k_0u_{nh}+k_1u_{(n+1)h}+\ldots+k_1u_{(2n-1)h}+k_0u_{2nh}, \text{ and } u_m=u_{nh},$$

we have

$$\begin{aligned} S_{2m}u_x = k_0u_0+k_1u_h+\ldots\ldots+k_1u_{(n-1)h}+(2k_0-1)u_{nh}+k_1u_{(n+1)h} \\ +\ldots\ldots+k_1u_{(2n-1)h}+k_0u_{2nh} \quad \ldots\ldots \quad (\lambda) \end{aligned}$$

and generally that

$$\begin{aligned} S_{rm}u_x = k_0u_0+k_1u_h+\ldots\ldots+(2k_0-1)u_{nh}+k_1u_{(n+1)h} \\ +\ldots\ldots+(2k_0-1)u_{2nh}+k_1u_{(2n+1)h}+\ldots\ldots+(2k_0-1)u_{3nh} \\ +\ldots\ldots+(2k_0-1)u_{(r-1)nh}+\ldots\ldots+k_0u_{rnh} \quad \ldots \quad (\mu) \end{aligned}$$

It is thus evident that, by resorting to the formula (μ), the computed coefficients in the tables may be universally resorted to for the summation of a series, however great the number of terms it may contain.

We now proceed to deduce the general expressions of the values of A_0, A_1, A_2, A_3, A_4, and A_5 from the formula (γ); then substituting in these expressions 3, 4, and 5, successively for n, and determining the expressions of the values of k_0, k_1, k_2, ...

by means of equation (ζ), we obtain the formula from which the values in the tables have been computed.

(13) Since $A_r = \dfrac{1}{1.2\,..\,rh^r}\Sigma x(x-h)(x-2h)\ldots\{x-(r-1)h\}$, substituting $(m+1)=nh+1$ for x after the integration, we have

$$A_0 = \Sigma x^0 = x = m+1 = nh+1\,;$$

$$A_1 = \frac{1}{h}\Sigma x = \frac{1}{h}\frac{x(x-1)}{2} = \frac{(m+1)m}{2h} = \frac{n(nh+1)}{2}\,;$$

$$A_2 = \frac{1}{1.2h^2}\Sigma x(x-h) = \frac{1}{2h^2}\Sigma\{x(x-1)-(h-1)x\}$$

$$= \frac{1}{2h^2}\left\{\frac{x(x-1)(x-2)}{3} - (h-1)\frac{x(x-1)}{2}\right\}$$

$$= \frac{n(nh+1)}{2h}\left\{\frac{nh-1}{3} - \frac{h-1}{2}\right\}\,;$$

$$A_3 = \frac{1}{1.2.3h^3}\Sigma x(x-h)(x-2h)$$

$$= \frac{1}{6h^3}\Sigma\{x(x-1)(x-2) - 3(h-1)x(x-1) + (h-1)(2h-1)x\}$$

$$= \frac{1}{6h^3}\left\{\frac{x(x-1)(x-2)(x-3)}{4} - 3(h-1)\frac{x(x-1)(x-2)}{3} + (h-1)(2h-1)\frac{x(x-1)}{2}\right\}$$

$$= \frac{n(nh+1)}{6h^2}\left\{\frac{(nh-1)(nh-2)}{4} - (h-1)(nh-1) + \frac{(h-1)(2h-1)}{2}\right\}\,;$$

$$A_4 = \frac{1}{1.2.3.4h^4}\Sigma x(x-h)(x-2h)(x-3h)$$

$$= \frac{1}{24h^4}\Sigma\{x(x-1)(x-2)(x-3) - 6(h-1)x(x-1)(x-2) + (h-1)(11h-7)x(x-1) - (h-1)(2h-1)(3h-1)x\}$$

$$= \frac{1}{24h^4}\left\{\frac{x(x-1)(x-2)(x-3)(x-4)}{5} - 6(h-1)\frac{x(x-1)(x-2)(x-3)}{4} + (h-1)(11h-7)\frac{x(x-1)(x-2)}{3} - (h-1)(2h-1)(3h-1)\frac{x(x-1)}{2}\right\}$$

$$= \frac{n(nh+1)}{24h^3}\left\{\frac{(nh-1)(nh-2)(nh-3)}{5} - \frac{3(h-1)(nh-1)(nh-2)}{2} + \frac{(h-1)(11h-7)(nh-1)}{3} - \frac{(h-1)(2h-1)(3h-1)}{2}\right\}\,;$$

$$A_5 = \frac{1}{1.2.3.4.5h^5}\Sigma x(x-h)(x-2h)(x-3h)(x-4h)$$

$$= \frac{1}{120h^5}\Sigma\{x(x-1)(x-2)(x-3)(x-4)$$
$$-10(h-1)x(x-1)(x-2)(x-3)+5(h-1)(7h-5)x(x-1)(x-2)$$
$$-5(h-1)(2h-1)(5h-3)x(x-1)+(h-1)(2h-1)(3h-1)(4h-1)x\}$$

$$= \frac{1}{120h^5}\left\{\frac{x(x-1)(x-2)(x-3)(x-4)(x-5)}{6}\right.$$
$$-10(h-1)x(x-1)(x-2)(x-3)(x-4)\div 5$$
$$+5(h-1)(7h-5)x(x-1)(x-2)(x-3)\div 4$$
$$-5(h-1)(2h-1)(5h-3)x(x-1)(x-2)\div 3$$
$$\left.+(h-1)(2h-1)(3h-1)(4h-1)x(x-1)\div 2\right\}$$

$$= \frac{n(nh+1)}{120h^4}\left\{\frac{(nh-1)(nh-2)(nh-3)(nh-4)}{6}\right.$$
$$-2(h-1)(nh-1)(nh-2)(nh-3)+\frac{5(h-1)(7h-5)(nh-1)(nh-2)}{4}$$
$$\left.-\frac{5(h-1)(2h-1)(5h-3)(nh-1)}{3}+\frac{(h-1)(2h-1)(3h-1)(4h-1)}{2}\right\}.$$

In like manner the values of A_6, A_7, A_8, A_9, &c., might, if required, be determined.

(14) First, let $n=3$, that is, let the function u_x be assumed such that the third order of differences is constant; then substituting 3 for n in the above general expressions of the values of A_0, A_1, A_2, and A_3, we have

$$A_0 = 3h+1;$$

$$A_1 = \frac{3(3h+1)}{2};$$

$$A_2 = \frac{3(3h+1)}{2h}\left\{\frac{3h-1}{3}-\frac{h-1}{2}\right\} = \frac{(3h+1)^2}{4h};$$

$$A_3 = \frac{3h+1}{2h^2}\left\{\frac{(3h-1)(3h-2)}{4}-(h-1)(3h-1)+\frac{(h-1)(2h-1)}{2}\right\}$$
$$= \frac{(3h+1)(h+1)}{8h}.$$

Hence we have

$$k_0 = k_3 = A_3 = \frac{(3h+1)(h+1)}{8h} \quad . \quad . \quad . \quad . \quad (a)$$

$$k_1 = k_2 = A_2 - 3A_3 = \frac{(3h+1)(3h-1)}{8h} \quad . \quad . \quad . \quad . \quad (a')$$

From the two formulas (a) and (a'), the values of $k_0=k_3$, and $k_1=k_2$, in Table I have been computed.

(15) Secondly, let $n=4$, that is, let the function u_x be assumed to be such that the fourth order of differences is constant; then substituting 4 for n in the above general expressions of the values of A_0, A_1, A_2, A_3, and A_4, we have

$$A_0=4h+1;$$

$$A_1=\frac{4(4h+1)}{2}=2(4h+1);$$

$$A_2=\frac{4(4h+1)}{2h}\left\{\frac{4h-1}{3}-\frac{h-1}{2}\right\}=\frac{(4h+1)(5h+1)}{3h};$$

$$A_3=\frac{4(4h+1)}{6h^2}\left\{\frac{(4h-1)(4h-2)}{4}-(h-1)(4h-1)+\frac{(h-1)(2h-1)}{2}\right\}$$
$$=\frac{(4h+1)(2h+1)}{3h};$$

$$A_4=\frac{4(4h+1)}{24h^3}\left\{\frac{(4h-1)(4h-2)(4h-3)}{5}-\frac{3(h-1)(4h-1)(4h-2)}{2}\right.$$
$$\left.+\frac{(h-1)(11h-7)(4h-1)}{3}-\frac{(h-1)(2h-1)(3h-1)}{2}\right\}$$
$$=\frac{(4h+1)(2h+1)(h+1)(7h-1)}{180h^3}.$$

Hence we have

$$k_0=k_4=A_4=\frac{(4h+1)(2h+1)(h+1)(7h-1)}{180h^3} \quad . \quad . \quad . \quad (b)$$

$$k_1=k_3=A_3-4A_4=\frac{(4h+1)(2h+1)(4h-1)(2h-1)}{45h^3}$$
$$=\frac{(16h^2-1)(4h^2-1)}{45h^3} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (b')$$

$$k_2=A_2-3A_3+6A_4=\frac{(4h+1)(4h-1)(h^2+1)}{30h^3}$$
$$=\frac{(16h^2-1)(h^2+1)}{30h^3} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (b'')$$

From the formulas (b), (b'), (b''), the values of $k_0=k_4$, $k_1=k_3$, and k_2 in Table II have been computed.

(16) Thirdly, let $n=5$, that is, let the function u_x be assumed such that the fifth order of differences is constant; then substituting 5 for n in the general expressions of the values of A_0, A_1, A_2, A_3, A_4, and A_5, we have

$$A_0 = 5h+1;$$

$$A_1 = \frac{5(5h+1)}{2};$$

$$A_2 = \frac{5(5h+1)}{2h}\left\{\frac{5h-1}{3} - \frac{h-1}{2}\right\} = \frac{5(5h+1)(7h+1)}{12h}$$

$$A_3 = \frac{5(5h+1)}{6h^2}\left\{\frac{(5h-1)(5h-2)}{4} - (h-1)(5h-1) + \frac{(h-1)(2h-1)}{2}\right\}$$

$$= \frac{5(5h+1)(3h+1)}{8h};$$

$$A_4 = \frac{5(5h+1)}{24h^3}\left\{\frac{(5h-1)(5h-2)(5h-3)}{5} - \frac{3(h-1)(5h-1)(5h-2)}{2} + \frac{(h-1)(11h-7)(5h-1)}{3} - \frac{(h-1)(2h-1)(3h-1)}{2}\right\}$$

$$= \frac{(5h+1)(85h^3+55h^2+5h-1)}{144h^3};$$

$$A_5 = \frac{5h+1}{24h^4}\left\{\frac{(5h-1)(5h-2)(5h-3)(5h-4)}{6} - \frac{2(h-1)(5h-1)(5h-2)(5h-3)}{1} + \frac{5(h-1)(7h-5)(5h-1)(5h-2)}{4} - \frac{5(h-1)(2h-1)(5h-3)(5h-1)}{3} + \frac{(h-1)(2h-1)(3h-1)(4h-1)}{2}\right\} = \frac{(5h+1)(h+1)(19h^2+6h-1)}{288h^3}.$$

Hence we have

$$k_0 = k_5 = A_5 = \frac{(5h+1)(h+1)(19h^2+6h-1)}{288h^3} \quad . \quad . \quad (c)$$

$$k_1 = k_4 = A_4 - 5A_5 = \frac{(5h+1)(5h-1)(5h^2-1)}{96h^3}$$

$$= \frac{(25h^2-1)(5h^2-1)}{96h^3} \quad . \quad . \quad . \quad . \quad . \quad (c')$$

$$k_2 = k_3 = A_3 - 4A_4 + 10A_5 = \frac{(5h+1)(5h-1)(5h^2+1)}{144h^3}$$

$$= \frac{(25h^2-1)(5h^2+1)}{144h^3} \quad . \quad . \quad . \quad . \quad . \quad (c'')$$

From the formulas (c), (c'), (c''), the values of $k_0 = k_5$, $k_1 = k_4$, and $k_2 = k_3$, have been computed.

TABLE I.—*3rd Differences Constant.*

h	$\frac{(3h+1)(h+1)}{8h}$		$\frac{(3h+1)(3h-1)}{8h}$				h
	$k_0=k_3$	$\text{Log}\,k_0=\text{Log}\,k_3$	$k_1=k_2$	$\text{Log}\,k_1=\text{Log}\,k_2$	$2k_0-1$	$\text{Log}(2k_0-1)$	
5	2·4	·3802112	5·6	·7481880	3·8	·5797836	5
6	2·77083	·4426104	6·72917	·8279613	4·54167	·6572146	6
7	3·14286	·4973246	7·85714	·8952647	5·28571	·7231027	7
8	3·515625	·5460025	8·[illegible]34375	·9534879	6·03125	·7804073	8
9	3·88889	·5898255	10·11111	1·0047989	6·77778	·8310873	9
10	4·2625	·6296644	11·2375	1·0506697	7·5250	·8765065	10

TABLE II.—*4th Differences Constant.*

h	$\frac{(4h+1)(2h+1)(h+1)(7h-1)}{180h^3}$		$\frac{(16h^2-1)(4h^2-1)}{45h^3}$		$\frac{(16h^2-1)(h^2+1)}{30h^3}$				h
	$k_0=k_4$	$\text{Log}\,k_0 = \text{Log}\,k_4$	$k_1=k_3$	$\text{Log}\,k_1 = \text{Log}\,k_3$	k_2	$\text{Log}\,k_2$	$2k_0-1$	$\text{Log}(2k_0-1)$	
5	2·0944	·3210596	7·0224	·8464856	2·7664	·4419150	3·1888	·5036273	5
6	2·39905	·3800390	8·45936	·9273376	3·28318	·5162946	3·7981	·5795664	6
7	2·70554	·4322539	9·89213	·9952897	3·80466	·5803164	4·41108	·6445449	7
8	3·01318	·4790256	11·32227	1·0539333	4·3291	·6363977	5·02636	·7012536	8
9	3·3216	·5213470	12·75065	1·1055323	4·85551	·6862348	5·6432	·7515254	9
10	3·63055	·5599724	14·1778	1·1516089	5·3833	·7310486	6·2611	·7966506	10

TABLE III.—*5th Differences Constant.*

h	$\frac{(5h+1)(h+1)(19h^2+6h-1)}{288h^3}$		$\frac{(25h^2-1)(5h^2-1)}{96h^3}$		$\frac{(25h^2-1)(5h^2+1)}{144h^3}$				h
	$k_0=k_5$	$\text{Log}\,k_0 = \text{Log}\,k_5$	$k_1=k_4$	$\text{Log}\,k_1 = \text{Log}\,k_4$	$k_2=k_3$	$\text{Log}\,k_2 = \text{Log}\,k_3$	$2k_0-1$	$\text{Log}(2k_0-1)$	
5	2·184	·3392526	6·448	·8094250	4·368	·6402826	3·368	·5273721	5
6	2·50809	·3993424	7·76046	·8898877	5·23145	·7186220	4·01618	·6038131	6
7	2·83382	·4523721	9·06997	·9576059	6·09621	·7850599	4·66764	·6690973	7
8	3·16058	·4997673	10·37762	1·0160980	6·96179	·8427210	5·32116	·7260063	8
9	3·48804	·5425809	11·68404	1·0675931	7·82792	·8936465	5·97608	·7764164	9
10	3·81597	·5816048	12·98959	1·1135956	8·69444	·9392415	6·63194	·8216404	10

Mr. Naylor gives the following as examples of his method :—

Example 1.—Suppose it is required to find the sum of the cubes, $1^3+2^3+3^3+\ldots\ldots+29^3+30^3$. Here we have third differences constant, and we therefore have to use the figures in Table I. Taking $h=10$, the formula for the sum of the cubes is

$$4{\cdot}2625(0^3+30^3)+11{\cdot}2375(10^3+30^3),$$
$$=115{,}087{\cdot}5+101{,}137{\cdot}5,$$
$$=216{,}225,$$

which is the exact sum required, as is easily proved from the fact that it is equal to the square of $(1+2+3+\ldots+29+30)$, that is, to the square of 465.

Example 2.—To find the value of an annuity-certain for 50 years at 4 per-cent interest. This is equal to

$$(1+v+v^2+\ldots\ldots+v^{50})-1,$$

where $v=1{\cdot}04^{-1}$; and by Mr. Naylor's method the sum of the series within the brackets

$$=k_0\times 1+k_1v^{10}+k_2v^{20}+k_3v^{30}+k_4v^{40}+k_5v^{50},$$

assuming fifth differences to be constant and taking $h=10$. In the actual working, it is not necessary to find the values of v^{10}, v^{20}, ..., and Mr. Naylor proceeds as follows, making use of the logarithms of the constants given in Table III :—

(1)	(2)	(1)+(2) (3)	Antilog (3) (4)
$\log v^0 = 0{\cdot}00000$	$\log k_0 = 0{\cdot}58160$	0·58160	3·8160
„ $v^{10} = \bar{1}{\cdot}82967$	„ $k_1 = 1{\cdot}11360$	0·94327	8·7755
„ $v^{20} = \bar{1}{\cdot}65933$	„ $k_2 = 0{\cdot}93924$	0·59857	3·9680
„ $v^{30} = \bar{1}{\cdot}48900$	„ $k_3 = 0{\cdot}93924$	0·42824	2·6806
„ $v^{40} = \bar{1}{\cdot}31867$	„ $k_4 = 1{\cdot}11360$	0·43227	2·7056
„ $v^{50} = \bar{1}{\cdot}14833$	„ $k_5 = 0{\cdot}58160$	$\bar{1}{\cdot}72993$	·5369
			22·4826

Subtracting unity from this, the value of the annuity desired is 21·4826, the correct value being 21·4822.

Example 3.—Mr. Naylor next applies his method to find the value of a 4 per-cent annuity-due upon a life of 10 according to a private mortality table which he was in the habit of employing. According to this table the separate payments of the annuity are of the values set forth in column (2) of the following table, and these have to be multiplied by the coefficients standing opposite to

them, fifth differences being assumed constant and the interval being 10.

r (1)	${}_r\mathrm{E}_{10}$ (2)	(3)	Log ${}_r\mathrm{E}_{10}$ (4)	(5)	(4)+(5) (6)	Antilog (6) (7)
0	1·00000	k_0 = 3·81597	0·00000	Log k_0 = 0·58160	0·58160	3·8160
10	·63489	k_1 = 12·98959	$\bar{1}$·80270	„ k_1 = 1·11360	0·96130	8·2471
20	·39434	k_2 = 8·69444	$\bar{1}$·59587	„ k_2 = 0·93924	0·53511	3·4285
30	·24142	k_3 = 8·69444	$\bar{1}$·38277	„ k_3 = 0·93924	0·32201	2·0990
40	·14457	k_4 = 12·98959	$\bar{1}$·16009	„ k_4 = 1·11360	0·27369	1·8780
50	·08070	$2k_0-1$ = 6·63194	$\bar{2}$·90686	„ $(2k_0-1)$ = 0·82164	$\bar{1}$·72850	·5352
60	·03703	k_1 = 12·98959	$\bar{2}$·56851	„ k_1 = 1·11360	$\bar{1}$·68211	·4810
70	·01051	k_2 = 8·69444	$\bar{2}$·02141	„ k_2 = 0·93924	$\bar{2}$·96065	·0913
80	·00091	k_3 = 8·69444	$\bar{4}$·9573	„ k_3 = 0·9392	$\bar{3}$·8965	·0079
90	·00000	k_4 = 12·98959	$\bar{6}$·416	„ k_4 = 1·114	$\bar{5}$·530	·0000
						20·5840

The separate payments in column (2), not being necessary for the work, are not shown by Mr. Naylor, who only uses the logarithms given in column (4); but we have added them for the sake of facilitating comparison with other mortality tables. The approximate value of the annuity-due, as calculated by the above method, is 20·5840; and Mr. Naylor states that the true value is 20·5802. We have also calculated the same annuity-value by means of Lubbock's formula of summation, as follows. Putting ${}_r\mathrm{E}_{10} = \mathrm{V}_r$, we have.

	Δ^1	Δ^2	Δ^3	Δ^4	Δ^5
$\mathrm{V}_0 =$ 1·00000	−·36511	+·12456	−·03693	+·00537	+·00310
$\mathrm{V}_1 =$ ·63489	·24055	·08763	·03156	·00847	
$\mathrm{V}_2 =$ ·39434	·15292	·05607	·02309		
$\mathrm{V}_3 =$ ·24142	·09685	·03298			
$\mathrm{V}_4 =$ ·14457	·06387				
$\mathrm{V}_5 =$ ·08070					
$\mathrm{V}_6 =$ ·03703					
$\mathrm{V}_7 =$ ·01051					
$\mathrm{V}_8 =$ ·00091					
$\mathrm{V}_9 =$ ·00000					
Sum = 2·54437					

Then, using Lubbock's formula as far as fifth differences, and taking the values of the coefficients from the table on page 313 of vol. xviii ($n=10$), we have, approximately,

$$
\begin{aligned}
\mathbf{a}_{10} &= 10 \times 2{\cdot}54437 - 4\tfrac{1}{2} - {\cdot}825 \times {\cdot}36511 - {\cdot}4125 \times {\cdot}12456 - {\cdot}261113 \\
&\qquad \times {\cdot}03693 - {\cdot}184519 \times {\cdot}00537 + {\cdot}141028 \times {\cdot}00310 \\
&= 25{\cdot}4437 - 4{\cdot}5 \\
&\qquad - {\cdot}3012 \\
&\qquad - {\cdot}0514 \\
&\qquad - {\cdot}0096 \\
&\qquad - {\cdot}0010 \\
&\qquad + {\cdot}0004 \\
&= 25{\cdot}4437 - 4{\cdot}8628 \\
&= 20{\cdot}5809,
\end{aligned}
$$

which is nearer the true value than Mr. Naylor's is. It will be observed that, if we stop at fourth differences, the approximate value by Lubbock's formula is 20·5805, which is still nearer the true value.

Should the State of Health be taken into account in Calculating the Value of an Interest that depends on the Duration of Life?

[From the *Insurance Monitor* of New York.]

A question of entail presenting features of novel interest, arose in Scotland some five years ago, and finally after various appeals has reached a conclusion in the English House of Lords, the detailed facts of which have but recently been made public through the *Institute of Actuaries' Journal.* The interesting feature of this case was the question as to the principle on which an individual life should be valued where the party was obligated to accept the value of his reversionary interest, and release an entail on demand. Must the life for legal purposes be regarded as of average value? or could the physical condition of the reversioner be taken into account? It will be seen that the issue involved, concerns not simply life estates, but a most important principle in the relation between a life company and its retiring members. Policyholders are accepted in this country at least on the basis of average lives. May the company fairly regard their individual condition when treating with them as to withdrawal, or should all individual circumstances be ignored? Such is really the much mooted question which this interesting Scottish suit (M'Donald *v.* M'Donalds) helps to decide. The doctrine finally established after repeated litigation was that, in the absence of any explicit legislation fixing the method of valuation, the life should be regarded, *prima facie*, as of average value, but that evidence was admissible to show any facts which would reduce this value, and all such facts should be properly estimated, in fixing the real value of the life, treating it as that of an individual and not that of a class. In other words, equity demanded that the person should be dealt with according to the actual facts, and not according to arbitrary statistics which would not meet the particular case.

It is obvious that the principle involved in the equitable surrender of a life policy is no different. For general business purposes, the company must deal with all its continuing members in a body as an average class. But when it comes to the question of withdrawal, it is no longer the class as a whole with the disabilities of one offset by the vigor of another, but the single member standing alone that is to be dealt with. As between the company and the member, the results of a life table have no other bearing than as evidence of the value of the withdrawing life if an average one. As was said in the House of Lords, nothing manifestly would be more inequitable than the attempt to apply such a table to the case of one *in extremis.*

We notice this decision at length not so much as a precedent, but because during a protracted litigation of four years, it involved the same fundamental principle of equity as that which is concerned in this question of surrender value.

A New Departure in American Life Insurance.

[Adapted from the *Spectator* of New York and Chicago.]

For a number of years the rate of interest in this country has been liberal, and life insurance companies have been able to invest their funds in such manner as to bring them large returns. They may continue to do so for some years to come, but there are many agencies at work that are calculated to reduce the rate of interest in the near future. The country is prosperous and the indications for continued prosperity are promising; the population is increasing rapidly, as is, also, the wealth of individuals. Money is plentiful and seeking investments that are safe at low rates. It therefore behoves the managers of life companies to carefully consider the promise of the future, and so shape their business now as to be prepared for a curtailment of their income in the future from their investments. No doubt the opportunities for investment will continue to be such that little difficulty will be experienced in obtaining a rate of interest that will provide for all existing contracts; but is it safe to make new ones on the same basis?

During the past few years *The Spectator* has discussed this question frequently, and has always maintained that managers of life insurance companies must prepare to meet this problem of a lower rate of interest on their investments. The fact that the rate of interest has been steadily decreasing for several years, demonstrates the correctness of our position. When the government can find a ready sale for its bonds at 3½ per-cent, it is folly to suppose that life insurance companies can find safe investments on long time at 6 or 7 per-cent, as they formerly could. Money is

too plenty now in private hands, seeking the same investments, to warrant such a supposition. When large capitalists are content with $3\frac{1}{2}$ to 5 per-cent interest, life insurance companies cannot expect to realize more than that. It becomes their duty, therefore, to base their calculations for the future upon getting smaller returns for their money. In this State the rate fixed by law at which they must estimate their reserve is $4\frac{1}{2}$ per-cent; in Massachusetts it is 4 per-cent. The Connecticut Mutual Life managers, however, have resolved that for future policies their reserve shall be calculated upon a basis of 3 per-cent. In a communication addressed to his board of directors, which formed the basis of their action in the matter, Col. Greene, president of the company, reviewed the financial situation in an able manner, clearly and concisely setting forth the reasons that induced him to recommend the change. Among other things he says:—

While still able to invest in many good securities at rates above 4 per-cent, and likely to be so for many years to come, many of those most desirable for long investment, on account of safety, are either yielding less, as United States and some State bonds of best repute; or from nothing above that rate to very little above, as most State bonds, municipal bonds of best repute, and railway bonds of the highest class; and the strength with which they are held is most significant. Real estate loans have experienced as great a relative change, the present actual rate on best securities ranging as low as 4 per-cent in the largest Eastern cities. The National debt is being rapidly paid; State debts as a whole are decreasing fast; municipal bonds of the best class are decreasing; and thus a vast body of funds is being forced out of those investments, and with the increase of wealth since liquidation, is seeking safe investments faster than they can be provided by legitimate enterprises. This intensifies the present effect of other causes upon the rate of interest, and is likely to have an important effect on prices of real estate. Outside our own peculiar resources is the important element of foreign capital, coming in increasing amounts. The whole world is becoming very familiar with us, and has a growing faith in the substantial basis of our legitimate undertakings, and in their comparative exemption from political dangers. This tends toward a reduction of our interest rate to that of the country with which we are in most intimate connection. The contingencies are not a few which may check the increase of our own surplus wealth and of foreign investment here, and the consequent downward drift of interest, and set it the other way for longer or shorter periods; and we cannot doubt their operation to a considerable extent. Our present basis is undoubtedly more than safe for the business now standing on it. But it is not certain to remain safe indefinitely for all the business that might be based upon it in future; and if a change ever be needed, it must be made long before the apprehended conditions actually arrive. When they are present in full force it will be too late.

The following is the announcement made by the company :—

The Connecticut Mutual Life Insurance Company, of Hartford, Conn., recognizing the possible continuance and future effect of the causes producing the recent great changes in the rate of interest on the best securities, and the necessity of basing all life insurance calculations upon a rate certain to be earned during the possible continuance of policies hereafter to be written, a period of more than two generations, because of the impossibility of changing their basis in the future as to business then existing should the assumed rate of interest fail to be realized; and that in a business dependent on future contingencies nothing is certainly safe that is not certainly more than apparently safe; in order to provide absolute safety in the basis of its business and to have the largest liberty in the future selection of investments, *will compute the net premiums and reserves upon future policies on the assumption of earning three per-cent interest on the best long investments, instead of four per-cent, which is now and will remain the basis of business now existing.*

As a further basis, it assumes the rate of mortality shown by the American Experience Table.

Commissioner Clarke, of Massachusetts, has always been a persistent and consistent advocate of life insurance reserves based on a low rate of interest. In his annual report for 1881 he discusses the subject at considerable length, predicting that 4 per-cent, which is the Massachusetts standard, would soon be found too high. We make the following extract :—

In order to disarm all possible implication of unfairness, and to present the most favorable exhibit in defence of lower reserves, we select the experience of twenty-seven life companies doing business in Massachusetts for five years consecutively (1876 to 1880 inclusive), taking for this purpose their mean amount of cash or income-producing assets, with interest received and accrued, also their mean amount of premium reserves, with interest to be accumulated thereon at 4 per-cent, and the surplus remaining for dividends to policyholders. Here are the results :—

Year.	Cash Assets, Mean Amount.	Interest Received and Accrued.	Rate per-cent.	Premium Reserve, Mean Amount.	Four per-cent Interest to be accumulated.	Surplus Interest for Dividends.	
						Amount.	Rate per-cent.
	£	£		£	£	£	
1876	66,775,448	4,433,586	6·64	61,319,893	2,452,796	1,980,790	3·23
1877	69,755,997	4,408,546	6·32	63,776,796	2,551,072	1,857,474	2·91
1878	71,774,290	4,243,991	5·91	65,150,478	2,606,020	1,637,971	2·51
1879	73,586,355	4,275,307	5·81	66,362,745	2,654,510	1,620,797	2·44
1880	76,240,087	4,094,477	5·37	68,323,624	2,732,945	1,361,532	1·99

This exhibit is significant and instructive. It will be observed that, with income-producing assets increased nearly nine per-cent since 1876, the rate per-cent of interest received and accrued has fallen more than one and a fourth per-cent, and the surplus of interest for dividends to policyholders in very nearly the same ratio, that for 1880 being only 1·99 against 3·23 in 1876. With this steady decline year by year in interest rates, falling from the eight to twelve per-cent claimed in 1871 by many lower-reserve advocates to 5·37 in 1880, followed by corresponding decrease of surplus for dividends, it is not difficult to foresee that there may be imminent approach to impairment if present conditions continue. Policyholders complaining of reduced dividends will find in these telling results substantial reasons for disappointed expectation.

In exceptional cases, however, a small number of companies, whose investments were placed on long terms and in better times, may continue for shorter or longer periods to show more favorable results; but with the present outlook, so suggestive of increasing taxation, of expenditures enhanced by new and varied contingencies, and of other financial burdens necessarily accruing, it must be apparent that, even in exceptional instances, average interest accumulated can exceed only by a mere fraction, if at all, the four per-cent required by law.

HOME AND FOREIGN INTELLIGENCE.

NEW ZEALAND GOVERNMENT INSURANCE DEPARTMENT.

Quinquennial Investigation, 1880.

First Division of Profits.

We have received from Mr. C. Godfrey Knight, the Actuary of the Department, a copy of the Second Quinquennial Report; also copies of the Annual Reports of the Government Insurance Commissioner for the years ending 30 June 1880 and 1881. From these documents we take the following particulars:—

Annual Premiums for £100 *(with Profits)*.

Table.	How Payable.	Age 25.			Age 30.			Age 35.			Age 40.			Age 45.			Age 50.		
		£	*s.*	*d.*	£	*s.*	*d.*	£	*s.*	*d.*	£	*s.*	*d.*	£	*s.*	*d.*	£	*s.*	*d.*
I	Payable during life	1	15	9	2	0	10	2	7	2	2	15	2	3	5	11	4	0	2
II	Limited to 20 payments	2	11	1	2	16	4	3	2	7	3	10	0	3	19	7	4	12	1
III*	Payable for 30 years, or till death if prior	2	13	9	2	16	1	2	19	4	3	4	2	3	11	9	...		
	Payable for 20 years, or till death if prior	4	3	3	4	4	11	4	7	0	4	10	2	4	15	2	...		

* Under Table III (Endowment Assurances), the sum assured is payable on the expiry of so many years, or at death if previous.

Liberal Regulations.

Policyholders after Three Years have an absolute right to a surrender value *defined by law*, and not subject to the discretion of Directors.

Policies cannot be Forfeited for Non-Payment of Premiums, *so long as the surrender value is sufficient to pay the arrears.* For instance, in the case of a policy where, from inadvertence or inability to pay, one or more premiums are overdue when death occurs—the total arrears not being in excess of the surrender value—the sum assured is at once paid to the representatives of deceased, after deducting the premiums in arrear and interest.

Policyholders can Borrow on Security of their Policies at 7 per-cent interest, to the extent of 90 per-cent of the cash surrender value.

Policyholders may Travel all over the world, or change their occupation, without extra charge.

Policies after Five Years' Duration are, if age has been admitted, *indisputable and unchallengeable*, being *freed from all conditions.* Holders of such policies may *reside in any part of the world* without license or extra charge, provided that they have resided continuously within ordinary limits from the date of the policy.

Policies are Protected from Bankruptcy, after two years, to the extent of £200; after five years, to £500; after seven years, to £1,000; after ten years, to £2,000.

New Business.

	1880.		1881.	
Proposals	3,032	£1,036,880	2,306	£735,920
New Policies	2,272	725,254	1,788	550,351
New Annual Premiums	—	23,211	—	17,515
Single Premiums	—	1,203	—	1,024

Mortality and Claims.

Year.	Deaths.	Policies.	Claims.
1880	67	78	£32,500
1881	70	80	28,425

Of the deaths, the numbers occasioned by accident were 16 and 13 respectively, including 8 and 9 deaths by drowning. There were 2 suicides in each year.

	Surrenders.		*Lapses.*	
Year.	Policies.	Sum Assured.	Policies.	Sum Assured.
1880	104	£42,840	649	£193,290
1881	146	61,301	797	265,878

Accumulated Funds and their Increase.

Year ending 30 June	Amount of Funds.	Increase on previous year.	Annual Revenue.
1876	£153,717	£43,750	£78,935
1877	219,394	65,677	91,994
1878	292,523	73,129	111,202
1879	371,889	79,366	132,262
1880	459,335	87,446	146,205
1881	557,230	97,895	156,615

Balance Sheet on 30 *June* 1880.

Liabilities.	£	s.	d.	Assets.	£	s.	d.
Total Assurance, Annuity, and Endowment Funds (as per Revenue Account)	459,335	17	0	Loans on Policies .	19,917	11	6
Claims announced, but not paid . . .	7,155	11	2	Government Securities	343,900	0	0
Surrenders . .	263	4	1	Railway Debentures (guaranteed by Government) . .	40,000	0	0
Annuities unclaimed .	800	0	0	Municipal Corporation Debentures . .	2,500	0	0
Commission (new) .	542	14	3	Office Furniture (Head Office and Agencies)	1,082	13	4
Medical Fees . .	579	12	0	Overdue Premiums on Policies in force .	6,040	7	0
Rent and Postage .	468	15	0	Outstanding Premiums due in June 1880 .	8,705	6	1
Sundry Accounts owing	146	8	11	Interest outstanding .	405	19	6
Premium Deposits .	617	5	4	Interest accrued, not due . . .	5,081	19	1
				Agents' balances (since accounted for) .	749	16	3
				Cash on Deposit .	37,000	0	0
				Cash on current account . . .	4,525	15	0
	£469,909	7	9		£469,909	7	9

From the Actuary's Report it appears that "the business of the Department is confined to a population of less than half a million, and that the competition is very keen, owing to the strenuous efforts made by foreign offices (there are no local companies) to secure the benefit of the low rate of mortality prevailing in the colony, and so counterbalance the necessarily higher rate of mortality on that portion of their sub-tropical business, on which no extra premium is charged."

About twenty-one per-cent (21·47) of the policies issued during existence of the Office have been discontinued. "The rate of discontinuance experienced by the three private offices doing business in the colony, as deduced from the data given in their last valuation reports, was 23·96, 24·27, and 29·27 respectively. These rates all exceed that of this Department, showing that its business is of a more permanent character than usual." It is stated, as marking the quality of the business transacted—

(1) That it does not include any policies reassured with other offices.

(2) That the agencies of the Department being confined to the Colony, the risks undertaken are those only which are accepted directly by the Head Office. The full benefit of the low rate of mortality in New Zealand is thus also secured, and the extra risk of inferior climates avoided.

(3) That there has been a strict adherence to the rule of paying commission to regularly appointed agents only.

Expenses of Management.

The whole of the expenses of management during the quinquennium, including commission and other agency expenses, amounted to 13·72 per-cent of the total income of the period, or to 15·95 of the

premium income. The provision made for such expenses in the premiums charged (or the loading, as it is technically called) averaged about 28 per-cent on the net premiums payable under the participating policies in force [rather less than 22 per-cent on the office premiums].

The following table gives the ratio of expenses to premium income in the four offices that are doing business in the colony:—

Comparative Return of Percentage of Expenses to Premium Income of Offices doing business in New Zealand.

Office.	Date of Establishment.	Age of Office.	Date of Valuation.	Ratio of Expenses.	Adjusted Ratio.
A	1849	30	1879	15·69	18·40
B	1869	11	1880	26·81	29·89
C	1869	11	1880	30·88	34·86
G.I.O.	1870	10	1880	15·95	15·95

The amounts from which the ratios have been calculated, have been obtained from the Consolidated Revenue Account of each office at the date of its last valuation.

"The adjusted ratio in the last column of the table is arrived at by estimating what would have been the effect on the premium income of the other offices, supposing that they had only charged the low rates of premium used by this office. For this purpose it is assumed that the difference between the premiums charged for a whole-life policy at age 35 (the average age at entry), is a reasonable measure of the reduction that should be made in the actual premium income, in order to arrive at this estimate."

Selection of Lives.

Out of 10,000 policies issued, 7,573 were accepted as select lives, and additions were made to the ages of 2,427 under-average lives, or, roughly speaking, three-fourths were taken as select, and the remaining quarter were considered under the average standard of health. The average addition made to the ages of the last-mentioned lives was about five years, but the greatest number were rated up three years. The proportion of select and under-average lives agrees substantially with that given in the quinquennial report of the Australian Mutual Provident Society for the year 1879.

Out of every 100 proposals received, more than 8 are absolutely declined; 17 are either deferred by the Board, or not completed by the proposer on account of the extra premium imposed or from other causes; and of the remaining 75 that become policies, about 19 are treated as under-average lives.

The following summary table shows the proportion of policies accepted at ordinary rates, and of those rated up, out of 10,000 issued:—

Age at Entry.	Ordinary Rates.	Accepted with the Addition of Undermentioned Years.											Total.
		1	2	3	4	5	6	7	8	9	10	11 & over	
24 & under	1,131·9	10·5	22·4	53·6	24·5	76·0	13·3	22·6	7·7	2·8	37·6	14·9	1,417·6
25 to 34	3,214·1	7·7	94·1	223·0	75·2	272·8	42·0	80·0	28·0	3·5	89·0	30·6	4,160·5
35 to 44	2·477·0	7·7	81·0	266·3	99·4	246·9	35·7	54·6	36·4	2·1	53·5	11·2	3,371·8
45 to 54	665·4	2·1	14·0	119·8	29·4	60·3	18·9	11·9	5·6	·7	3·5	2·1	933·7
55 & over	84·9	1·4	2·1	9·1	5·6	8·4	2·1	2·1	...	...	·7	...	116·4
	7,573·3	29·4	213·9	671·8	234·1	664·4	112·0	171·2	77·7	9·1	184·3	58·8	10,000·0

Mortality Experience.

The following table shows the causes of all deaths that have taken place since the establishment of the office, classified in accordance with the system adopted by the Registrar-General in England, and indicates what has been the effect of medical selection upon the deaths occurring in the early years of insurance. It might be presumed that persons actually suffering from disease at the time of examination would be weeded out, consequently that the deaths in the first years of insurance would be principally caused by acute

Causes of Death during the ten years ending the 30th June 1880, classified according to duration of Policy.

Class.	Causes of Death.	Duration of Policy. Number of Deaths.—Male and Female. 1st Year. M.	1st Year. F.	2nd Year. M.	2nd Year. F.	3rd Year. M.	3rd Year. F.	4th Year. M.	4th Year. F.	5th Year and over. M.	5th Year and over. F.	Percentage of Deaths. Males only. 1st Year. M.	2nd Year. M.	3rd Year. M.	4th Year. M.	5th Year and over. M.	Total Number of Deaths. Male and Female. Number. M.	Number. F.	Percentage. M.	Percentage. F.
	All Causes	56	5	40	4	38	1	43	2	123	2	18·7	13·3	12·7	14·3	41·0	300	14	100·0	100·0
	Classes:—																			
I	Zymotic Diseases	11	2	8	1	5	1	5	...	8	1	29·8	21·6	13·5	13·5	21·6	37	5	12·3	35·7
II	Constitutional Diseases	3	...	3	...	6	...	9	1	15	...	8·3	8·3	16·7	24·9	41·8	36	1	12·0	7·1
III	Local Diseases	20	2	21	2	19	...	20	1	74	...	13·0	13·6	12·3	13·0	48·1	154	5	51·4	35·7
IV	Developmental Diseases	..	..	...	1	..	...	...	...	...	1	...	...	...	...	...	...	2	...	14·4
V	Violent Deaths	22	1	8	...	8	...	9	...	23	...	31·4	11·4	11·4	12·9	32·9	70	1	23·3	7·1
	Unspecified	...	...	...	...	...	...	...	...	3	...	...	...	...	...	100·0	3	...	1·0	...

diseases of short duration and by accidents. The table shows that accidents, miasmatic and other diseases, which could not have been foreseen by medical examiners, are the chief causes of death during the first few years of insurance, and that very few deaths have arisen from consumption, and other diseases against which it is possible to

guard by medical skill. In some few cases, there is no doubt that deaths have occurred from causes that might have been foreseen. Whether this has arisen through the neglect of the local medical examiner, or through the fraudulent mis-statements or concealment of the proposer and of his referees, it is difficult to determine. Intemperate habits are generally the cause of this class of deaths, and the experience of our Department on this point agrees with that of private offices. All have to deplore the fact that claims arise from such causes, in spite of the precautions taken to prevent them, and notwithstanding the greatest care on the part of Directors and Managers.

The Sums Assured and Limit of Risk.

The sum for which the largest number of policies has been issued is £500, £200 comes next, then £100; after that £300, and then other sums in diminishing numbers. 4,949 policies were issued for sums varying from £100 to £200; 5,265 for sums from £200 to £500; and 1,263 from sums above £500 and not exceeding £3,000, which is the largest amount that the office will take upon one life. The Department has been repeatedly urged to extend this limit to £4,000; but the fact that only 22 policies have been issued for £3,000 since the limit was extended some five years ago from £2,000 to £3,000, shows that it would not have been wise to again increase it; and until there is a reasonable prospect of issuing yearly at least five policies of £4,000 each, the actuary recommends that the present limit should not be extended.

Premiums Charged on Young Lives.

On this subject Mr. Knight says:—"The Consulting Actuaries "stated in their last Quinquennial Report for 1875, that the "premiums charged were too low at the higher ages. I, on the "other hand, thought that the rates required increasing at the "younger ages, more than at the higher. In accordance with their "recommendation, the rates were increased at the older ages, and in "accordance with my advice the premiums were re-computed on the "basis of H^M Table of mortality, instead of Carlisle; and at the same "time it was determined that lives under 25 should be charged the "same premium as that for age 25. This was a rough method of "giving effect to the opinion entertained by many actuaries, that "the H^M premium was too low at the younger ages. Since then, "the accuracy of this opinion has been thoroughly established by "elaborate investigations, made by Mr. Sprague and others, into the "actual mortality experienced by persons entering at each age. It "is now found that the H^M premiums are far too low at the younger "ages; so much so, that the true premium for age 20 is higher than "that of the H^M premium for 24, thus proving that a large increase "in the premiums originally charged for young lives was necessary, "and that the rough rule followed by the Department, has not on "the whole been unfair in its operation."

Results of the Valuation.

The valuation brings out a surplus of . .	£73,670 17 0
of which Mr. Knight recommends that a portion, equivalent to 5 per-cent upon the net liability, should be retained to form a Reserve Fund, to meet unforeseen contingencies, and to equalize future bonuses. This would amount to about	17,670 17 0
leaving a divisible surplus of	£56,000 0 0

He recommends that the surplus should be distributed to the policyholders in proportion to the accumulated loading.

That participation in the surplus be confined to whole life and endowment assurance policies (Tables I, II, and III). No profit is likely to arise from policies issued under the other tables, such as Annuities, Children's Endowments, &c. In the prospectuses issued since the passing of the Act in 1874, authorizing a division of the surplus, these last-mentioned tables have always been headed as "Non-participating".

That all holders of whole-life and of endowment assurance policies, who have paid two years' premium in full, and whose policies are in force at the date of valuation, should be entitled to participate. Many offices stipulate that the bonus shall not be payable in the event of death happening before the policy has endured for five years, that is, the bonus does not *vest* until the expiration of five years from the date of the policy; but he thinks that this stipulation is a fruitful source of disappointment to the representatives of the assured, owing to their not understanding the contingent nature of the bonus.

That each policyholder's share of the surplus should be converted into its reversionary equivalent by a table of Office Single Premiums.

That the policyholder be entitled, at any time, to exchange the reversionary bonus actually allotted to him for a Cash Bonus, or a Permanent Reduction of Future Premiums, or a Temporary Reduction of Future Premiums. This right of exchanging one benefit for another will probably be exercised in a manner prejudicial to the office; therefore it will be necessary to calculate these commutations in such a manner as to neutralize any adverse selection. This will be easily effected by using a higher rate of interest in the calculations than that adopted in the valuation.

Adjustment of New Business Expenses between Old and New Policyholders.

" In calculating the share of each policyholder in the surplus, an " adjustment should be made in the case of policies recently effected, " on account of the cost of obtaining new business, which not only " exceeds the whole of the loading or provision made for expenses, " but even absorbs a considerable portion of the net premium during " the first year.

" The interests of the old policyholders would suffer materially " if an amount of new business were not obtained, sufficient to fill up

" the void caused by death and the numerous lapses and surrenders " that are always taking place. Even if there were no lapses or " surrenders, and the only cause of a policy being discontinued was " the death of the assured, the ratio of expenses to premium income " would gradually become heavier; as it would not be possible to " reduce the expenditure as rapidly as the premium income would be " decreased, by deaths occurring at a continually accelerating rate as " the lives became older year by year. But if lapses and surrenders, " which are far more numerous than deaths, are taken into account, " then not only would the ratio of expenses to premium income be " still further increased, but the rate of mortality would also become " heavier than the normal rate, as it is well known that the majority " of those who voluntarily discontinue their policies are select lives, " whereas the sick and unhealthy make every effort to keep their " policies in force. Consequently, unless fresh and healthy lives can " be secured to take the place of those who leave or die, there would " be a constant impairment of the average vitality of those who " remain, and the ratio of expenses to income would continue to " increase, and the surplus would be diminished year by year; and " thus the interests of old policyholders would, as before stated, suffer " materially by any failure to repair the continual waste that takes " place in a life insurance office. It is clear, then, that it would not " be equitable to charge the whole of the cost of new business against " the loading contributed by the new policyholders. On the other " hand, with vigorous management an office will not only repair its " waste, but continue to expand; and then it becomes necessary to " protect the interests of the older policyholders, and to prevent the " surplus earned by their premiums from being shared by new policy-" holders who have not contributed anything to the surplus.

" The amount of the adjustment that will be necessary to secure " a fair apportionment of the cost of new business between the old " and the new policyholders, will vary with the cost of new business, " which is more likely to increase than diminish in the future. As " it is not probable that the amount of this adjustment will ever be " less than the first year's loading, I have recommended that a " policyholder shall not participate until he has paid two years' " premiums, leaving any further adjustment that may be necessary " to be determined at each valuation. In calculating the share of " new policyholders in the surplus that has now accrued, I propose " that a year-and-a-half's loading should be deducted from the " accumulated loading contributed by those new policyholders who " have entered during the last quinquennium, and who will become " entitled to participate if my recommendations are approved."

Adjustment of the Loading on Paid-up and Limited Premium Policies.

" In cases where the policy is purchased by one payment, or the " payment is spread over a few years, instead of over the whole " duration of the policy, it is necessary to calculate how much of " the loading actually contributed properly belongs to the period " under review, and how much should be reserved to meet future

" expenses, when there will be no further loading contributed, on " account of the premiums having been paid up in full.

" This adjustment of the loading is made by first equalizing it " over the whole duration of the policy, and then taking only the " equalized loading into account. By this means only so much of " the loading contributed as actually belonged to the past, is dealt " with in the distribution of profits, and that belonging to the future " is left intact, and the share of each policyholder is equitably deter- " mined according to his contribution towards the profits that have " been made."

THE HIGH RESERVES MADE FOR LIABILITIES.

" The highest standard of valuation in use by life insurance com- " panies is that known as the 'Combined H^{M}, $H^{M(5)}$'. Under this " system of valuation the H^{M} pure premiums are used together with " $H^{M(5)}$ annuities and reversions. The effect of this combination is " to make a much larger reserve than that produced by the H^{M} Table " alone in the case of policies that have endured for five years and " upwards, higher, in fact, than is necessary, and at the same time " it does not make a sufficient reserve for policies recently effected.

" For these reasons it was considered that the 'Combined H^{M}, " $H^{M(5)}$' system of valuation was not applicable to an office whose " policies had not, on the average, endured for four years.

" The H^{M} rate of mortality was adopted instead throughout the " calculations, and the necessary reserves were made for lives recently " assured, and for those which had been assured for five years and " upwards. In addition to these reserves, which recent investigations " have shown are necessary to make good the insufficient reserve " brought out by the use of the H^{M} Table, I have recommended a " special reserve of 5 per-cent upon the net liability. These extra " reserves are together of such magnitude that they raise the valuation " which has been made at H^{M} 4½ per-cent, to an equality with a " 'Combined H^{M}, $H^{M(5)}$' 4 per-cent net premium valuation. I do not " know of any office that has attained such a high standard of valua- " tion in its tenth year, and there are very few old offices able to " make such strong reserves as those I have recommended; as a " further illustration of the stringency of the valuation that has been " made, I would state that if the liabilities of the office had been " valued on the same basis as that upon which its premiums were " originally calculated, namely Carlisle 4 per-cent, the divisible " surplus would have been about £100,000, instead of £56,000."

From the schedules it appears that the following additions have been made to the net liability brought out by the formula used, namely,—

A reserve amounting to £3,990, and equal to five months' interest on the net liability under whole-life policies, on account of claims being paid one month after death, instead of six as assumed in the tables used, for the valuation.

A reserve amounting to £7,900, to provide for future expenses and profits on limited premium and paid-up policies, after the further payment of premiums has ceased.

A reserve amounting to £1,850, to cover the unexpired portion of the current risk on under-average lives, which have been charged extra premiums.

A further reserve of £19,000, to provide against the suspended mortality of lives recently assured, and the over-average mortality that is known to prevail amongst assured lives, after the effect of the medical selection has been exhausted.

The following table shows the approximate bonus that would be allotted under the system of distribution recommended :—

Age at Entry.	Duration, 5 Years.				Duration, 10 Years.			
	Reversionary Bonus.	Cash Bonus.	Reduction of Premium. For 5 Years.	Reduction of Premium. For Life.	Reversionary Bonus.	Cash Bonus.	Reduction of Premium. For 5 Years.	Reduction of Premium. For Life.
	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.
25	4 0 10	0 19 7	0 4 3	0 1 1	8 5 10	2 5 3	0 10 0	0 2 8
*30	4 1 3	1 2 3	0 4 11	0 1 4	*8 6 0	*2 11 8	*0 11 5	*0 3 2
35	3 14 3	1 3 0	0 5 1	0 1 6	7 10 7	2 13 10	0 11 11	0 3 8
40	3 11 0	1 3 5	0 5 8	0 1 8	7 4 5	2 19 5	0 13 3	0 4 5
45	2 17 2	1 3 5	0 5 3	0 1 9	5 16 8	2 14 11	0 12 4	0 4 7
50	2 11 10	1 4 5	0 5 6	0 2 0	5 6 3	2 16 10	0 13 0	0 5 5
55	2 19 7	1 11 10	0 7 4	0 3 0	6 3 2	3 14 0	0 17 6	0 8 4

* Example.—A person having a policy for £1,000, effected at age 30, and which has been 10 years in force, would receive a reversionary bonus of about £83, which he could surrender for £25. 16*s*. 8*d*. in cash, or apply to reduce his future premiums for 5 years to the extent of £5. 14*s*. 2*d*. per annum, or £1. 11*s*. 8*d*. per annum for the whole of life.

Policyholders, when comparing this bonus with that given by private offices, must bear in mind that the difference in the premiums charged by this Department, at the age quoted (amounting to about £4 less per annum than that required by private companies) is equivalent to a reversionary bonus of £200, which, together with the £83 above-mentioned, gives a total bonus of £283.

The following is taken from the Report by the Consulting Actuaries, Mr. Bailey and Mr. Hardy :—

Summary of the Financial Transactions in the Quinquennial Period ending on 30 June 1880.

Fund, 30 June 1875		£109,967 12 1
Add Income from :—		
Premiums	£482,392 15 1	
Annuity Purchase Money .	12,228 7 5	
Interest	66,238 7 3	
Sundries	99 8 8	
		560,958 18 5
		£670,926 10 6
Deduct Outgo for :—		
Claims	£114,850 0 0	
Surrenders	11,058 18 6	
Annuities	8,716 16 6	
Expenses, including commission	76,964 17 8	
		211,590 12 8
Fund, 30 June 1880		£459,335 17 10

Comparing the interest brought to account each year with the mean between the fund at the beginning and end of the year, we find that the rate of interest realized in the quinquennium has been as follows, namely:—

	£	s.	d.	
1875–6 . . .	5	1	3	Per-cent per annum.
1876–7 . . .	4	17	11	
1877–8 . . .	4	18	5	
1878–9 . . .	5	0	10	
1879–80 . . .	5	1	7	

Analysis of the Risks shown under Table I—(Whole-Life Uniform Premiums).

Premium Payable.	Males (6,542).			Females (202).		
	Average Sum Assured.	Proportionate Distribution.		Average Sum Assured.	Proportionate Distribution.	
		Number of Policies.	Sums Assured.		Number of Policies.	Sums Assured.
	£			£		
Yearly	414	21·17	23·43	286	21·78	22·71
Half-yearly . . .	372	45·11	44·78	275	43·56	43·53
Quarterly . . .	356	27·22	25·91	268	34·66	33·76
Monthly	339	6·50	5·88	...	...	...
All Classes .	£374	100·00	100·00	£275	100·00	100·00

From a further grouping of the sums assured, according to the ages at entry, we ascertained that the mean age at entry was—for males, 36·74 years; for females, 38·77; the mean present age being—for males, 40·45 years; females, 42·07.

It will be seen, therefore, that the number of females is too small to justify their being treated separately; also, the average duration of the contracts is too short, and the average age of the lives too low, to enable us to arrive at any trustworthy conclusions as to the rate of mortality that will prevail in future among the assured.

For Tables II (Limited Payments) and III (Endowment Assurances), no distinction of the sexes was attempted; but the facts were re-arranged in groups, according to the number of premiums remaining to be paid, in order to obtain some idea of the period during which the assumed rates of mortality and interest might be affected, and of the time when the contributions for expenses in Class II would expire.

As on the occasion of the last valuation, there is no definite information respecting the death-rate that may be expected to prevail amongst assured lives in New Zealand, nor any mortality statistics for the Colony generally, available for the purposes of the present investigation. From such tests as we have been able to make, there would not appear to be any great difference between the general rate of mortality experienced in the last quinquennium amongst the lives assured under Table I, and similar lives in this country. We do not, upon general grounds, see much reason for expecting a contrary

result; but we consider that it would be wise to collect and tabulate all available information on this head, especially in regard to the death-rate of the older lives. After full consideration, we think that the Institute of Actuaries' H^M Table is the most suitable table of mortality to adopt for the present valuation. Nevertheless, we have thought it prudent (as will be hereafter shown) to make a further provision beyond what an estimate on this basis would provide.

We observe that, in a large number of cases, additional premiums have been charged, the lives having been assumed to be a certain number of years older, upon the ground that they were deemed to be below the average in point of future longevity. In these cases, the risks have been classified according to the true, and not the assumed ages, which we consider to be the most suitable method of procedure. The effect is that the surcharges, taken in the aggregate, form an annual fund applicable to meet the additional claims arising from the expected increased mortality in this class of lives.

At the last valuation, the rate of interest assumed was $4\frac{1}{2}$ per-cent per annum; and from what precedes, it will be seen that rather more than this rate was realized in the last five years. The question for consideration, however, is not what has been or is; but what will be the rate of interest realized by the fund during a period of time covered by the present generation. This point has received our very full consideration, and we have come to the conclusion that it would not be consistent with prudence to assume that the fund will earn a higher rate than $4\frac{1}{2}$ per-cent, which is fully $\frac{1}{2}$ per-cent beyond that obtained in the mother country. Indeed, we do not consider it free from doubt whether in after years this rate will not be too high, having regard to the marked and inevitable tendency to an equalization of the rate of interest throughout the world. Bearing in mind, however, that a fall in the rate of interest must be gradual, and that, if a reduction of $\frac{1}{2}$ per-cent were made on the present occasion, the difference in the net liability would not be great, we think that this valuation, like the last, may be made upon a $4\frac{1}{2}$ per-cent basis.

Appended hereto is a summary of the several assurance and annuity contracts in force as at 30 June 1880, with a valuation at $4\frac{1}{2}$ per-cent interest, of the assurances by the H^M Table of Mortality, and of the annuities by the Carlisle Table.

An additional reserve, amounting to £19,302, has been made, because the sums assured are payable one month after death, and also in respect of the 817 assurances under Table II, where the premiums are payable for a limited term only, so as to make provision for the expenses of management and for profits, after the premiums have ceased to be payable.

The financial position is as follows:—

The realized assets on 30 June 1880, were	£459,335
The estimated net liability, as per valuation, was . .	381,740
Surplus	£77,595

Having regard to all circumstances, and especially to the necessity of proceeding cautiously for some time until the prevailing rates of mortality and interest have been better ascertained, we suggest that

the whole of this amount be not divided on this occasion, but that the distribution should be restricted to a sum not exceeding £60,000.

Apart from other considerations, we feel satisfied that some portion of the surplus has arisen through the operation of what is sometimes termed " Suspended Mortality ", or the lightened death-rate resulting from the recent selection of so large a proportion of the lives. Experience has shown that an acceleration of the rate of mortality exhibited by any general table, takes place when the lives are no longer of recent selection; and we think it is expedient that some further reserve should be made on this account. This, we recommend, should, on the present occasion, be done by leaving undivided some portion of the estimated surplus, rather than by assessing a part of the liabilities by another table of mortality.

The next matter submitted to our consideration is, " the best and fairest system by which the divisible profits should be distributed."

Various methods, more or less elaborate, have been suggested and adopted by several companies, the applicability of which to the present case has received our full consideration. There can be little doubt, however, that the surplus proposed for division has been derived mainly from what is called the " loading " upon the premiums having been found to be more than sufficient for the expenses incurred. And, therefore, we consider that it will do substantial justice to the policyholders, without a needless amount of labour in calculation, to divide that surplus in proportion to the accumulated " loading ". We deem it an unnecessary refinement to introduce the element of interest in this process, and that it will be sufficient to ascertain the amount of " loading " on the premiums received on each policy.

This being the first division of profits, the accumulation, on this occasion, should be taken as from the dates of the policies.

It should be noted that the " loading " used for this purpose, in the case of the whole-term policies, should be that corresponding to the annual premiums under Table I (taking the increased age where the lives are under-average); so that, *cæteris paribus*, the sum apportioned as the bonus in cash will be the same, whether the policy be effected under Tables I or II, or the premiums be receivable by annual or more frequent instalments.

The same principles are applicable to endowment assurances, and to policies of other classes.

If, as is commonly the case, it is desired to convert the cash bonuses into corresponding additions to the sums assured, a lower rate of interest than that adopted in the valuation should be assumed for the purpose.

We have examined the tables of rates given in the prospectus, and we consider them to be sufficient under the present circumstances. They are undoubtedly low, and especially so at the older ages; and if the rate of interest in the colony were to fall materially, we think that the rates in question would require revision before new transactions were entered into.

We have also tested the table of surrender values, given in the prospectus; and we consider that, on the basis of the present valuation, the amounts promised to be allowed, whether in cash or as paid-up policy, can be granted with safety. We do not think that

either should be increased; and we recommend that, in applying them, the real, and not the increased, ages of the lives assured be taken.

As both rates of premium and amounts of surrender-allowance must be ultimately determined by competition on the spot, we consider that, so long as the first are sufficient, and the second not too high, it is not necessary to propose, upon purely theoretical grounds, any revision of either, on the present occasion. Theoretical perfection on such points as these must be subordinated to the practical considerations involved in meeting the public demands.

The results of our valuation show that the finances of the Department are in a sound condition; and, considering the low rates of premium paid by the assured, the amount of the divisible surplus appears to us very satisfactory, and the business of the Department to have been well conducted, and in such a manner as to reflect great credit on those who have been responsible for its management.

Summary *and* Valuation (*see* p. 453).

The Commissioner states in his Annual Report for 1880-81 :—

Section 45 of the Act, under which the Consulting Actuaries were appointed, provides that the sum recommended for division, or such less sum as shall be fixed by resolution of both Houses of the General Assembly as the amount for division, shall be divided amongst policyholders according to such scheme as shall be approved by such resolution. Immediately the amount to be divided has been determined, and the scheme of distribution approved by Parliament, the requisite calculations for allotment will be proceeded with.

General Remarks.

"The New Zealand Government Insurance Department is the first institution of the kind originated by a Government that has achieved a marked success.* Causes similar to those which led to the establishment of Post Office Savings Banks in Great Britain induced the Home Government to set on foot a system of Government Life Insurance. A recent writer, referring to the action of the British Postmaster-General with respect to increasing facilities for inducing habits of provident saving among the people of Great Britain, speaks of the necessity for devising means for popularizing life insurance. He says:—

"The fault of the Post Office hitherto is that it has not been sufficiently enterprising in this direction. Its attitude towards insurers has been one of entire indifference—the indifference of a Government Department. If something of the temper of an insurance office which has its way to make in the world, could be infused into it, some better result might perhaps be arrived at. If it is worth the while of the Government to invite insurers at all, it must be worth its while to secure them as well as to bid for them."

* We think it probable that the Commissioner would have modified this statement if he had been aware of the success obtained by the assurance and annuity offices conducted and guaranteed for many years past by the Government of Denmark.—Ed. *J.I.A.*

SUMMARY and VALUATION of the ASSURANCE AND ANNUITY CONTRACTS *as at* 30 *June* 1880.

Table.	DESCRIPTION OF TRANSACTION.	PARTICULARS OF THE POLICIES FOR VALUATION.				VALUATION. Assurances: Institute of Actuaries' H^M $4\frac{1}{2}$ per-cent. Annuities: Carlisle Table of Mortality, $4\frac{1}{2}$ per-cent.			
		Number of Policies.	Sums Assured.	Annual Premiums.		Value of Sums Assured.	Value of Annual Premiums.		Net Liability.
				Office.	Net H^M $4\frac{1}{2}$ per-cent.		Office.	H^M $4\frac{1}{2}$ per-cent Net.	
	ASSURANCES.		£	£	£	£	£	£	£
I	Whole Term: Uniform Annual Premiums	6,744	2,504,775	69,863	53,405	893,182	975,885	741,264	151,918
II	Do. Limited and Commuted Premiums	817	417,199	14,786	11,524	144,147	106,827	83,775	60,372
III	Endowment Assurances at various Ages	3,916	1,221,845	45,720	37,098	553,091	533,387	431,004	122,087
IV	Joint Lives	14	5,150	200	172	2,470	2,266	1,957	513
V	Children's Endowments, without return of Premium	5	1,500	76	...	575	...	...	575
VI	Do. with return of Premium	113	13,585	682	...	5,314	...	...	5,314
A	Temporary Risks	11	6,950	123	...	62	...	...	62
	Extra Premiums	...	...	607	...	304	...	...	304
	Additional Reserve, for Payment of Claims one Month after Death, and for Profits and Expenses of Assurances under Table II	...	...	...	...	...	...	...	19,302
	Total Amount of Assurances	11,620	4,171,004	132,057	102,199	1,599,145	1,618,365	1,258,000	360,447
	ANNUITIES.								
VII	Immediate	30	2,074	...	...	18,948	...	...	18,948
VIII	Contingent	2	335	9	8	1,303	113	94	1,209
IX	Deferred: Without return of Premium	2	200	70	70	926	181	181	745
X	Do. With return of Premium	2	85	11	...	391	...	...	391
	Total Amount of Annuities	36	2,694 per annum.	90	78	21,568	294	275	21,293
	Total of Results	11,656	4,171,004 2,694 per annum.	132,147	102,277	1,620,713	1,618,659	1,258,275	381,740

"The spirit and temper here referred to have characterized the action of this Department; and, although that action has undergone antagonistic criticism,—all the less considerate because a Government Department was deemed a fair mark for opposition—the results, as displayed by the valuations now published, might fairly be left to bear their own unaided testimony.

"Evidences are not wanting to show the interest with which the success of such an experiment as that of the establishment of this Department is elsewhere regarded. Other Governments have heard of its progress, and from time to time official enquiries have been made with respect to its condition and working. Amongst these, were enquiries on behalf of the Government of the German Empire, the Dominion of Canada, and more recently of the Kingdom of Belgium, and the sister colonies of South Australia and Tasmania. Copies of the principal forms, the laws, regulations, tables, reports, and other documents of the Department, have been supplied in response to these applications."

NATIONAL MUTUAL LIFE ASSOCIATION OF AUSTRALASIA, LIMITED.

Established 1869.

THIRD INVESTIGATION, 1880.

EXTRACTS FROM THE REPORT BY THE ACTUARY, MR. J. M. TEMPLETON.

The Third Investigation Report embraces the period from 1 October 1877 to 30 September 1880. This period has been marked by the establishment of Branch Offices in the neighbouring colonies and in New Zealand. Up to the date of the Second Investigation (that is, from 1869 to 1877), the business of the Association was almost wholly confined to the colony of Victoria; one or two attempts to do business by means of agents in the other colonies met with very slight success, and it became evident that the only way to secure a good foothold was to provide the means of transacting business in each colony without having to refer to the Head Office. This could not be done without considerable expense, as it necessitated a complete staff of officers in each colony; but the results of the business, as disclosed in the Second Investigation Report, enabled the Directors to set apart a sum for the establishment of Branch Offices in Sydney and Adelaide for New South Wales and South Australia respectively. These Branches were established in the latter end of 1878, an influential Board of Directors having been secured in each place; and the immediate result was a steady influx of new business, which has continued to the present time.

Encouraged by these results, the Directors resolved to open another Branch for the colony of New Zealand. Here the Association experienced difficulties not to be met with in any other part of the world, namely, the active competition of the Government Service.

The Government of New Zealand has a department for the assurance of lives, and, in addition to the postmasters throughout the colony who act as local agents, a great number of travelling canvassing agents and lecturers are employed at high rates of commission to urge the people to assure their lives with the Government. Wellington was selected as the head-quarters of the Branch, and the Association had the good fortune to secure a Local Board of Directors selected from among the leading men of the colony. Each of the Branch Offices is under the control of a Local Board of Directors, empowered to transact the whole of the business of the Association, including the investment of moneys in the colony where the Branch is situated. Local agents have been appointed in nearly all the centres of population in the various colonies, and a staff of travelling agents is attached to the Head Office and to each Branch Office, for the purpose of supplementing the efforts of local agents in country towns, and of canvassing outlying districts.

As the result of the adoption of the policy above indicated, a very large increase in the amount of new business annually transacted has been experienced.

The following table exhibits the number and amount of proposals received, and the number and amount of proposals declined or not completed, during each of the years now under review:—

Year ending 30 September	Received.		Declined, or not Completed.	
	No. of Proposals.	Amounts Proposed.	No. of Proposals.	Amounts Proposed.
		£		£
1878	1,670	475,650	663	230,095
1879	3,322	755,752	1,031	267,332
1880	3,547	888,652	1,065	301,065
Total .	8,539	2,120,054	2,759	798,492

The number of policies issued, the amount assured, and the annual premium income derivable therefrom, are exhibited in the following summary:—

Year ending 30 September	No. of Policies issued.	Amount Assured.	Annual Premiums.	Single Premiums.
		£	£ s. d.	£ s. d.
1878	1,007	245,555	8,353 14 6	Nil.
1879	2,291	488,420	16,622 12 3	Nil.
1880	2,482	587,587	18,989 18 6	350 0 0

The following is a summary for the three investigation periods:—

Period.	No. of Policies issued.	Amount Assured.	Annual Premiums.	Single Premiums.
		£	£ s. d.	£ s. d.
1st Period, 5 years	1,141	379,500	12,945 2 0	57 9 0
2nd Period, 3 years	2,513	634,092	23,618 15 10	138 18 7
3rd Period, 3 years	5,780	1,321,562	43,966 5 3	350 0 0
Total, 11 years	9,434	2,335,154	80,530 3 1	546 7 7

By the establishment of the three Branch Offices above referred to, a very considerable addition has been made to the expenses of the triennium. It has been customary in many offices to treat expenses of this kind as an asset, writing the amount off gradually during a series of years, on the ground that such expenses do not properly form a charge on the business of the year in which they are incurred. Viewing this kind of expenditure as a capital sunk in a profitable business, with the intention of having it recouped out of the additional profit of future years secured thereby, no objection need be taken to such a manner of treating it. The Directors, however, have preferred the more stringent and conservative course of writing off the whole amount of the expenses as they are incurred year by year. The effect of this is to impose a heavier charge on the business of the period now under review, and so decrease the surplus available for division at the present time; but there will be a corresponding reduction in the charges on the business of future years, and, therefore, an increase in the amount of surplus divisible in the time to come.

Notwithstanding this extra charge on the business of the triennium, there is scarcely any perceptible increase in the percentage of expenses to income over that of the previous triennium. If, however, a comparison be made with the first five years during which similar expenses were incurred in connection with the establishment of the office in Victoria, there is a reduction of more than 30 per-cent in the percentage of expenses to income.

The Consolidated Revenue Account shows that the expenses of management during the three years amounted to £15,222. 1*s.* 5*d.*, being less than ten and a-half per-cent of the gross income of the period. The other items of expenditure are almost wholly connected with the obtaining of new business, and do not, with respect to existing policies, recur in future years. They amount to £24,400. 17*s.* 8*d.*; and this expenditure is less than 56 per-cent of the new premium income (£43,966. 5*s.* 3*d.* per annum) on the policies issued during the triennium.

The following table exhibits the amount of the funds and the annual revenue at the date of last investigation, and at the end of each subsequent year:—

Year ending 30 September	Funds.	Annual Revenue.
	£ s. d.	£ s. d.
1877	53,470 8 10	31,712 16 3
1878	71,210 9 11	37,565 3 10
1879	95,201 16 10	52,026 3 8
1880	129,698 4 6	67,295 12 10

Since the date of the last investigation 93 members have died, involving claims to the amount of £26,255. The number of deaths to be expected during the triennium among the lives at risk was, according to the table of mortality used by the Association, 131; and the amount of claims to be expected was, on the same basis of calculation, £37,472. Since the Association was established, its members have passed through nearly twenty thousand years of life in the aggregate; and the mortality during the whole eleven years has been less than 80 per-cent of that expected and provided for.

On this occasion the valuation of policies has been made by the same method of calculation as that adopted at last investigation; but a special reserve of £2,481. 6*s*. has been made in respect of policies more than six years in force. This amount is equal to the difference between the values of these policies by the H^{M} Table and their values by the $H^{M(5)}$ Table, using the same net premiums. The result is therefore the same as it would have been if the more stringent table had been used in the valuation of these policies. The object of this reserve is to provide for possible increase in the mortality of lives, after the influence of selection has worn off. As the investigations of the Association are made triennially, the term of six years (being two investigation periods) forms a more natural division than five years; hence the application of the reserve to policies more than six years in force.

Of the surplus of £21,663. 1*s*. 10*d*. in the Assurance Branch, the Directors have decided to set apart the sum of £5,000 as a Guarantee Fund for the further security of policyholders; and the share of every member in this Guarantee Fund will be ascertained and reserved, so that, in the event of death, it may be paid to his personal representatives, if no unforeseen contingency arise. By treating the Guarantee Fund in this way, every member has the assurance that he or his heirs will receive the full value of his contribution thereto. The Directors have also decided to reserve the sum of £1,663. 1*s*. 10*d*. to meet possible adverse fluctuations in the value of the investments of the Association, and unforeseen contingencies. The balance of the surplus, £15,000, will be divided among the members whose policies are dated previous to 1 October 1879. The policies which participate are 4,397, insuring £1,100,283. 12*s*. 6*d*.

Of the surplus of £240. 15*s*. in the Endowment Branch, £90. 15*s*. has been reserved to meet unforeseen contingencies; and the balance, £150, will be divided among the members of the Branch whose policies are dated previous to 1 October 1879.

The small surplus of £4. 7*s*. 2*d*. in the Annuity Branch has been carried forward to the next triennium.

The following table gives a comprehensive view of the progress of the Association in each of the three investigation periods:—

Term.	Total Premiums Received.			Claims and Surrenders.			Saved and Invested.			Profit Divided.		
	£	*s.*	*d.*	£	*s.*	*d.*	£	*s.*	*d.*	£	*s.*	*d.*
1st Period, 5 yrs.	28,671	0	10	3,282	7	11	15,464	16	9	3,098	13	4
2nd Period, 3 yrs.	71,935	3	10	22,408	4	5	38,005	12	1	10,000	0	0
3rd Period, 3 yrs.	128,682	0	8	28,587	7	7	76,227	15	8	20,150	0	0
Total, 11 years	229,288	5	4	54,277	19	11	129,698	4	6	33,248	13	4

The funds accumulated by the Association now amount to more than 70 per-cent of the total premiums received on account of existing policies since the Association was established.

EXTRACTS FROM THE REPORT OF THE LATE PROFESSOR PELL, CONSULTING ACTUARY TO THE ASSOCIATION.

"The formula which your Actuary has adopted in the valuation of ordinary whole-life policies is recommended by eminent authorities on the subject, and after much consideration I certify that it is rightly applied in the present instance. I find that on the basis of the H^M Tables at 4 per-cent, the use of the formula adopted is equivalent to supposing that the preliminary expenses amount to a little less than one-half of the first year's premium, and to charging this amount upon future loading, instead of upon realized assets. It is, in fact, the anticipation of a certain portion of the future loading intended to cover all expenses, to meet the exceptionally large expenses necessarily involved in the initiation of the policy.

"It is generally allowed, and indeed it is obvious, that such an anticipation may rightly and safely be made within proper limits, where the gross premiums payable are of adequate amount. The proper limits have certainly not been exceeded in the method adopted by your Actuary; for one-half of a year's premium is, if anything, less on the average than the usual preliminary expenses incurred.

"The policies paid for by a limited number of premiums, and the endowment assurance policies, have been valued on similar principles; the charge upon future loading for preliminary expenses being in every case very moderate, and commensurate with the amount of the gross premiums actually payable.

"I approve of the method of distribution of profit recommended by your Actuary. It has been recommended by high authorities, and its application appears to have given satisfaction to the public. It seems to me to be as good as any attempt which has yet been made to divide profits on principles generally approved; and I have no hesitation in commending its adoption."

The following additional particulars are taken from the returns under the Eighth and Ninth Schedules of "The Life Assurance Companies Act, 1873 (Victoria)":—

The principles upon which the valuation and distribution of profits are to be made, are left to the discretion of the Actuary, in consultation with the Consulting Actuary of the Association. The Articles of Association, however, require that the profits (after reserving such an amount as the Directors may consider necessary to meet unforeseen contingencies) shall be rateably apportioned as a bonus among the policies entitled to participate in profits, in proportion to the amounts contributed towards such profit on account of such policies since the next preceding investigation.

The policies for the whole term of life were valued in classes, according to the year of birth of the persons assured; thus, for example, all persons born between 1 April 1830 and 31 March 1831 have been treated as of one age, and as if all born on 30 September 1830. The future premiums were valued by annuities deferred six months, that being the average time at which the next renewal premiums would fall due; and a check valuation was made in detail, every policy being valued separately, according to its office age at entry and its duration in years and months. This was made by an independent method, the values being deduced from a table of the values of annuities; and the results agreed very closely with that of the valuation in classes.

Policies on Joint Lives and on Last Survivor were valued each separately, the ages being calculated to the nearest birthday as above described; and the future premiums were valued by annuities deferred the exact proportions of a year to the dates of next renewal respectively.

All other policies were valued separately, the values both of sums assured and of future premiums being calculated to the nearest month; and in these cases it has been assumed that the persons assured attained their office ages exactly on the days on which their policies were issued.

In cases where the premiums were payable by half-yearly or quarterly instalments, the corresponding annual premiums according to the Tables of the Association were valued; the value (after providing for cost of collection and risk of lapse) of the instalments necessary to complete the current year of policies, is included in the list of assets entered in the Balance Sheet, as the payment thereof is secured by the conditions of the policy.

The liability of the Association is determined by taking the difference between the present value of the sums assured and the present value of the future net premiums. The net premiums valued were deduced from the pure premiums according to the Table of Mortality and Rate of Interest used in the valuation, by a small addition to cover the expenses connected with the initiation of policies. The capitalized value of this small addition, in the case of [ordinary] policies for the whole term of life is, in every instance, less than half-a-year's premium; and in the case of other policies the average is much less. The liability for policies under Table II [Limited Payments] has been increased by adding a proportionate part of the loading paid in the past, as a reserve for future expenses and profits.

The profits will be divided among the members entitled to participate, in the following manner:—

1st. The profit arising from excess of interest realized over the rate of interest assumed in the last valuation, in respect of the funds in hand at the date of such valuation, will be divided among those members whose policies are still in force, in proportion to their respective interests in the Assurance Fund at that date.

2nd. The remaining surplus will be divided in proportion to the loading contributed by members during the triennium, eliminating the loading contained in the premium for the first year of Assurance.

The tables of mortality used in the valuation were—for assurances, the Institute of Actuaries H^M Table—for Children's Endowments, the Peerage Table—and for Annuities, the Government Experience Tables.

The rates of interest assumed in the calculations were—for the valuation of Assurances, 4 per-cent; for the valuation of Children's Endowments and Annuities, 5 per-cent.

The amount reserved out of the annual premium income for future expenses and profits is, on the average, 20·79 per-cent of the annual premiums payable on Assurance policies, and 11·32 per-cent of the annual premiums payable on Children's Endowment policies; and the present value of this reserve is £156,816. 8*s.* 5*d.* In addition to this, the difference between the actual premiums payable (half-yearly, quarterly, and monthly) and the corresponding annual premiums, has not been credited to the Association. This constitutes an additional reserve of over £1,200 per annum.

Consolidated Revenue Account *for the period commencing* 1 *October* 1877 *and ending* 30 *September* 1880.

		£	*s.*	*d.*
Amount of Funds on 1 October 1877	£53,470 8 10			
Less amount reserved for establishing Branches	1,907 6 0			
		51,563	2	10
Premiums (no Reassurance Premiums), Assurance Branch		125,916	6	4
Premiums (no Reassurance Premiums), Endowment Branch		2,414	10	4
Consideration for Annuities granted		351	4	0
Interest (no Dividends)		18,019	5	11
Fees, &c.		459	14	6
		£198,724	3	11

	£	*s.*	*d.*
Claims under Policies, including Bonus Additions	26,355	0	0
Surrenders	2,232	7	7
Annuities	15	14	4
Bonuses paid in Cash	799	18	5
Advertising	1,604	19	5
Medical Fees	6,398	3	0
Commission	5,334	4	0
Branch and Agency Expenses	5,970	0	1
Expenses of Travelling Agents	5,093	11	2
Expenses of Management	15,222	1	5
Amount of Funds on 30 September 1880	129,698	4	6
	£198,724	3	11

Balance Sheet *on* 30 *September* 1880.

LIABILITIES.

			£ s. d.
Assurance Fund		£126,781 15 3	
Endowment Fund . . .		2,588 0 7	
Annuity Fund		328 8 8	
Total Funds			129,698 4 6
Claims admitted or announced, but not paid . .			1,338 3 0
Other Sums Owing by the Association—			
Amounts held on Deposits			3,497 9 6
Deposits on Proposals			355 8 2
Medical Fees		£120 0 6	
Directors' Fees		175 0 0	
Rent of Offices		87 10 0	
Sundry Accounts		82 10 0	
			465 0 6
			£135,354 5 8

ASSETS.

			£ s. d.
In Victoria—			
Debentures under Local Government Act . .			3,722 14 4
Mortgages			58,656 10 8
Loans on Policies within their Surrender value .			6,646 8 10
Loans on Policies with Personal Security . .			18,417 7 5
Loans on Building Society Shares . . .			46 5 10
Accrued Interest			1,795 14 8
Outstanding Premiums on Policies in force .			2,259 15 6
Deferred Instalments of Annual Premiums on Policies in force			9,038 0 4
Agents' Balances			523 8 7
Office Furniture			437 1 6
Cash in hand and on Current Account		£1,806 5 8	
On Fixed Deposit . . .		7,000 0 0	
			8,806 5 8
Total Assets in Victoria			£110,349 13 4
Elsewhere than in Victoria—			
Cash in hand and on Current Account, Sydney, Adelaide, and Wellington . .	£754 9 1		
On Fixed Deposit .	3,000 0 0		
		£3,754 9 1	
Branch Office Furniture . .		551 9 11	
New Zealand Government Debentures		910 0 0	
Outstanding Premiums on Policies in force		2,037 10 8	
Deferred Instalments of Annual Premiums on Policies in force		4,652 6 7	
Mortgages		4,960 0 0	
Loans on Policies . . .		7,502 9 3	
Agents' Balances		636 6 10	
Total Assets elsewhere than in Victoria .			25,004 12 4
Total Assets . . .			£135,354 5 8

Valuation Balance Sheet *as at* 30 *September* 1880.

Dr.

		£ s. d.
To Net Liability under Assurance and Annuity transactions (as per Summary Statement, page 463) .		107,790 0 6
To Surplus—Assurance Branch . .	£21,663 1 10	
Endowment Branch .	240 15 0	
Annuity Branch . .	4 7 2	
		21,908 4 0
		£129,698 4 6

Cr.

	£ s. d.
By Life Assurance Fund (as per Balance Sheet) . .	126,781 15 3
By Endowment Fund (as per Balance Sheet) . .	2,588 0 7
By Annuity Fund (as per Balance Sheet) . . .	328 8 8
	£129,698 4 6

Summary *and* Valuation (*see* p. 463).

A policy must be in force twelve months in order to entitle it to share in the profits.

Specimen Bonuses to Policies of £100 *each (ordinary Whole-term).*

Age at Entry.	Policies in Force for						Age at Entry.
	Five Years.			Ten Years.			
	Reversionary Bonus.	Cash Value.	Reduction of Premium for whole term of Life.	Reversionary Bonus.	Cash Value.	Reduction of Premium for whole term of Life.	
	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	
20	5 1 2	1 1 3	0 1 5	6 14 3	1 12 2	0 2 2	20
30	4 12 11	1 5 4	0 1 10	5 15 11	1 16 1	0 2 8	30
40	3 19 7	1 8 5	0 2 3	5 9 6	2 4 11	0 3 10	40
50	3 10 2	1 12 11	0 3 1	4 18 6	2 12 8	0 5 7	50

The average rate of interest at which the life assurance fund of the company was invested at the close of each year during the period since last investigation, has been as follows:—

On 30 September 1875	£7 1 4	per-cent.
„ „ 1876	6 19 2	„
„ „ 1877	7 5 7	„

Ninety per-cent of the office value, as ascertained by the method of valuation adopted at the next previous investigation, is allowed for the surrender of policies which have been in force for any period exceeding five years. Eighty per-cent of the office value is given for the surrender of policies over two years in force.

SUMMARY *and* VALUATION *of the Policies as at* 30 *September* 1880.

DESCRIPTION OF TRANSACTIONS.	PARTICULARS OF THE POLICIES FOR VALUATION.					VALUATION. Value of Assurances by the "Inst. of Act. HM Table". Interest 4 %. Value of Children's Endowments by the "Peerage Table of Mortality". Interest 5 per-cent. Value of Annuities by the "Govt. Experience Table". Interest 5%				
	Number of Policies.	Sums Assured and Bonuses.	Office Yearly Premiums.	Loading Contained in Office Yearly Premiums.	Net Yearly Premiums.	Sums Assured and Bonuses.	Office Yearly Premiums.	Loading contained in Office Yearly Premiums.	Net Yearly Premiums.	Net Liability.
ASSURANCE BRANCH.										
ASSURANCES.										
1.—*With Participation in Profits.*		£	£	£	£	£	£	£	£	£
For Whole Term of Life	3,948	1,093,041	31,774	6,728	25,046	422,152	466,822	102,946	363,876	58,276
Do. Limited Number of Premiums	69	27,267	1,120	258	862	9,868	8,109	(a) 1,668	6,441	3,427
Do. Fully Paid-up	4	438	...	...	...	(a) 171	...	...	...	171
Endowment Assurances—One Life	2,215	462,747	19,176	3,773	15,403	223,067	232,065	45,822	186,243	36,824
Do. do. Joint Lives	3	1,009	51	8	43	600	497	87	410	190
Joint Lives	238	53,383	2,251	379	1,872	26,160	27,375	4,692	22,683	3,477
Last Survivor	1	632	18	4	14	285	256	53	203	82
Extra Premiums Payable	...	...	204	204	...	102				
Special Reserve for Policies more than Six							...	...	...	(b) 102
years in force	...	...	...	...	...	2,481	...	...	...	2,481
Total Assurances With Profits	6,478	1,638,517	54,594	11,354	43,240	684,886	735,124	155,268	579,856	105,030
2.—*Without Participation in Profits.*							511			
For Whole Term of Life	3	1,100	42	3	39	518	193	8	503	15
Short Term Policies	7	4,200	54	17	37	211	...	38	155	56
Endowment Assurances—One Life, Fully Paid-up	1	40	...	...	...	18	...	...	...	18
Total Assurances Without Profits	11	5,340	96	20	76	747	704	46	658	89
Total Assurances	6,489	1,643,857	54,690	11,374	43,316	685,633	735,828	155,314	580,514	105,119
No Re-assurances	...	...	...	...	...	...	...	...	...	...
ENDOWMENT BRANCH.										
Children's Endowments	183	23,500	1,339	152	1,187	11,967	11,123	1,503	9,620	2,347
ANNUITY BRANCH.										
Immediate	2	per annum 60	...	...	...	324	...	...	...	324
Total of the Results	6,674	1,667,357	56,029	11,526	44,503	697,924	746,951	156,817	590,134	107,790

(*a*) This amount contains a proportionate part of the loading contained in premiums already paid. (*b*) The liability for extra risk is estimated at half a year's extra premium.

Minimum Surrender Values of [ordinary Whole-term] Policies of £100, exclusive of Bonus Additions.

Ages at Entry.	Minimum Surrender Value at the end of					Ages at Entry.
	2 Years.	5 Years.	7 Years.	10 Years.	15 Years.	
	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	
25	0 13 10	3 4 5	4 18 7	7 13 10	12 15 6	25
30	0 16 2	3 16 0	5 17 1	9 2 5	15 6 2	30
35	0 19 6	4 11 0	7 0 10	11 1 8	18 6 3	35
40	1 4 1	5 13 7	8 14 0	13 8 0	21 17 0	40
45	1 9 4	6 15 2	10 6 8	15 18 10	25 13 7	45
50	1 15 6	8 3 10	12 8 11	18 19 0	29 14 10	50
55	2 2 10	9 15 2	14 13 6	21 18 11	34 1 0	55

This Association has not transacted business at other than European rates. But if members go to reside between the 22nd parallels of north and south latitude, an extra premium varying from 10*s.* to £2 per-cent per annum is charged, according to the locality of residence. This extra premium is remitted when the members come to reside south of the 22nd parallel of south latitute, but the amount of extra premium thus paid is not taken into consideration in fixing the surrender values.

This Association does not grant policies on unhealthy lives; but when the personal or family history, the occupation, or any other circumstances connected with the persons proposed for assurance, lead the Directors to believe that their prospects of longevity are not quite so good as those of persons in every respect unexceptionable, their proposals are accepted at rates for ages somewhat in advance of their actual ages. These persons, when accepted, however, have the right to complete their policies at the minimum rates for their actual ages, subject to a contingent debt, equal to the value of the additional premium sought to be charged, to be deducted from the amount of the policy if the person assured die under the average age attained by persons of his age; but when he attains the said average age the contingent debt is expunged, and no additional premium is charged. If such persons elect to pay the rates for such advanced ages, the surrender values of their policies are fixed as if such advanced ages were their actual ages; and the same principle was adopted in ascertaining the liability of the Association.

ERRATUMS IN THIS VOLUME.

Page 352, line 10, for $14u_7$ read $14u_9$.

,, ,, ,, 14, ,, $-2u_{11}-3u_{12}$ read $-2u_{12}-3u_{13}$.

INDEX TO VOL. XXIII.

END OF VOL. XXIII.

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